EVALUATION AND SUBSTITUTION

Summary

When letters in a formula are replaced by numbers, it's called substitution.

EXAMPLES:

1. *Find the value of* $\sqrt{x^2 + y^2}$ *when* ***x = 8*** *and* ***y = –6***

2. *Find the value of* $x^2 - x + 6$ *when* ***x = –3***

3. *Find the value of* ***2(4 + 3y) + y(3 – 2t)*** *when* ***y = 2*** *and* ***t = 3***

4. *Given that* ***P = 200, r = 50*** *and* ***n = 2,*** *find the value of* $A = P\left(1 + \frac{r}{100}\right)^n$

5. *If* ***a = 6, b = 8, c = 10*** *and* ***s = 12,*** *find the value of* $A = \sqrt{s(s-a)(s-b)(s-c)}$

6. *Given that* $v^2 = u^2 + 2as$, *find the values of* ***v*** *when* ***u = 6, a = 3.5*** *and* ***s = 4.***

7. *Given that* ***a = 2, b = –4*** *and* ***c = –6,*** *find the values of* $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

8. *Given that* $y = \frac{17pr - 2r^2}{3pr}$, *find the value of* ***y*** *when* ***p = r.***

9. *Given that* ***a = 3, b = 4*** *and* ***c = 5,*** *find the value of*
$A = \sqrt{s(s-a)(s-b)(s-c)}$, *where* $s = \frac{1}{2}(a + b + c)$.

OPERATIONS

Summary:

An operation is a rule connecting two terms using sign language.

EXAMPLES:

1. *Given that* $a * b = ab^2$, *find the value of:*

(i) $1 * 3$

(ii) $(1 * 3) * 2$

(iii) a *if* $a * 3 = 63$

2. *Given that* $m \uparrow n = \frac{m^2 - n^2}{m + n}$, *find the value of:*

(i) $5 \uparrow 3$

(ii) $8 \uparrow (5 \uparrow 3)$

(iii) $9 \uparrow (6 \uparrow 2)$

(iv) $(12 \uparrow 4) \uparrow (7 \uparrow 2)$

3. *Given that* $p * q = \frac{pq}{p + q}$, *find the value of:*

(i) $3 * (10 * 15)$

(ii) $6 * (20 * 30)$

(iii) $15 * (30 * 15)$

(iv) p *if* $p * 3 = 2$

4. *Given that* $a \wedge b = ab - 5$, *find the value of* y *if* $3 \wedge y = 6 \wedge 4$

EER:

1. *Given that* $\boldsymbol{x \wedge y} = x^{y} - y$, *find the value of:*

(i) $3 \wedge 2$

(ii) $(3 \wedge 2) \wedge 2$

2. *Given that* $\boldsymbol{a * b} = \dfrac{a^{2} + b^{2} - 2ab}{a - b}$, *find the value of:*

(i) $4 * 3$

(ii) $8 * (4 * 3)$

3. *Given that* $\boldsymbol{a * b} = \dfrac{a + b}{a - b}$, *find the value of* $(5 * 3) * -2$

4. *Given that* $\boldsymbol{m * n} = \dfrac{m^{2} + n^{2}}{10n}$, *find the value of:*

(i) $4 * -8$

(ii) $7 * (4 * -8)$

5. *Given that* $\boldsymbol{a * b} = a^{2} + \sqrt{ab}$, *find the value of* $(1 * 4) * 3$

6. *Given that* $\boldsymbol{a * b} = \dfrac{a}{b} + \dfrac{b}{a}$, *find the value of* $\dfrac{1}{2} * \dfrac{2}{3}$

7. *Given that* $\boldsymbol{a * b} = a^{2} + \sqrt{3ab}$, *find the value of:*

(i) $3 * 4$

(ii) $5 * (3 * 4)$

8. *Given that* $\boldsymbol{a * b} = a^{2} - 2b$, *find the value of:*

(i) $-3 * -4$

(ii) y *if* $y * 2 = 21$

9. *Given that* $\boldsymbol{a * b} = a(b+2)$*, find the value of:*

(i) $1 * (2 * 5)$

(ii) *y if* $4 * y = 16$

COLLECTING LIKE TERMS

Summary

1. *In collecting like terms, the expression is re–arranged so that like terms are next to each other.*

2. *The products* xy *and* yx *are the same. Thus* xy *and* yx *are like terms.*

3. *Like terms with powers can be added or subtracted if the powers are the same. Thus* $4p + p^2$ *cannot be simplified because* ***4p*** *and* p^2 *are not like terms.*

EXAMPLES:

Simplify the following expressions:

(i) $6x + 8 + x^2 - 2x + 2x^2 - 3$ ***(ii)*** $5 + 4x^2 - 3 + 2x + 3x^2 - 4x$

(iii) $3x^2 - 7 + 4x^3 - x^2 + 2$ ***(iv)*** $4p + 6pq - 2q + 8p - 11qp + 10q$

REMOVING BRACKETS

Summary

1. *When removing brackets, each term inside the brackets is multiplied by the quantity outside the brackets.*

2. *If the sign in front of the bracket is negative, the signs inside the bracket* *are changed.*

EXAMPLES:

1. *Remove the brackets and simplify the following expressions:*

(i) $3(x + 2) + 2(x - 4)$ **(ii)** $2(4x - 2) - (5x - 3)$ **(iii)** $4(1 - 2x) - 3(6x - 5)$

(iv) $5(3x + 2) - 2(4p + 3)$ **(v)** $4x(x + 2) + x(x - 4)$ **(vi)** $3x(5 + 2x) - 2x(3x - 7)$

(vii) $a(b + c) - 2b(c + a) + c(a + b)$ **(viii)** $4x\left(1 - \frac{2}{x}\right) + 5(2x - 3)$

EER:

1. *Remove the brackets and simplify the following expressions:*

(i) $2(5a + 3b)+3(a - 2b)$ **(ii)** $4(1 - 2x) - 3(3x - 4)$
(iii) $4(1 - 2x) - 3(3x - 4)$ **(iv)** $3(x + 1) + 2(x + 4)$
(v) $5((4y - 9) - (10y - 5))$ **(vi)** $9(2x + 3) - 3(5x+1)$
(vii) $-4(a - 3b) - 3(-3a - b)$ **(viii)** $2p(q + r) - p(3q - 2r)$
(ix) $2x(x + 3) + x(x - 2)$ **(x)** $5[4(y - 4) + 15] - [2(5y - 3)+1]$

SOLVING LINEAR EQUATION

Summary:

1. *An equation consists of two expressions separated by an equal sign*

2. *In solving linear equations the following apply:*

(i) *When moving any term from one side of the equation to the other, its sign changes*

(ii) *If the equation contains brackets, first remove the brackets and then workout*

(iii) *If the equation contains fractions, multiply each term by the* ***LCM*** *of the denominators to remove the fractions*

EXAMPLES:

1. *Solve for* ***x*** *in the following equations:*

(i) $6x - 22 = 3x - 10$ ***(ii)*** $2(3x - 6) = 3(5 + x)$

(iii) $5 - 2(x - 1) = 4(3 - x) - 2x$ ***(iv)*** $(5x - 4) - (3x - 1) = 3$

(v) $(8x + 3) - 3(x - 1) = x - 2$

2. *Solve for* ***x*** *in the following equations:*

(i) $\frac{3x - 6}{31x - 22} = \frac{1}{13}$ ***(ii)*** $\frac{2}{x + 3} = \frac{3}{5 - x}$

3. *Given that* $(7x - 5) : (2 + 4x) = 8:7$*, find the value of* ***x***

4. *Solve for* ***x*** *in the following equations:*

(i) $\frac{2x - 3}{2} - \frac{5 - x}{4} = \frac{x}{3}$ ***(ii)*** $\frac{x - 3}{4} + \frac{x - 1}{5} - \frac{x - 2}{3} = 1$

(iii) $\frac{2x - 1}{9} - \frac{x - 1}{4} = 0$ ***(iv)*** $\frac{5x - 4}{8} - \frac{x - 3}{5} = \frac{x + 6}{4}$

(v) $\frac{5x - 4}{7} - \frac{1}{2} = \frac{3x - 1}{14}$ ***(vi)*** $\frac{3}{5}(5x - 4) = 1 - \frac{2}{5}(x - 1)$

(v) $(x - 1) = \frac{3}{4}(x + 1) - \frac{1}{2}$ ***(vi)*** $\frac{1}{3}(x - 1) - \frac{1}{4}(x - 2) = 1$

EER:

1. *Solve for* ***x*** *in the following equations:*

(i) $4(2x + 3) = 31 - 3(x - 1)$ ***(ii)*** $15(x - 7) - 3(x - 9) + 5(x + 6) = 0$

(iii) $10 - 2(x - 4) = 2(x - 1) - 6x$ ***(iv)*** $5(x - 1) = 3(2x - 5) - (1 - 3x)$

(v) $9 - 2(x - 5) = x + 10$

2. *Solve for **x** in the following equations:*

(i) $\frac{8x - 5}{7x + 1} = -\frac{4}{5}$ *(ii)* $\frac{7}{2 + 4x} = \frac{8}{7x - 5}$

3. *Given that **(x + 3) : x = 8:5,** find the value of **x***

4. *Given that **(x + 5) : 2 = 18:3,** find the value of **x***

5. *Solve for **x** in the following equations:*

(i) $\frac{x + 7}{6} - \frac{3x - 2}{3} = \frac{2x + 3}{3}$ *(ii)* $\frac{x - 1}{3} - \frac{x - 2}{4} = 1$

(iii) $\frac{3x - 1}{5} - \frac{x}{7} = 3$ *(iv)* $4 - \frac{x - 3}{2} = 3$

(v) $\frac{x}{3} - \frac{x - 2}{2} = \frac{7}{3}$ *(vi)* $\frac{x}{2} = 3 + \frac{x}{3}$

(vii) $\frac{x}{2} - \frac{1}{4} = \frac{x}{3} + \frac{1}{2}$ *(viii)* $\frac{x}{2} + \frac{x}{3} = x - 7$

WORD PROBLEMS ON LINEAR EQUATION

Summary:

In solving word problems on linear equation, read the problem carefully and form an equation using the conditions given in the problem

EXAMPLES:

1. *Find two numbers such that one exceeds the other by **9** and their sum is **25***

Soln:

*If the numbers are **x** and **(x + 9),***

$\Rightarrow x + (x + 9) = 25$

x = 8

⇒ (x + 9) = 8 + 9 = 17

∴ *The numbers are **8** and **17***

2. *The length of a rectangle is twice its width. If the perimeter of the rectangle is **72m,** find the length and width of the rectangle*

Soln:

If the width = ***x***

⇒ *length =* ***2x***

If 2(l + w) = 72

⇒ *2(2x + x) = 72*

x = 12m

∴ *Width* ***= 12m,*** *length* **=** *2(12)* ***= 24m***

3. *The sum of three consecutive multiples of **5** is **90.** Find these multiples*

Soln:

*If the multiples are **x, (x + 5)** and **(x + 10),***

⇒ x + (x + 5) + (x + 10) = 90

x = 25

⇒ (x + 5) = 25 + 5 = 30, (x + 10) = 25 + 10 = 35

∴ *The multiples are **25, 30** and **35***

4. *Three–fifth of a number is **4** more than one–half of the number. Find the number*

Soln:

If the required number = ***N***

$\Rightarrow \frac{3}{5}N - \frac{1}{2}N = 4$

$\therefore$ ***N = 40***

5. *The denominator of a fraction is greater than its numerator by* ***3.*** *If the numerator is increased by* ***7*** *and the denominator is decreased by* ***1,*** *the fraction becomes* $\frac{3}{2}$*. Find the original fraction*

Soln:

If the original fraction = $\frac{x}{x+3}$

$\Rightarrow \frac{x + 7}{x+2} = \frac{3}{2}$

$\Rightarrow$ ***x = 8***

$\therefore$ *Original fraction =* $\frac{8}{8+3} = \frac{8}{11}$

6. *In a multiple choice test of* ***90*** *questions, each correct answer carries* ***5*** *marks and each wrong answer leads to a loss of* ***2*** *marks. If a student scored a total of* ***387*** *marks from the test, find how many questions were answered correctly*

Soln:

If the correct answers ***= x,*** *then wrong answers* ***= (90 – x)***

$\Rightarrow$ ***correct score – incorrect score = total score***

$\Rightarrow$ ***5 x – 2(90 – x) = 387***

x = 81

7. *The length of a rectangle is* ***4m*** *less than* ***3*** *times its width. If the* *perimeter of the rectangle is **32m,** find the length and width of the rectangle*

Soln:

If the width = $\boldsymbol{x}$

$\Rightarrow$ *length =* $\boldsymbol{3x - 4}$

If $2(l + w) = 32$

$\Rightarrow 2(3x - 4 + x) = 32$

$\boldsymbol{x = 5m}$

$\therefore$ *Width* $\boldsymbol{= 5,}$ *length* $\boldsymbol{=} 3(5) - 4 \boldsymbol{= 11m}$

***8.** A man is **24** years older than his son. In two years time, his age will be twice the age of his son. Find the present age of the son*

Soln:

Son's present age = $\boldsymbol{x,}$ *Father's present age =* $\boldsymbol{x + 24}$

$\Rightarrow$ *After two years, Son's age =* $\boldsymbol{x + 2,}$ *Father's age =* $\boldsymbol{x + 26}$

$\Rightarrow x + 26 = 2(x + 2)$

$\therefore \boldsymbol{x = 22}$ ***years***

***9.** Tom's present age is two–fifth of his father's age. After **8** years he will be one–half of his father's age then. Find how old is his father*

Soln:

Father's present age = $\boldsymbol{x,}$ *Tom's present age =* $\frac{2}{5}x$

$\Rightarrow$ *After **8** years, Father's age =* $\boldsymbol{x + 8}$ *Tom's age =* $\frac{2}{5}x + 8$

$\Rightarrow \frac{2}{5}x + 8 = \frac{1}{2}(x + 8)$

$\therefore \boldsymbol{x = 40}$ ***years***

10. *The sum of the present ages of a man and his son is **60** years. Six years ago, the man's age was **5** times the son's age. Find the son's present age*

Soln:

Son's present age = $\boldsymbol{x}$, *Father's present age =* $\boldsymbol{60 - x}$

$\Rightarrow$ *Six years ago, Son's age =* $\boldsymbol{x - 6}$, *Father's age =* $\boldsymbol{(60 - x) - 6 = 54 - x}$

$\Rightarrow$ $54 - x = 5(x - 6)$

$\therefore$ $\boldsymbol{x = 14}$ ***years***

11. *The sum of the present ages of a man and his son is **45** years. Five years ago, the product of their ages was **4** times the man's age at that time. Find their present ages*

Soln:

Son's present age = $\boldsymbol{x}$, *Father's present age =* $\boldsymbol{45 - x}$

$\Rightarrow$ *Five years ago, Son's age =* $\boldsymbol{x - 5}$, *Father's age =* $\boldsymbol{(45 - x) - 5 = 40 - x}$

$\Rightarrow$ $(x - 5)(40 - x) = 4(40 - x)$

$\Rightarrow$ $(x - 5) = 4$

$\Rightarrow$ $\boldsymbol{x = 9}$ ***years***

$\therefore$ *Son's age* ***= 9 years,*** *Father's age =* $\boldsymbol{45 - 9 = 36}$ ***years***

12. *Ten years ago, a man's age was thrice as old as his son. In ten years time, the man's age will be twice as old as his son. Find their present ages*

Soln:

Ten years ago, Son's age = $\boldsymbol{x}$, *Father's age =* $\boldsymbol{3x}$

$\Rightarrow$ *Son's present age =* $\boldsymbol{x + 10}$, *Father's present age =* $\boldsymbol{3x + 10}$

*After **10** years, Son's age =* $\boldsymbol{x + 20}$, *Father's age =* $\boldsymbol{3x + 20}$

$\Rightarrow$ $3x + 20 = 2(x + 20)$

$\Rightarrow$ ***x = 20 years***

$\therefore$ *Son's present age = 20 + 10* ***= 30 years,*** *Father's age = 3(20) + 10* ***= 70 years***

13. *Tom is* ***9*** *years older than Bob. In ten years time Tom will be twice as old as Bob was* ***10*** *years ago. Find Bob's present age*

Soln:

Bob's present age = ***x,*** *Tom's present age =* ***x + 9***

$\Rightarrow$ *In ten years, Tom's age = (x + 9) + 10* ***= x + 19,***

Ten years ago, Bob's age = ***(x – 10)***

$\Rightarrow$ $x + 19 = 2(x - 10)$

$\therefore$ ***x = 39 years***

14. *A man said to his son, "I was as old as you are at present at the time of your birth". If the man's age is* ***38*** *years now, find the son's age* ***5*** *years ago*

Soln:

Son's present age = ***x,***

x *years ago, father's age =* ***38 – x***

$\Rightarrow$ $38 - x = x$

$\Rightarrow$ ***x = 19 years***

$\therefore$ ***5*** *years ago, son's age = 19 – 5* ***= 14 years***

15. *The ratio between two numbers is* ***7:5.*** *If the difference between them is* ***48,*** *find the numbers*

Soln:

If the common ratio is ***x,***

$\Rightarrow$ *The ratio* ***7 : 3 = 7x:3x (****The ratio* ***7x:3x*** *simplifies to* ***7:3)***

$\Rightarrow$ *The numbers are* ***7x*** *and* ***3x***

$\Rightarrow 7x - 3x = 48$

$x = 12$

$\therefore\ 7x = 7(12) = 84, \quad 3x = 3(12) = 36$

Thus the numbers are ***84*** *and* ***36***

16. *Tom is* ***6*** *years older than Bob. The ratio between the present ages of Tom and Bob is* ***7:5.*** *Find their present ages*

Soln:

The ratio ***7 : 5 = 7x:5x***

$\Rightarrow$ *Tom's present age =* ***7x,*** *Bob's present age =* ***5x***

$\Rightarrow 7x - 5x = 6$

$x = 3$

$\therefore$ *Tom's age = 7(3)* ***= 21 years,*** *Bob's age = 5(3)* ***= 15 years***

METHOD II

Bob's age = ***x***

Tom's age = ***x + 6***

$$\Rightarrow \frac{x + 6}{x} = \frac{7}{5}$$

$\Rightarrow\ x = 15$

$\therefore$ *Bob's age =* ***15 years,*** *Tom's age = 15 + 6* ***= 21 years***

17. *The present ages of Tom and Bob are in the ratio of* ***5:4.*** *In three years time, the ratio of their ages will become* ***11:9****respectively. Find Bob's present age*

Soln:

The ratio ***5 : 4 = 5x:4x***

$\Rightarrow$ *Tom's present age =* ***5x,*** *Bob's present age =* ***4x***

$\Rightarrow$ *After* ***3*** *years, Tom's age =* ***5x + 3,*** *Bob's age =* ***4x + 3***

$\Rightarrow \frac{5x + 3}{4x + 3} = \frac{11}{9}$

$\Rightarrow x = 6$

$\therefore$ *Bob's present age = 4(6)* ***= 24 years***

18. *Four years ago, the ratio of the ages of Tom and Bob was* ***3:2.*** *In six years time, the ratio of their ages will become* ***14:11*** *respectively. Find Bob's present age*

Soln:

Four years ago, Tom's age = ***3x,*** *Bob's age =* ***2x***

$\Rightarrow$ *Tom's present age =* ***3x + 4,*** *Bob's present age =* ***2x + 4***

After ***6*** *years, Tom's age =* ***(3x + 4) + 6 = 3x + 10,***

Bob's age = ***(2x + 4) + 6 = 2x + 10***

$\Rightarrow \frac{3x + 10}{2x + 10} = \frac{14}{11}$

$\Rightarrow x = 6$

$\therefore$ *Bob's present age = 2(6)* ***+*** *4* ***= 16 years***

19. *The present ages of Tom, Bob and Ben are in the ratio* ***4:7:9.*** *Eight years ago, the sum of their ages was* ***76.*** *Find their present ages*

Soln:

Tom's present age = ***4x,*** *Bob's age =* ***7x,*** *Ben's age =* ***9x***

$\Rightarrow$ ***8*** *years ago, Tom's age =* $\boldsymbol{4x - 8}$***,*** *Bob's age =* $\boldsymbol{7x - 8}$***,*** *Ben's age =* $\boldsymbol{9x - 8}$

$\Rightarrow \ (4x - 8) + (7x - 8) + (9x - 8) = 76$

$\Rightarrow \ \boldsymbol{x = 5}$

$\therefore$ *Tom's present age = 4(5)* ***= 20 years,*** *Bob's age = 7(5)* ***= 35 years***

Ben's age = 9(5) ***= 45 years***

20. *Ten years ago, the ratio of the ages of Tom, Bob and Ben was* ***2:3:7.*** *The sum of their present ages is* ***90.*** *Find Bob's present age*

Soln:

Ten years ago, Tom's age = $\boldsymbol{2x}$***,*** *Bob's age =* $\boldsymbol{3x}$***,*** *Ben's age =* $\boldsymbol{7x}$

$\Rightarrow$ *Tom's present age =* $\boldsymbol{2x + 10}$***,*** *Bob's age =* $\boldsymbol{3x + 10}$***,*** *Ben's age =* $\boldsymbol{7x + 10}$

$\Rightarrow \ (2x + 10) + (3x + 10) + (7x + 10) = 90$

$\Rightarrow \ \boldsymbol{x = 5}$

$\therefore$ *Bob's present age = 3(5) + 10* ***= 25 years***

EER:

1. *Find two numbers such that one exceeds the other by* ***11*** *and their sum is* ***73***

2. *The sum of three consecutive multiples of* ***4*** *is* ***444.*** *Find these multiples*

3. *The cost of two tables and three chairs is* ***Shs 705,000.*** *If the table costs* ***Shs 40,000*** *more than the chair, find the cost of each table and a chair*

4. *The length of a rectangle is* ***4*** *times its width. If the perimeter of the rectangle is* ***80m,*** *find the length and width of the rectangle*

***5**. Three–fourth of a number is more than one–fourth of a number by **2.** Find the number*

***6.**The numerator of a fraction is **5** less than its denominator. If **2** is added to both the numerator and denominator, the fraction becomes* $\frac{4}{5}$*. Find the original fraction*

***7.** In a multiple choice test of **30** questions, each correct answer carries **5** marks and each wrong answer leads to a loss of **2** marks. If a student scored a total of **80** marks from the test, find how many questions were answered correctly*

***8.**The numerator of a fraction is **1** less than its denominator. If the numerator is decreased by **2** and the denominator is increased by **3,** the fraction becomes* $\frac{1}{4}$*. Find the original fraction*

***9.**The numerator of a fraction is **3** less than its denominator. If the numerator is tripled and the denominator is increased by **7,** the resulting fraction is* $\frac{3}{2}$*. Find the original fraction*

***10.** The sum of the ages of **5** children born at intervals of **3** years each is **50** years. Find the age of the youngest child*

***11.** The present ages of **P, Q** and **R** are in the ratio **4:7:9.** Eight years ago, the sum of their ages was **56.** Find their present ages*

***12.** A man is **4** times as old as his son. Five years ago, the man was **9** times as old as his son. Find the man's present age*

***13.** Tom is **4** times as old as Bob. In **4** years time, the sum of their ages will be **43** years. Find Tom's present age*

***14.** Five years ago, a man's age was seven times as old as his son. In five years time, the man's age will be thrice as old as his son. Find their present ages*

15. *In* ***15*** *years time, Tom's age will be* ***6*** *times his age* ***5*** *years back. Find his present age*

16. *Tom is* ***12*** *years younger than Bob. The ratio between the present ages of Tom and Bob is* ***2:5.*** *Find their present ages*

17. *Ten years ago,* ***P*** *was twice as old as* ***Q.*** *The ratio between the present ages of* ***P*** *and* ***Q*** *is* ***4:3.*** *Find their present age*

18. *Six years ago, Tom was thrice as old as Bob. In six years time, Tom will be one and a third times as old as Bob. Find Bob's present age*

19. *The present ages of Tom and Bob are in the ratio* ***4:3.*** *In six years time, Tom's age will be* ***26*** *years. Find Bob's present age*

20. *The present ages of Tom and Bob are in the ratio* ***11:5.*** *If the difference between their ages is* ***24*** *years, find their present ages*

21. *The present ages of a man and his son are in the ratio* ***4:1.*** *If the product of their ages is* ***196*** *years, find the ratio of their ages in* ***5*** *years time*

22. *The ratio between the present ages of* ***P*** *and* ***Q*** *is* ***2:5.*** *In eight years time, their ages will be in the ratio* ***1:2.*** *Find their present ages*

23. *Six years ago, the ratio of the ages of Tom and Bob was* ***6:5.*** *In four years time, the ratio of their ages will become* ***11:10*** *respectively. Find Bob's present age*

24. *The ages of two people differ by* ***16*** *years. Six years ago, the elder one was* ***3*** *times as old as the younger one. Find the elder's present age*

ALGEBRAIC FRACTIONS

Summary:

(i) *A fraction with any letter in its numerator or denominator is called an algebraic fraction*

(ii) *A fraction is undefined **(meaningless)** when its denominator is equal to zero.*

(iii) *To reduce an algebraic fraction in its simplest form, factorization is sometimes necessary*

EXAMPLES:

1. *Find the value of **x** that makes the fraction* $\frac{5x+9}{4x-12}$ *undefined*

2. *Find the values of **x** for which the fraction* $\frac{3x+4}{x^2-25}$ *is undefined*

3. *Find the values of **x** for which the fraction* $\frac{5x}{2x^2-18}$ *is undefined*

4. *Express the following as a single fraction:*

(i) $\frac{x}{3}+\frac{x}{4}+\frac{x}{5}$ ***(ii)*** $\frac{x-2}{4}+\frac{2}{5}$

(iii) $\frac{3x-5}{10}+\frac{2x-3}{15}$ ***(iv)*** $\frac{x-5}{3}-\frac{x-2}{4}$

(v) $3(x+2)-\frac{4x-5}{4}$ ***(vi)*** $\frac{2x-5}{5}-\frac{3x+2}{4}+\frac{7x+15}{10}$

5. *Reduce the following fractions to their lowest form:*

(i) $\frac{3xy^2}{6x^2y}$ ***(ii)*** $\frac{18x^2y}{27xy^2z}$ ***(iii)*** $\frac{(x+5)(x-1)}{(x+3)(x+5)}$ ***(iv)*** $\frac{(2x-1)^5}{(2x-1)^3}$

(v) $\frac{x^5(x+1)^2}{x^3(x+1)^8}$ ***(vi)*** $\frac{12x}{20x+8}$ ***(vii)*** $\frac{27x+15}{9x}$ ***(viii)*** $\frac{3x-6}{(x+5)(x-2)}$

(ix) $\frac{2x+4}{3x+6}$ **(x)** $\frac{5-3x}{3x-5}$ **(xi)** $\frac{3x}{x^2-5x}$ **(xii)** $\frac{5x-x^2}{(x+2)(x-5)}$

(xiii) $\frac{(x+4)(x-3)}{3x-x^2}$ **(xiv)** $\frac{12abc-4a^2bc^2}{8abc^2}$

(i) Soln:

$$\frac{3xy^2}{6x^2y} = \frac{3x\cdot y\cdot y}{6x\cdot x\cdot y} = \frac{y}{2x}$$

6. *Simplify the following fractions as far as possible:*

(i) $\frac{3a^2b}{2b^3c} \times \frac{2c^2}{5a^3} \div \frac{6ac}{10b^2}$ **(ii)** $\frac{a^2}{(2a)^2} \times \frac{(3b)^2}{27a} \div 3b$

(iii) $\frac{7a}{2a+14} \times \frac{3a+21}{9a}$ **(iv)** $\frac{y^2+3y}{6y^2} \div \frac{5y+15}{4}$

LCM OF ALGEBRAIC EXPRESSIONS

Summary:

*The **LCM** of given expressions is obtained as follows:*

(i) *Determine the prime factor form of each expression*

(ii)*The product of each factor with the highest power is the required **LCM***

EXAMPLES:

1. *Find the **LCM** of* $16y^3$ *and* $24y^5$

Soln:

$16y^3 = 2^4 \times y^3$

$24y^5 = 2^3 \times 3 \times y^5$

$LCM = 2^4 \times 3 \times y^5$

$= 48y^5$

2. *Find the* ***LCM*** *of* $8a^2bc^3$ *and* $20ac^4$

Soln:

$8a^2bc^3 = 2^3 \times a^2 \times b \times c^3$

$20ac^4 = 2^2 \times 5 \times a \times c^4$

$LCM = 2^3 \times 5 \times a^2 \times b \times c^4$

$= 40a^2bc^4$

3. *Find the* ***LCM*** *of* $4a^2b$, $6ab$ *and* $8ab^3$

Soln:

$4a^2b = 2^2 \times a^2 \times b$

$6ab = 2 \times 3 \times a \times b$

$8ab^3 = 2^3 \times a \times b^3$

$LCM = 2^3 \times 3 \times a^2 \times b^3$

$= 24a^2b^3$

4. *Find the* ***LCM*** *of* $y(y + 2)$ *and* $(y + 2)(y - 3)$

Soln:

$y(y+2) = y \times (y+2)$

$(y+2)(y-3) = (y+2) \times (y-3)$

$$\begin{aligned} LCM &= y \times (y+2) \times (y-3) \\ &= y(y+2)(y-3) \end{aligned}$$

5. *Find the **LCM** of* $(y-5)(y+3)$ *and* $(y+3)^2$

Soln:

$(y-5)(y+3) = (y-5) \times (y+3)$

$(y+3)^2 = 1 \times (y+3)^2$

$$\begin{aligned} LCM &= (y-5) \times (y+3)^2 \\ &= (y-5)(y+3)^2 \end{aligned}$$

6. *Find the **LCM** of* $ab(a-b)$, $4(a-b)$ *and* $6(a-b)^2$

Soln:

$ab(a-b) = a \times b \times (a-b)$

$4(a-b) = 2^2 \times (a-b)$

$6(a-b)^2 = 2 \times 3 \times (a-b)^2$

$$\begin{aligned} LCM &= 2^2 \times 3 \times a \times b \times (a-b)^2 \\ &= 12ab(a-b)^2 \end{aligned}$$

7. *Simplify the following fractions as far as possible:*

(i) $\frac{2}{3x} + \frac{1}{2x} + \frac{3}{4x}$ (ii) $\frac{4}{5x} + \frac{2}{x^2} - \frac{1}{3x}$ (iii) $2 + \frac{3}{x+1}$

(iv) $x - \frac{5}{x}$ (v) $\frac{5}{x-3} - \frac{1}{x}$ (vi) $\frac{3}{x+2} + \frac{2}{x-1}$ (vii) $\frac{5x}{x-1} - \frac{2x}{x+1}$ (viii) $\frac{13x+1}{2x+1} - \frac{7x-2}{2x+1}$ (ix) $\frac{2x^2}{x+4} + \frac{8}{x+4}$ (x) $\frac{9-5x}{3x-2} - \frac{7-2x}{3x-2}$ (xi) $\frac{3x}{(x-2)(x+5)} - \frac{6}{(x-2)(x+5)}$ (xii) $\frac{2}{x(x+2)} + \frac{3}{(x+2)(x-3)}$

(xiii) $\frac{2x}{(x-3)(x+3)} - \frac{1}{(x+3)} - \frac{2}{(x-3)}$

EER:

1. Find the values of x that make the fraction $\frac{14x}{x^2 - 9}$ undefined

2. Find the values of x for which the fraction $\frac{5x+3}{9x^2 - 4}$ is undefined

3. Find the values of x for which the fraction $\frac{10x}{x^2 - 4}$ is undefined

4. Find the **LCM** of $6a^2b^3c^2$, $20ab^2c^5$ and $18a^4b^2c$

5. Find the **LCM** of $ab(a-b)^2$, $6a^2(a-b)$ and $8(a-b)^3$

6. Simplify as far as possible: $\frac{6x^2y}{13p^2q} \times \frac{39q^2r}{18y^2z} \div \frac{x^2p^2}{yqr}$

7. Express $1 - \frac{2x}{5} + \frac{x}{8}$ as a single fraction

8. Express $\frac{x+3}{3} - \frac{x-7}{4} + \frac{x-5}{2}$ as a single fraction

9. Express $4x - \left(\frac{3x + y}{8} - \frac{3x + y}{4}\right)$ as a single fraction

10. Express $\frac{3}{x+1} - \frac{x + 4}{(x + 1)(x - 2)}$ as a single fraction

11. Given that $y = \frac{7}{x-2} - \frac{x + 5}{(x - 1)(x - 2)}$, express **y** in the form $\frac{a}{x + b}$.

Hence find the value of **x** for which **y** is undefined

12. Express $\frac{2}{x+5} + \frac{1}{x-3} - \frac{5}{(x+5)(x-3)}$ as a single fraction

13. Express $\frac{4}{x+5} + \frac{1}{x-2} - \frac{7}{(x+5)(x-2)}$ as a single fraction

14. Reduce the following fractions to their lowest form:

(i) $\frac{14a^2bc}{42ab^3c^4}$ **(ii)** $\frac{-6mn^4}{-24m^5n^2}$ **(iii)** $\frac{(x + 3)(x - 7)}{6(x + 3)}$ **(iv)** $\frac{4(p + 7)}{(p + 7)^3}$

(v) $\frac{x+2}{(x + 5)(x + 2)}$ **(vi)** $\frac{6m}{m^2 - 4m}$ **(vii)** $\frac{2x + 10}{x^2 + 5x}$ **(viii)** $\frac{x-5}{5-x}$

(ix) $\frac{3x - x^2}{(x + 5)(x - 3)}$ **(xi)** $\frac{15a^2bc^2 - 12abc}{18abc^2}$

15. Simplify the following fractions as far as possible:

(i) $\frac{x - 1}{x + 2} \div \frac{5(x - 1)}{x + 3}$ **(ii)** $\frac{14a^2bc}{42ab^3c^4}$

(iii) $\frac{3a}{6a + 6b} \times \frac{a+b}{5a^2}$ **(iv)** $\frac{5y + 25}{y^3} \div \frac{2y + 10}{7y}$

16. Simplify the following fractions as far as possible:

(i) $\frac{1}{x} + \frac{2}{x^2} - \frac{1}{x^3}$ **(ii)** $3 + \frac{2}{x + 2}$ **(iii)** $x - \frac{5}{x}$

(iv) $\frac{3}{x - 3} - \frac{2}{x}$ **(v)** $\frac{6}{x - 1} + \frac{3}{x + 1}$ **(vi)** $\frac{5x}{x - 1} - \frac{2x}{x + 1}$

(vii) $\frac{6x+1}{x-3} - \frac{4x-2}{x-3}$ (viii) $\frac{3x^2}{x+2} + \frac{6}{x+2}$ (ix) $\frac{9-5x}{3x-2} - \frac{7-2x}{3x-2}$

COORDINATES

Summary:

1. (i) *A pair of values written in the form* ***(x, y)*** *is called coordinates*

(ii) *A point with given coordinates can be plotted on the* ***x–y*** *plane*

2. (i) *The* ***x–y*** *plane is the same as the coordinate plane* ***or*** *the Cartesian plane*

(ii) *On the* ***x–y*** *plane, the horizontal axis is called the* ***x–axis*** *and the vertical axis is called the* ***y–axis.***

(iii) *The* ***x–axis*** *meets the* ***y–axis*** *at a point called the* ***origin.*** *The coordinates of the origin are* ***(0, 0)***

(iv) *On the* ***x–axis,*** *values to the right of the origin are positive and those to the left are negative*

(v) *On the* ***y–axis,*** *values above the origin are positive and those below are negative*

(vi) *For each axis a suitable scale is chosen and then marked off at equal intervals*

3. *In coordinate geometry, area is stated in terms of square units* ***(sq units)***

EXAMPLES:

1. *On the same axes, plot the following points* ***A(4, 5), B(–2, –4), C(3, 0), D(0, 3), E(–3, 0)*** *and* ***F(0, –2)***

2. (i) *On the same axes, plot the points* ***A(3, 2), B(7, 2)*** *and* ***C(7, 8)***

(ii) *Join the points and name the formed figure* ***ABC.***

(iii) *Calculate the area of the formed figure* ***ABC.***

Soln:

(ii) *Figure **ABC** is a right angled triangle*

***(iii) Area** =* $\frac{1}{2}bh = \frac{1}{2} \times 4 \times 6 = 12 \;\; squnits$

METHOD II

***Area** =* $\frac{1}{2} \times \left| major \quad product \quad sums - minor \quad product \quad sums \right|$

$$\begin{array}{c} A(3, 2) \\ B(7, 2) \\ C(7, 8) \\ A(3, 2) \end{array}$$

Area A_1 ***=*** *3(2) + 7(8) + 7(2)* ***= 76***

Area A_2 ***=*** *2(7) + 2(7) + 8(3)* ***= 52***

***Required Area** =* $\frac{1}{2} \times \left| 76 - 52 \right| = 12 \;\; squnits$

3. (i) *On the same axes, plot the points* ***P(–2, 3), Q(–8, 3), R(–8, 8)*** *and* ***S(–2, 8)***

(ii) *Join the points and name the formed figure* ***PQRS.***

(iii) *Calculate the area of the formed figure* ***PQRS.***

Soln:

(ii) *Figure **PQRS** is a rectangle*

***(iii) Area** =* $lw = 6 \times 5 = 30 \;\; squnits$

4. (i) *On the same axes, plot the points* ***A(3, 6), B(8, –5)*** *and* ***C(–4, –2)***

(ii) *Join the points and name the formed figure* ***ABC.***

(iii) *Calculate the area of the formed figure* ***ABC.***

Soln:

(ii) *Figure* ***ABC*** *is a triangle*

(iii) Area = $\frac{1}{2} \times \left| major \quad product \quad sums \ - \ minor \quad product \quad sums \right|$

$$\begin{array}{c} A(3, 6) \\ B(8, -5) \\ C(-4, -2) \end{array}$$

Area A_1 = $3(-5) + 8(-2) + -4(6) = -55$

Area A_2 = $8(6) + -5(-4) + -2(3) = 62$

Required Area = $\frac{1}{2} \times \left| -55 - 62) \right| = 58 \cdot 5$ *squnits*

METHOD II

Area = $12(11) \ - \ \{ \frac{1}{2}(12)(3) \ + \ \frac{1}{2}(7)(8) \ + \frac{1}{2}(5)(11) \ \} = 58 \cdot 5$ *squnits*

5. (i) *On the same axes, plot the points* ***P(3, 4), Q(5, 4), R(6, 2)*** *and* ***S(2, 2)***

(ii) *Join the points and name the formed figure* ***PQRS.***

(iii) *Calculate the area of the formed figure* ***PQRS.***

Soln:

(ii) *Figure* ***PQRS*** *is a trapezium*

(iii) Area = $\frac{1}{2}(a + b)h \ = \ \frac{1}{2} \times (4 + 2) \times 2 \ = \ 6$ *squnits*

6. *On the same axes, plot the following points* ***A(4, 10), B(–2, –40), C(3, 0), D(0, 30), E(–3, 15)*** *and* ***F(0, –20).*** *[Use a scale of* ***1cm*** *to represent* ***1 unit*** *on the* ***x– axis*** *and* ***1cm*** *to represent* ***5 units*** *on the* ***y– axis]***

7. *On the same axes, plot the following points* ***A(0·5, 1·5), B(3, –2·5), C(1·3, 3),***

D(1·7, 2·2), E(–1·8, 0·6) *and* ***F(0, –1·6) [****Use a scale of* ***2cm*** *to represent* ***1 unit*** *on both axes****]***

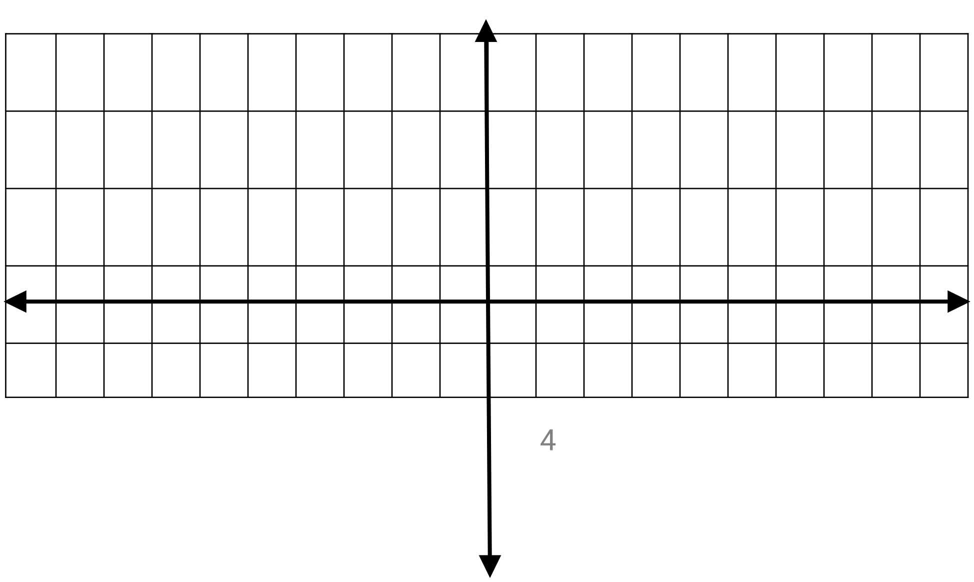
8. *Write down the coordinates of each of the following points shown on the graph below:*

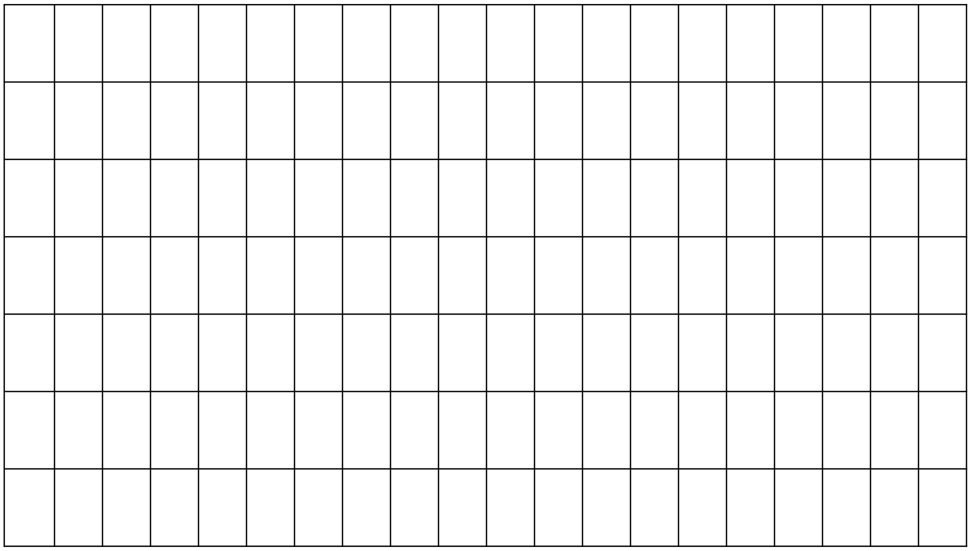
EER:

1. *On the same axes, plot the following points* ***A(3, 2), B(–2, 3), C(–3, –2)*** *and* ***D(2, –3)***

2. *On the same axes, plot the following points* ***A(0, 4), B(–3, 0), C(0, –2)*** *and* ***D(2, 0)***

3. *Write down the coordinates of the points marked* ***A, B, C, D, E, F, G*** *and* ***H*** *on the graph below:*





Write down the coordinates of the points marked ***A, B, C, D, E, F, G*** *and* ***H***

4. *A quadrilateral has vertices* ***A(1, 20), B(–3, 30), C(–2, –10)*** *and* ***D(2, –20).***

(i) *Plot the points of the quadrilateral and identify it.*

(ii) *Find the coordinates of the point of intersection of the diagonals of the*

Quadrilateral

5. *Find the area of the quadrilateral with vertices* ***A(–2, 2), B(3, 5), C(10, 5)*** *and*

D(5, 2)

6. (i) *Find the area of a triangle with vertices* ***P(-2, -2) Q(2 , 4)*** *and* ***R(5 , 0).***

(ii) *Construct a circle circumscribing triangle* ***PQ R.*** *Hence calculate the area of the segments cut off by triangle* ***PQ R.***

7. (i) *Find the area of the quadrilateral with vertices* ***A(–2, 1), B(1, 3), C(4, 1)*** *and*

D(1, –1)

(ii) *Find the coordinates of the point of intersection of the diagonals of*

the

quadrilateral

8. *A quadrilateral has vertices* ***A(–10, 0), B(–10, 25), C(15, 25)*** *and* ***D(25, –10).***

(i) *Plot the points of the quadrilateral and identify it.* ***[****Use a scale of* ***2cm*** *to*

represent ***10 units*** *on both axes****]***

(ii) *Find the coordinates of the point of intersection of the diagonals of the*

Quadrilateral

THE SLOPE OF A LINE (GRADIENT OF A LINE)

Summary:

1. (i) *The gradient of a line =* $\frac{Change \quad in \;\; y \; values}{Change \quad in \;\; x \; values}$

(ii) *A line joining two points is called a line segment*

2. *The following relationships apply to a line segment with endpoints* $A(x_1, \; y_1)$

and $B(x_2, \; y_2)$*:*

(i) *Gradient of* ***AB*** $= \frac{y_2 - y_1}{x_2 - x_1}$

(ii) *Midpoint of* **AB** $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

(iii) *Length of* **AB =** $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EXAMPLES:

1. *Find the gradient of a line passing through the points* ***P(1, 4)*** *and* ***Q(5, 12)***

2. *Find the gradient of a line passing through the origin and the point* ***P(2, 10)***

3. *Find the gradient of a line passing through the origin and the point* ***P(a, b)***

4. *A line of gradient* ***–3*** *passes through the points* ***P(3, 8)*** *and* ***Q(k, 2).*** *Find the value of* ***k***

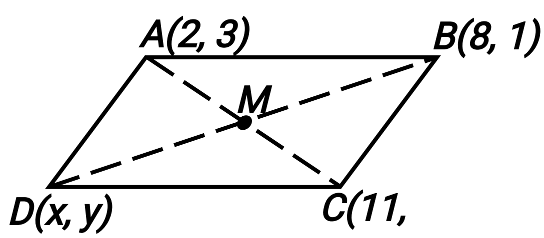
5. *Find the midpoint of a line segment with end points* ***(3, –7)*** *and* ***(9, –1)***

6. *Find the coordinates of the point that is halfway between* ***(–1, 4)*** *and* ***(3, 6)***

7. *The vertices of a parallelogram are* ***A(2, 3), B(8, 1), C(11, 5)*** *and* ***D.*** *Find the coordinates of* ***D***

Soln:

HINT: *The diagonals of a parallelogram bisect each other*



If D **= (x, y)**

$\Rightarrow \left(\dfrac{x + 8}{2}, \dfrac{y + 1}{2}\right) = \left(\dfrac{2 + 11}{2}, \dfrac{3 + 5}{2}\right)$

$$\Rightarrow \left(\frac{x+8}{2}, \frac{y+1}{2}\right) = (6 \cdot 5, \ 4)$$

$$\frac{x+8}{2} = 6 \cdot 5 \qquad \text{and} \qquad \frac{y+1}{2} = 4$$

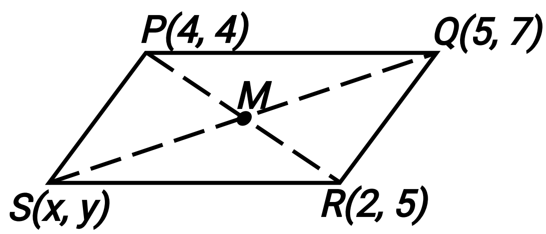
x = 5, y = 7

$\therefore$ *Vertex **D** is **(5, 7)***

8. *The vertices of a parallelogram are **P(4, 4), Q(5, 7), R(2, 5)** and **S.** Find the coordinates of **S***

Soln:

HINT: *The diagonals of a parallelogram bisect each other*



*If S **= (x, y)***

$$\Rightarrow \left(\frac{x+5}{2}, \frac{y+7}{2}\right) = \left(\frac{4+2}{2}, \frac{4+5}{2}\right)$$

$$\Rightarrow \left(\frac{x+5}{2}, \frac{y+7}{2}\right) = (3, \ 4 \cdot 5)$$

$$\frac{x+5}{2} = 3 \qquad \text{and} \qquad \frac{y+7}{2} = 4 \cdot 5$$

x = 1, y = 2

$\therefore$ *Vertex **S** is **(1, 2)***

9. *The midpoint of a line segment **PQ** is **(4, –1).** If the coordinates of **P** are **(–4, 3),** find the coordinates of **Q***

Soln:

If Q ***= (x, y)***

$$\Rightarrow \left(\frac{-4+x}{2}, \frac{3+y}{2}\right) = (4, -1)$$

$$\frac{-4+x}{2} = 4 \quad \text{and} \quad \frac{3+y}{2} = -1$$

x = 12,   y = –5

$\therefore$ *The other end* ***= (12, –5)***

10. *The midpoint of a line segment is* ***(6, 2).*** *If one endpoint of the line segment is* ***(3, –3),*** *find the coordinates of the other endpoint*

11. *Point* ***M(1, 5)*** *is halfway between* ***(–1, 4)*** *and another point* ***Q.*** *Find the coordinates of* ***Q***

12. *Find the distance between the points* ***P(–8, 2)*** *and* ***Q(4, 7)***

Soln:

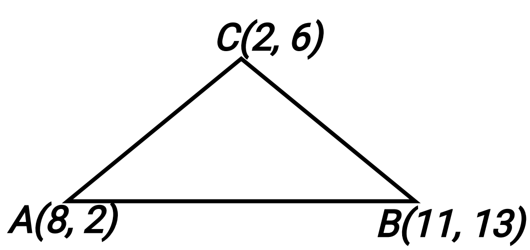
$$|PQ| = \sqrt{(4 - {}^{-}8)^2 + (7 - 2)^2} = 13 \text{ units}$$

13. *Find the length of a line segment with end points* ***(1, 9)*** *and* ***(–2, 5)***

14. *Show that the points* ***A(8, 2), B(11, 13)*** *and* ***C(2, 6)*** *are vertices of an isosceles triangle.*

Soln:

HINT: *The two sides of an isosceles triangle are equal*



$$|\boldsymbol{AB}| = \sqrt{(11-8)^2 + (13-2)^2} = \sqrt{130} \text{ units}$$

$$|\boldsymbol{BC}| = \sqrt{(11-2)^2 + (13-6)^2} = \sqrt{130} \text{ units}$$

$$|\boldsymbol{AC}| = \sqrt{(8-2)^2 + (2-6)^2} = \sqrt{52} \text{ units}$$

$\therefore$ *Since* ***AB = BC,*** *then the triangle is isosceles*

15. *The distance of point* ***P(12, k)*** *from the origin is* ***13 units.*** *Find the possible values of* ***k***

Soln:

If $\sqrt{(12-0)^2 + (k-0)^2} = 13$

$$\Rightarrow \; 144 + k^2 = 169$$

$$k = \pm\sqrt{25}$$

$$\therefore \; k = 5 \quad or \quad -5$$

16. *The distance between the points* ***(4, 8)*** *and* ***(1, k)*** *is* ***5 units.*** *Find the possible values of* ***k***

Soln:

If $\sqrt{(1-4)^2 + (k-8)^2} = 5$

$$\Rightarrow \; 9 + (k-8)^2 = 25$$

$$(k-8) = \pm\sqrt{16}$$

$$\therefore \; k = 4 \quad or \quad 12$$

17. *Point* ***P(3, 0)*** *is equidistant from the points* ***A(5, k)*** *and* ***B(–4, 6).*** *Find the* *possible values of* ***k***

Soln:

If distance ***AP*** *= distance* ***BP***

$$\Rightarrow \sqrt{(5-3)^2+(k-0)^2} = \sqrt{(-4-3)^2+(6-0)^2}$$

$$\Rightarrow 4+k^2 = 85$$

$$k = \pm\sqrt{81}$$

$$\therefore\ k = 9 \quad \text{or} \quad -9$$

18. *The end points of the diameter of a circle are* ***(1, –2)*** *and* ***(7, 6).*** *Find the:*

(i) *coordinates of the centre of the circle*

(ii) *radius of the circle*

Soln:

Hint: *Centre is the midpoint of the diameter and radius is half the diameter*

(i) Centre = $\left(\frac{1+7}{2}, \frac{-2+6}{2}\right) = (4, 2)$

(ii) Radius = $\sqrt{(4-1)^2+(2-{}^{-}2)^2} = 5 \text{ units}$

19. *Find the centre and radius of a circle that has a diameter with endpoints* ***(–11, 19)*** *and* ***(21, –5)***

Soln:

Centre = $\left(\frac{-11+21}{2}, \frac{19+-5}{2}\right) = (5, 7)$

Radius = $\sqrt{(5-{}^{-}11)^2+(7-19)^2} = 20 \text{ units}$

20. *One end of the diameter of a circle with centre* ***(–3, 4·5)*** *is* ***(–1, 3).*** *Find the:*

(i) *coordinates of the other end of the diameter*

(ii) *radius of the circle*

Soln:

If the other end ***= (x, y)***

$$\Rightarrow \left(\frac{-1+x}{2}, \frac{3+y}{2}\right) = (-3, 4\cdot 5)$$

$$\frac{-1+x}{2} = -3 \qquad \text{and} \qquad \frac{3+y}{2} = 4\cdot 5$$

x = –5,   y = 6

$\therefore$ *The other end* ***= (–5, 6)***

Radius $= \sqrt{(-3-{}^{-}1)^2 + (4\cdot 5-3)^2} = 2\cdot 5$ **units**

EER:

1. *Find the gradient of a line joining the points* ***P(2, 10)*** *and* ***Q(–4, –8).***

2. *Find the distance between the points* ***P(–8, 2)*** *and* ***Q(4, 7)***

3. (i) *Show that the points* ***A(–2, 3), B(–5, 4)*** *and* ***C(2, –1)*** *are vertices of an isosceles triangle.*

(ii) *Find the area of the triangle*

4. *Find the midpoint of a line segment with end points* ***(3, –1)*** *and* ***(9, 5)***

5. *The vertices of a parallelogram are **P(2, 1), Q(4, 7), R(6, 5)** and **S.** Find the coordinates of **S***

6. *The midpoint of a line segment **PQ** is **(4, 2)**. If the coordinates of **P** are **(7, 6),** find the:*

(i) *coordinates of **Q***

(ii) *length of the line segment **PQ***

7. *The end points of the diameter of a circle are **(1, 6)** and **(7, –2)**. Find the:*

(i) *coordinates of the centre of the circle*

(ii) *radius of the circle*

8. *The vertices of a parallelogram are **A(5, 2), B(2, 6), C(–8, –3)** and **D**. Find the coordinates of **D***

9. *A line passes through the points **P(–1, 8)** and **Q(2, 12)**. Find the:*

(i) *gradient of the line*

(ii) *coordinates of the midpoint of the line segment **PQ***

(ii) *length of the line segment **PQ***

10. *A triangle has vertices **A(4, 2), B(8, 6)** and **C(5, 9)**. Find the gradient of each side of the triangle*

11. *The points **P(5, 2)** and **Q(2, 4)** are in the **x–y** plane. Find the:*

(i) *coordinates of **M,** the midpoint of **PQ***

(ii) *|**OM**|, where **O** is the origin*

12. *A line of gradient* $\frac{2}{3}$ *passes through the points **P(5, 7)** and **Q(k, 13)**. Find the value of **k***

13. *Find the midpoint of a line segment with end points **(3a, b)** and **(a, c)***

14. *Find the length of a line segment with end points **(8, –1)** and **(5, 3)***

15. Find the distance between the points ***P(b, 0)*** *and* ***Q(0, a)***

16. Point ***P(7, 6)*** *lies on a circle whose centre is* ***(4, 2).*** *Find the radius of the circle*

17. The distance of point ***A(k, 4)*** *from the origin is* ***5 units.*** *Find the possible values of* ***k***

18. Find the distance between the points ***P(8, –1)*** *and* ***Q(5, 3)***

19. The vertices of a rectangle are ***A(0, 2), B(4, 8), C(7, 6)*** *and* ***D(3, 0).*** *Show that its diagonals are equal in length*

20. The distance between the point ***(16, k)*** *and the origin is* ***20 units.*** *Find the possible values of* ***k***

21. Point ***P(0, –2)*** *is equidistant from the points* ***A(k, –2)*** *and* ***B(3, 2).*** *Find the possible values of* ***k***

22. The distance between the point ***(0, –5)*** *and* ***(k, 0)*** *is* ***13 units.*** *Find the possible values of* ***k***

23. The distance between the point ***(k, k+2)*** *and the origin is* ***10 units.*** *Find the possible values of* ***k***

24. The midpoint of a line segment ***AB*** *is* ***(3, 6).*** *If the coordinates of* ***A*** *are* ***(–1, 1),*** *find the coordinates of* ***B***

25. Find the centre and radius of a circle that has a diameter with endpoints

(8, 6) *and* ***(–4, –3)***

26. One end of the diameter of a circle with centre ***(2, 1·5)*** *is* ***(8, –3).*** *Find the:*

(i) *coordinates of the other end of the diameter*

(ii) *radius of the circle*

27. *One end of the diameter of a circle with centre* ***(5, 7)*** *is* ***(21, –5).*** *Find the:*

(i) *coordinates of the other end of the diameter*

(ii) *radius of the circle*

THE EQUATION OF A LINE

Summary:

(i) $y = mx + c$ *is the equation of the line with gradient* ***m*** *and* ***y****–intercept* ***c***

(ii) *In the above formula,* ***m*** *and* ***c*** *are constants to be determined*

(iii) *The* ***y****–intercept of a line is a point where the line cuts the* ***y–axis***

(iv) *If a point lies on a line, its coordinates must satisfy the equation of the line*

(v) *Points that lie on a straight line are called* ***collinear points***

(vi) *A line that divides a figure in to two equal parts is called* ***a line of symmetry***

EXAMPLES:

1. *State the gradient and* ***y****–intercept of the following lines:*

(i) $y = 3x - 4$ **(ii)** $y - 5x - 9 = 0$ **(iii)** $3y + x = 10$ **(iv)** $3y + 2x = 12$

(v) $20x - 4y - 3 = 0$ **(vi)** $2y = \frac{1}{3}x + 7$ **(vii)** $\frac{x}{4} + \frac{y}{3} = 1$

Soln:

(i) *Since* $y = 3x - 4$ *is comparable to* $y = mx + c$

$\Rightarrow$ *Gradient* $m = 3$, *y–intercept* $c = -4$

(ii) *Express* $\boldsymbol{y - 5x - 9 = 0}$ *in the form* $\boldsymbol{y = mx + c}$

If $y - 5x - 9 = 0$

$\Rightarrow \boldsymbol{y = 5x + 9}$

$\therefore$ *Gradient* $\boldsymbol{m = 5}$, *y–intercept* $\boldsymbol{c = 9}$

(iii) *Express* $\boldsymbol{3y + x = 10}$ *in the form* $\boldsymbol{y = mx + c}$

If $3y + x = 10$

$\Rightarrow \boldsymbol{y} = -\frac{1}{3}x + \frac{10}{3}$

$\therefore$ *Gradient* $\boldsymbol{m} = -\frac{1}{3}$, *y–intercept* $\boldsymbol{c} = \frac{10}{3}$

(iv) *Express* $\boldsymbol{3y + 2x = 12}$ *in the form* $\boldsymbol{y = mx + c}$

If $3y + 2x = 12$

$\Rightarrow \boldsymbol{y} = -\frac{2}{3}x + 4$

$\therefore$ *Gradient* $\boldsymbol{m} = -\frac{2}{3}$, *y–intercept* $\boldsymbol{c = 4}$

(v) *Express* $\boldsymbol{20x - 4y - 3 = 0}$ *in the form* $\boldsymbol{y = mx + c}$

If $20x - 4y - 3 = 0$

$\Rightarrow \boldsymbol{y} = 5x - \frac{3}{4}$

$\therefore$ *Gradient* $\boldsymbol{m = 5}$, *y–intercept* $\boldsymbol{c} = -\frac{3}{4}$

(vi) *Express* $\boldsymbol{2y} = \frac{1}{3}x + \boldsymbol{7}$ *in the form* $\boldsymbol{y = mx + c}$

If $2y = \frac{1}{3}x + 7$

$\Rightarrow$ $y = \frac{1}{6}x + \frac{7}{2}$

$\therefore$ Gradient $m = \frac{1}{6}$, **y**–intercept $c = \frac{7}{2}$

2. Find the equation of a line whose gradient is **3** and **y**–intercept **7.**

Soln:

If $y = mx + c$, where $m = 3$ and $c = 7$

$\Rightarrow$ $y = 3x + 7$

3. Find the equation of a line whose gradient is **2** and passes through the point **P(3, 1).**

Soln:

If $y = mx + c$, where $m = 2$

$\Rightarrow$ $y = 2x + c$

Substituting $x = 3$ and $y = 1$ gives

$1 = 2(3) + c$

$c = -5$

$\therefore$ $y = 2x - 5$

4. Find the equation of a line whose **y**–intercept is **5** and passes through the point **P(–3, –7).**

Soln:

If $y = mx + c$, where $c = 5$

$\Rightarrow$ $y = mx + 5$

Substituting $x = -3$ *and* $y = -7$ *gives*

$-7 = -3m + 5$

$m = 4$

$\therefore \; y = 4x + 5$

5. *Find the equation of a line passing through the points* ***P(3, 7)*** *and* ***Q(5, 11)***

Soln:

If $y = mx + c,$

$\Rightarrow \; m = \frac{11 - 7}{5 - 3} = 2$

$\Rightarrow \; y = 2x + c$

Substituting $x = 3$ *and* $y = 7$ *gives*

$7 = 2(3) + c$

$c = 1$

$\therefore \; y = 2x + 1$

6. *Find the equation of a line passing through the points* ***A(–2, –2), B(2, 0), C(4, 1)*** *and* ***D(6, 2)***

Soln:

Hint: (i) *choose any two points on the line and work out its equation*

(ii) *The gradient of a line must be the same between any two points*

The chosen points are ***(4, 1)*** *and* ***(6, 2)***

If $y = mx + c,$

$\Rightarrow m = \frac{2-1}{6-4} = \frac{1}{2}$

$\Rightarrow y = \frac{1}{2}x + c$

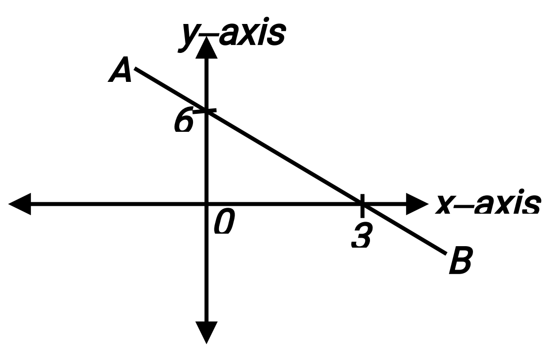
Substituting $x = 4$ *and* $y = 1$ *gives*

$1 = \frac{1}{2}(4) + c$

$c = -1$

$\therefore\ y = \frac{1}{2}x - 1$

7. Find the equation of line ***AB*** *shown on the graph below:*



Soln:

Hint: (i) *choose any two points on the line and work out its equation*

(ii) *The chosen points are* ***(3, 0)*** *and* ***(0, 6)***

If $y = mx + c,$

$\Rightarrow$ ***m =*** $\frac{6-0}{0-3} = -2$

$\Rightarrow$ ***y = –2x + c***

*Substituting **x = 3** and **y = 0** gives*

0 = –2(3) + c

c = 6

$\therefore$ ***y = 6 – 2x***

8. *Find the value of **k** for which the points **A(1, 3), B(2, 5)** and **C(k, 9)** are collinear*

Soln:

Hint: *The gradient of a line must be the same between any two points*

*Gradient of **AB =*** $\frac{5-3}{2-1} = 2$

*Gradient of **AC =*** $\frac{9-3}{k-1} = \frac{6}{k-1}$

$$\Rightarrow \frac{6}{k-1} = 2$$
$$\therefore \ k = 4$$

9. *Find the equation of a line passing through the origin and the point **P(3, 12)***

Soln:

If ***y = mx + c,***

$\Rightarrow$ ***m =*** $\frac{12 - 0}{3 - 0} = 4$

$\Rightarrow$ ***y = 4x + c***

*Substituting **x = 3** and **y = 12** gives*

12 = 4(3) + c

c = 0

$\therefore$ ***y = 4x***

10. *Find the equation of a line passing through the points **P(0, 0)** and **Q(a, b)***

11. *A quadrilateral has vertices **A(–2, 1), B(–1, 4), C(2, 3)** and **D(3, –4).***

(i) *Plot the points of the quadrilateral and identify it*

(ii) *Draw the line of symmetry of the quadrilateral*

(iii) *Find the equation of the line of symmetry*

Soln:

Hint: (i) *plot the points of the quadrilateral and identify it*

(ii) *Use two points from the line of symmetry to find it equation*

*Quadrilateral **ABCD** is a kite*

*Using **B(–1, 4)** and **D(3, –4)***

If ***y = mx + c,***

$\Rightarrow$ ***m =*** $\frac{-4 - 4}{3 - {}^{-}1} = 2$

$\Rightarrow$ ***y = 2x + c***

Substituting ***x = –1*** *and* ***y = 4*** *gives*

$4 = 2(-1) + c$

c = 6

$\therefore$ ***y = 2x + 6***

12. *A triangle has vertices* ***A(1, 2), B(6, 3)*** *and* ***C(4, 11).*** *Find the equation of the line passing through* ***A*** *and the midpoint of* ***BC***

13. *A line whose equation is* ***2y = 3x – 8*** *passes through the points* ***P(k, 5)*** *and* ***Q(–2, h).*** *Find the values of* ***k*** *and* ***h***

Soln:

If ***2y = 3x – 8***

Substituting ***x = k*** *and* ***y = 5*** *gives*

$2(5) = 3(k) - 8$

k = 6

Similarly, substituting ***x = –2*** *and* ***y = h*** *gives*

$2(h) = 3(-2) - 8$

h = –7

14. *Show that the points* ***A(3, 7), B(–1, –5)*** *and* ***C(2, 4)*** *lie on a straight line*

Soln:

Hint: *If a point lies on a line, its coordinates must satisfy the equation of the line*

Using ***A(3, 7)*** *and* ***B(–1, –5)***

If ***y = mx + c,***

$\Rightarrow$ ***m =*** $\frac{-5-7}{-1-3} = 3$

$\Rightarrow$ ***y = 3x + c***

Substituting ***x = 3*** *and* ***y = 7*** *gives*

7 = 3(3) + c

c = –2

$\therefore$ ***y = 3x – 2***

If a point ***C(2, 4)*** *lies on the line* ***y = 3x – 2,*** *its coordinates must satisfy this equation*

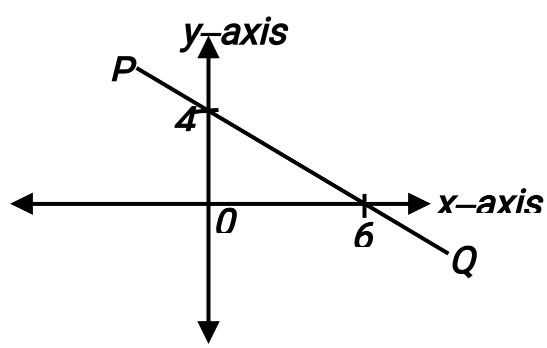
Thus when ***x = 2, y =*** *3(2) – 2* ***= 4***

$\therefore$ *The three given points lie on a straight line*

EER:

1. *Find the equation of the line joining the points* ***P(2, 7)*** *and* ***Q(5, 13)***

2. *Find the equation of line* ***PQ*** *shown on the on the graph below:*



3. *Find the equation of a line whose gradient is* $\frac{5}{2}$ *and passes through the point* ***P(5, 4).***

***4**. Find the equation of a line with gradient **3** and **y**–intercept **–5**.*

5**. A triangle has vertices **A(4, 2), B(8, 6)** and **C(5, 9)**. Find the equation of each of the lines **AB, BC** and **AC

6.** Find the equation of a line passing through the points **P(0, a)** and **Q(b, 0)

7.** A line whose equation is **y = 7x – 8** passes through the point **(k, 6)**. Find the value of **k

8.** Find the equation of the line with gradient $\frac{3}{4}$ and passing through the midpoint of the line joining **(–3, –4)** and **(–5, 6)

***9.** Find the value of **k** for which the points **A(8, 2), B(–1, k)** and **C(4, 8)** are collinear*

***10.** Show that the points **A(1, 2), B(3, 6)** and **C(2, 4)** are collinear*

***11.** Show that the line joining the points **A(4, –3)** and **B(–8, 6)** passes through the origin*

***12.** Find the value of **k** for which the points **A(2, 2), B(–1, k)** and **C(1, 6)** are collinear*

***13**. A triangle has vertices **P(3, -1), Q(7,6)** and **R(0, 2)**. Find the equation of its line of symmetry.*

14.** A quadrilateral has vertices **A(4, 3), B(4, 7), C(10, 1)** and **D(6, 1).

***(i)** Plot the points of the quadrilateral and identify it*

***(ii)** Draw the line of symmetry of the quadrilateral*

***(iii)** Find the equation of the line of symmetry*

15.** Find the equation of the line of symmetry of a quadrilateral whose vertices are **A(–5, 7), B(2, 6), C(5, –3)** and **D(–4, 0).

16.** Find the equation of the line of symmetry of a geometrical figure whose vertices are **A(8, 11), B(8, 7), C(4, 3), D(0, 3), E(0, 7)** and **F(4, 11).

INTERCEPTS OF A LINE

Summary:

1. (i) *Intercepts are points where the line cuts the coordinate axes*

(ii) *The **y**–intercept of a line is a point where the line cuts the **y–axis***

(iii) *The **x**–intercept of a line is a point where the line cuts the **x–axis***

2. (i) *The equation of the **x–axis** is **y = 0.** Thus across the **x**–axis**, y = 0***

(ii) *The equation of the **y–axis** is **x = 0.** Thus across the **y**–axis**, x = 0***

EXAMPLES:

1. *Find the intercepts of a line whose equation is given by **5y – 3x + 30 = 0***

2. *A line is given by the equation **45 – 15x + 3y = 0.** Find the coordinates of:*

(i) *its **x**–intercept*

(ii) *its **y**–intercept*

3. *A line whose equation is **5y – 3x + 30 = 0** cuts the **x–axis** and **y–axis** at points **P** and **Q** respectively. Find the coordinates of **P** and **Q***

4. *A line whose equation is **6y – 8x = 24** cuts the **x–axis** and **y–axis** at points **P** and **Q** respectively. Find the length of **PQ***

5. *A line of gradient $\frac{2}{3}$ passing through point **P(9, 4),** cuts the **y–axis** at point **Q.** Find the coordinates of **Q***

6. *Find the equation of a line whose **x**–intercept is **5** and passes through point **P(3, 2).***

Soln:

Hint: *This line meets the x–axis at point **(5, 0)***

*$\Rightarrow$ The line passes through **(5,0)** and **(3, 2)***

7. *Find the equation of a line with gradient **5** and **x**–intercept **6.***

8. *Find the equation of a line with **x**–intercept **2** and **y**–intercept **9.***

EER:

1. *Find the **x** and **y**–intercepts of the line whose equation is given by*

$\frac{x}{3} + \frac{y}{2} = 1$

2. (i) *Find the gradient of a line whose equation is **2y + 3x − 8 = 0.***

(ii) *Find the the coordinates of the point where the line in **(i)** above cuts the*

y–axis.

3. *A line whose gradient is **3** and y–intercept **–12** cuts the **x**–axis at point **P.***

(i) *State the equation of the line.*

(ii) *Find the coordinates of **P.***

4. *A line of gradient **3** passes through the point **(2, 1).** Find the:*
(i) *equation of the line*
(ii) *coordinates of point **R** where it cuts the **y–axis.***

5. *The line $2x + 3y = 10$ cuts the **x**-axis at point **D.** Find the equation of the line passing through point **D** and whose gradient is $\frac{1}{2}$*

__6.__ A line of gradient __–1__ passes through point __P(3, –2).__ Find the:

__(i)__ equation of the line

__(ii)__ coordinates of the point where the line cuts the __y__–axis

__7.__ A line of gradient $\frac{7}{9}$ passing through point __P(3, 4),__ cuts the __y–axis__ at point __Q.__ Find the coordinates of __Q__

INTERSECTION OF TWO LINES

Summary:

__1. (i)__ Point of intersection of two lines is a point where the two lines meet.

__(ii)__ When the lines __y = mx +c__ and __y = nx +d__ intersect, they have the same __x__ and __y__ values at that point. Thus __mx +c = nx +d (__the two equations are equal__)__

__(iii)__ The intersection point can also be obtained graphical ly

EXAMPLES:

__1.__ Find the coordinates of the point of intersection of the lines __y + 2x + 1 = 0__ and __x – 5y = 16__

__2.__ Find the coordinates of the point of intersection of the lines __y = 2x + 3__ and $y = 8 - \frac{1}{2}x$

__4.__ Find the coordinates of the point of intersection of the line __y + 2x + 1 = 0__ and the line joining the points __A(3, 4)__ and __B(8, 5).__

__5.__ The lines __5y + mx = 17__ and __ny – x = 6__ intersect at point __P(–4, 1).__ Find the values of __m__ and __n__

*7. Find the equation of the line that passes through the point **(4, 3)** and the point of intersection of the lines **$y + 2x = 8$** and **$2y - x = 6$***

*8. Find the equation of a line with gradient $-\frac{1}{2}$ and passing through the point of intersection of the lines **$y + 2x = 10$** and **$3y - x - 2 = 0$***

*9. Find the coordinates of the point of intersection of line **$y - 5x + 9 = 0$** and the curve $y = x^2 - 3$*

EER:

*1. Find the points of intersection of the line **$y = mx + c$** with the axes*

*2. Find the equation of the line whose y–intercept is **– 13** and passes through the point of intersection of the lines **$y + x = 5$** and **$x - y = 1$***

*3. The lines **$ax + 2y = 3$** and **$ax - by = 6$** intersect at point **P(1, 2).** Find the values of **a** and **b***

GRAPHING LINEAR EQUATIONS

Summary:

***(i)** In graphing a line using its equation, assume two values of **x** and work out their **y** values*

***(ii)** Plot the two points and draw a line that connects them*

EXAMPLES:

1. (i)** Draw a line whose equation is **$y = 3 - x$** for values of **x** from **–3** to **3

(ii) *Find the coordinates of the intercepts of the line*

(iii) *Calculate the area enclosed between the line and the coordinate axes*

2. *On different axes, draw the lines with the following equations:*

(i) $y = 2x + 1$ ***(ii)*** $y = \frac{4}{3}x + 4$ ***(iii)*** $2x - 3y - 12 = 0$ ***(vi)*** $x - y = 0$

(v) $x + y = 0$ ***(vi)*** $y = 3$ ***(vii)*** $y = -2$ ***(viii)*** $x = 4$ ***(ix)*** $x = -3$

Soln:

(ii) Hint: (i) *since* $y = \frac{4}{3}x + 4$, *use two value of* x *that are divisible by* **3** *to get the points to be plotted*

X	0	3	6
Y	4	8	12

3. (i) *On the same axes, draw the lines with equations* $y = 2x + 3$ *and* $y = 8 - \frac{1}{2}x$

(ii) *Find the coordinates of the point of intersection of the two lines*

4. (i) *On the same axes, draw the lines with equations* $y = x - 2$, $y + x = 14$ *and* $y = 7x - 26$

(ii) *Find the coordinates of the vertices of the region enclosed by the three lines*

(iii) *Calculate the area enclosed between the three lines*

OTHER FORMS OF LINEAR EQUATIONS

Summary:

1. (i) *All points on a horizontal line have the same* ***y****–coordinate*

(ii) y = 0 *is the equation of the* ***x****–axis*

(iii) y = b *is the equation of a horizontal line whose* ***y****–intercept is* ***b***

2. (i) *All points on a vertical line have the same* ***x****–coordinate*

(ii) x = 0 *is the equation of the* ***y****–axis*

(iii) x = a *is the equation of a vertical line whose* ***x****–intercept is* ***a***

EXAMPLES:

1. *On the same axes, draw the graphs of the equations:*

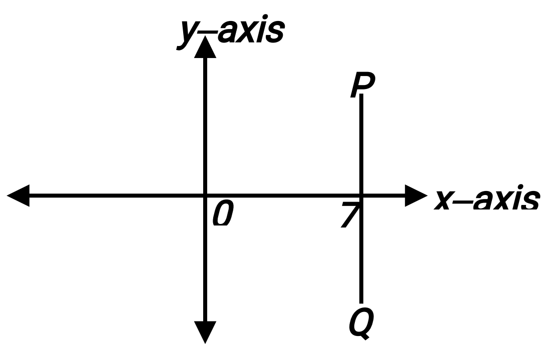
(i) x = 2   (ii) x = 5   (iii) x = –2 (iv) x = –4

2. *On the same axes, draw the graphs of the equations:*

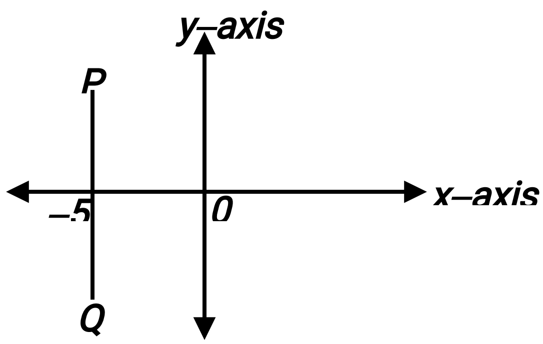
(i) y = 2   (ii) y = 5   (iii) y = –2 (iv) y = –4

3. *Write down the equation of each of the following lines* ***PQ*** *on the graphs below:*

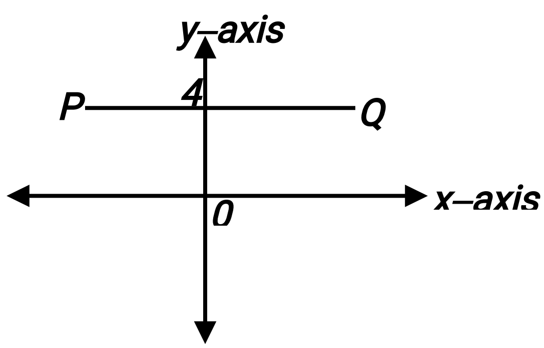
(i)



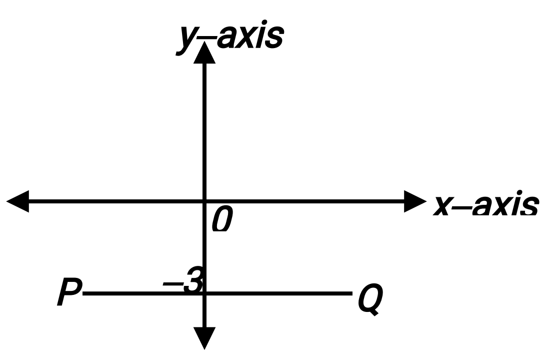
(ii)



(iii)



(iv)



4. *Write down the equation of a horizontal line passing through* ***(2, –3)***

Soln:

Hint: *The equation of the* ***x****–axis is* ***y = 0***

∴ *Required equation is* ***y = –3***

5. *Write down the equation of a vertical line passing through* ***(4, 6)***

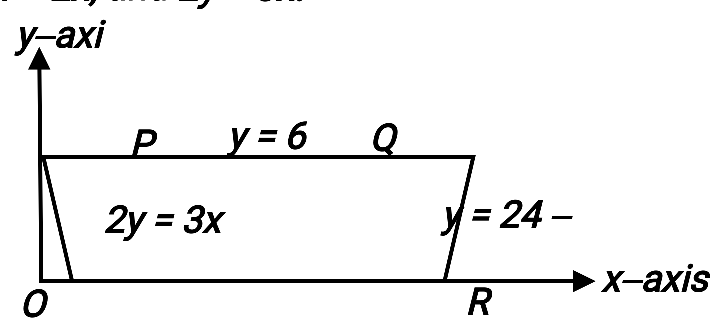
Soln:

Hint: *The equation of the* ***y****–axis is* ***x = 0***

$\therefore$ *Required equation is* ***x = 4***

6. *Find the coordinates of the point of intersection of the lines* ***y = 3*** *and* ***y = x – 1***

7. *In the figure below,* ***OPQR*** *is a trapezium formed by the* ***x****–axis, the lines* ***y = 6,*** ***y = 24– 2x,*** *and* ***2y = 3x.***



Find the:

(i) *coordinates of* ***P, Q*** *and* ***R***

(ii) *area of the trapezium*

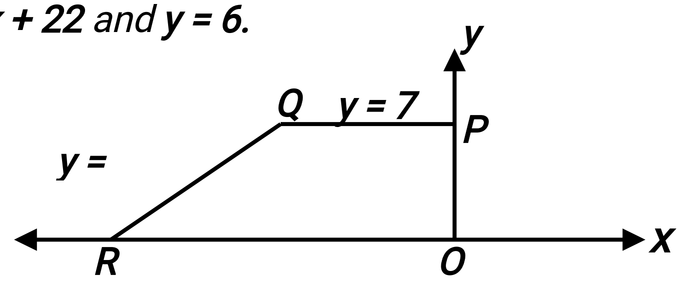
EER:

1. **(i)** Draw a line whose equation is $3y = 4x + 12$

(ii) Find the coordinates of the intercepts of the line

(iii) Calculate the area enclosed between the line and the coordinate axes

2. In the figure below, **OPQR** is a trapezium formed by the **x**–axis, **y**–axis, the lines $y = 3x + 22$ and $y = 6$.



Find the:

(i) coordinates of **Q** and **R**

(ii) area of the trapezium

3. Two lines with gradients **0** and **3** each pass through point **P(8, 4)**. Find the:

(i) equation of each line

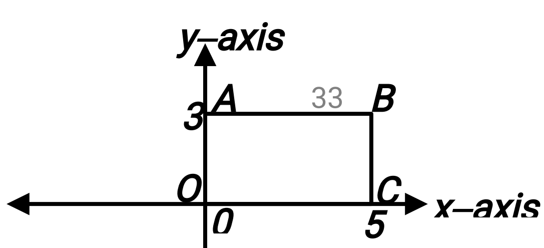
(ii) area enclosed between the two lines and the coordinate axes

4. **(i)** On the same axes, draw the lines with equations $y = 2x - 3$ and $y = -x - 3$

(ii) Find the coordinates of the point of intersection of the two lines

(iii) Calculate the area enclosed between the two lines and the **x**–axis

5. Write down the equations of the lines forming rectangle **OABC** shown below:

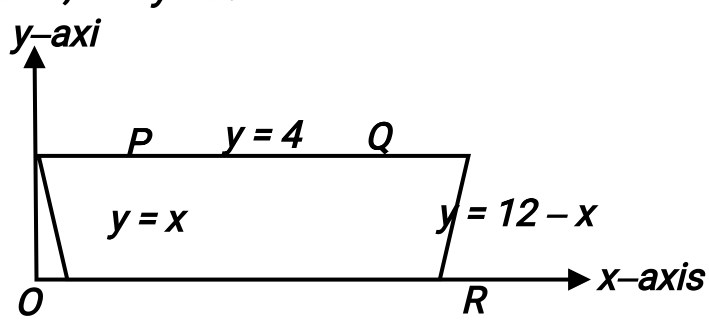


6. (i) *On the same axes, draw the lines with equations* $y = 3$ *and* $y = x - 1$

(ii) *Find the coordinates of the point of intersection of the two lines*

(iii) *Calculate the area enclosed between the two lines and the* **y**–*axis*

7. *In the figure below,* **OPQR** *is a trapezium formed by the* **x**–*axis, the lines* $y = 4$, $y = 12 - x$, *and* $y = x$.



Find the:

(i) *coordinates of* **P, Q** *and* **R**

(ii) *area of the trapezium*

8. (i) *Use a graphical method to find the coordinates of the point of intersection of the lines* $y = 2x - 3$ *and* $y = -x - 3$

(ii) *Calculate the area enclosed between the two lines and the* **x**–*axis*

PARALLEL LINES

Summary:

*Parallel lines have the same gradient. Thus if the lines **y = mx +c** and **y = nx +d** are parallel, then **m = n.***

EXAMPLES:

***1.** Find the gradient of the line that is parallel to the line with gradient* $-\frac{2}{3}$

2.** Find the gradient of a line that is parallel to the line whose equation is **3y + 2x = 0.

***3.** Find the value of **k** for which the lines with gradients* $\frac{k}{4}$ *and* $\frac{k+5}{6}$ *are parallel to each other*

4.** Find the equation of the line that is parallel to the line **3y – 2x = 6** and whose **y**–intercept is **–5

5.** Find the equation of the line that is parallel to the line **2x + y = 6** and passes through the point **(1, 10)

***6.** Find the equation of the line that is parallel to the line* $\frac{x}{4} + \frac{y}{3} = 1$ *and passes through the origin*

7.** Find the equation of the line passing through **(2, –4)** and parallel to the line joining the points **(2, 3)** and **(–4, 5)

8. *Find the equation of the line that is parallel to the line* ***12x – 3y = 2*** *and passes through the point of intersection of the lines* ***y + x = 5*** *and* ***x – y = 1***

9. *A line passing through the points* ***A(3, k)*** *and* ***B(2,7)*** *is parallel to the line through the points* ***P(–1, 4)*** *and* ***Q(0, 6).*** *Find the value of* ***k***

10. *A line passing through the points* ***A(–1, 3k)*** *and* ***B(k,3)*** *is parallel to the line whose equation is* ***2y – 3x = 9.*** *Find the coordinates of* ***A*** *and* ***B***

11. *Show that the line passing through the points* ***A(6,4)*** *and* ***B(7, 11)*** *is parallel to the line through the points* ***P(0,0)*** *and* ***Q(2, 14).***

Soln:

Gradient of ***AB =*** $\frac{11-4}{7-6} = 7$

Gradient of ***PQ =*** $\frac{14-0}{2-0} = 7$

Since Gradient of ***AB =*** *Gradient of* ***PQ,*** *then the lines are parallel*

EER:

1. *Find the equation of the line that is parallel to the line* ***2y + x = 3*** *and passes through the point* ***(2, 3)***

2. *Find the equation of the line passing through* ***(–2, 4)*** *and parallel to the line joining the points* ***(2, –5)*** *and* ***(4, 1)***

2. *Find the equation of the line that is parallel to the line* ***3y + 2x = 1*** *and passes through the midpoint of the line joining* ***(–2, 8)*** *and* ***(–4, 6)***

9. *Find the equation of a line parallel to the* ***x****–axis and passing through the* *point of intersection of the lines* $y + 2x = 10$ *and* $3y - x - 2 = 0$.

19. *Line* l_1 *passes through point* **P(–4, 1)** *and is parallel to the line joining the*

points **A(1, 3)** *and* **B(–1, 5)**. *Find the:*

(i) *equation of line* l_1

(ii) *coordinates of the point of intersection of line* l_1 *and the line* $y = 2x - 3$

(iii) *area enclosed between the two lines in* **(ii)** *above and the* **x**–*axis*

3. *Line* l_1 *passes through point* **P(–2, –5)** *and is parallel to the line joining the points* **A(6, 6)** *and* **B(–6, –10)**. *Find the:*

(i) *equation of line* l_1

(ii) *coordinates of the point of intersection* **Q** *of line* l_1 *and the line* $x - 2y + 2 = 0$

(iii) *length of the line segment* **PQ**

PERPENDICULAR LINES

Summary:

Two lines are perpendicular if the product of their gradients is equal to -1. Thus if the lines ***y = mx +c*** *and* ***y = nx +d*** *are perpendicular, then* ***mn = –1.***

EXAMPLES:

1. *Find the value of* ***k*** *for which the lines with gradients $\frac{3}{4}$ and $\frac{8}{k-6}$ are perpendicular to each other*

2. *Find the value of* ***m*** *for which the lines* ***3x + my + 7 = 0*** *and* ***9x – 2y + 5 = 0*** *are perpendicular to each other*

3. *Find the gradient of the line that is perpendicular to the line whose equation is* ***6x + 9y + 4 = 0***

4. *Find the equation of the line that is perpendicular to the line* ***2y + 3x = 6*** *and whose* ***y****–intercept is* ***–4***

5. *Find the equation of the line that is perpendicular to the line* ***6y – 2x = 7*** *and passes through* ***(1, 2)***

6. *Find the equation of the line that is perpendicular to the line $\frac{x}{3} + \frac{y}{2} = 1$ and passes through the origin*

7. *Find the equation of the line passing through* ***(2, 0)*** *and perpendicular to the line joining the points* ***A(4, 9)*** *and* ***B(1, 3)***

8. *Find the equation of the line that is perpendicular to the line* ***12y + 4x = 9*** *and passes through the point of intersection of the lines* ***y = 2x – 5*** *and* ***x + y = 1***

9. *A line passing through the points* ***A(2, k)*** *and* ***B(4,6)*** *is perpendicular to the line through the points* ***P(–1, 7)*** *and* ***Q(2, 1).*** *Find the value of* ***k***

10. *Two perpendicular lines intersect at* ***(3, 5).*** *If one of the lines passes through* ***(2, 3),*** *find the equation of the other line*

11. *Two perpendicular lines intersect at* ***(–2, 0).*** *If one of the lines is* ***15y + mx = 6,*** *find the:*

(i) *value of* ***m***

(ii) *equation of the other line*

12. *The line* ***y = 3x – 4*** *is perpendicular to the line passing through* ***(6, 4).*** *Find the coordinates of their point of intersection*

13. *Show that the line passing through the points* ***A(6, 0)*** *and* ***B(0, 12)*** *is perpendicular to the line through the points* ***P(8, 10)*** *and* ***Q(4, 8).***

Soln:

Gradient of ***AB =*** $\frac{12-0}{0-6} = -2$

Gradient of ***PQ =*** $\frac{8-10}{4-8} = \frac{1}{2}$

Gradient of ***AB*** × *Gradient of* ***PQ =*** $-2 \times \frac{1}{2} = -1$

Hence the lines are perpendicular

14. *A line* l_1 *passes through the point of intersection of the lines* ***x – y – 2 = 0*** *and* ***3x + 4y + 15 = 0*** *and is perpendicular to the line passing through the points* ***P(2, 3)*** *and* ***Q(1, 1).*** *Find the equation of line* l_1

15. (i) *Show that the points* ***A(–2, –4), B(2, –1)*** *and* ***C(5, –5)*** *are vertices of a right triangle*

(ii) *Find the area of the triangle*

Soln:

Hint: *This could be done using gradient method* ***or*** *Pythagoras theorem*

Gradient of ***AB =*** $\frac{-1-{}^{-}4}{2-{}^{-}2} = \frac{3}{4}$

Gradient of ***BC =*** $\frac{-5-{}^{-}1}{5-2} = -\frac{4}{3}$

*Since $\frac{3}{4} \times -\frac{4}{3} = -1$, then **AB** is perpendicular **BC.** Thus the triangle is right angled since $\angle B = 90°$*

EER:

***1.** Find the gradient of the line that is perpendicular to the line with gradient $-\frac{2}{3}$*

2.** Find the gradient of the line that is perpendicular to the line whose equation is **3y + 2x = 6

3.** Find the equation of the line that is perpendicular to the line **2y – 4x = 7** and whose **y**–intercept is **5

4.** Find the equation of the line perpendicular to the line **2y + 3x = 7** and passing through **(1, 3)

5.** The line joining the points **A(k, 2)** and **B(–4, 4)** is perpendicular to the line **2y + 3x – 8 = 0.** Find the value of **k

6.** Find the gradient of the line that is perpendicular to the line joining the points **(a, 3a)** and **(2a, –a)

7.** Find the equation of the line perpendicular to the line **2y – 4x = 7** and passing through **(1, 2)

8.** Find the equation of the line that is perpendicular to the line $\frac{x}{4} + \frac{y}{3} = 1$ and passes through **(4, –5)

***9.** State with a reason whether the lines **3y – 4x + 10 = 0** and **12y + 9x – 8 = 0** are parallel **or** perpendicular*

***10.** State with a reason whether the lines **2y + 8x + 1 = 0** and **3y + 12x – 7 = 0** are parallel **or** perpendicular*

11.** Find the equation of the line that is perpendicular to the line **6y + 9x – 7 = 0** and passes through **(–3, 2)

*12. Find the equation of the line that is perpendicular to the line **$3x - y - 6 = 0$** and passes through **(6, 5)***

*13. Find the equation of the line that is perpendicular to the line **$2y + x = 5$** and passes through **(3, –2)***

*14. Line l_1 is perpendicular to the line **$2y + 8x - 10 = 0$** and intersect at point **P(–3, k)**. Find the:*

(i)** value of **k

***(ii)** equation of line l_1*

*15. Line l_1 is perpendicular to the line **$3y + 2x = 7$** and passes through the points **A(2, 2)** and **B(4, k)**. Find the:*

(i)** value of **k

***(ii)** equation of line l_1*

***(i)** coordinates of the point where lines l_1 cuts the **x**–axis*

*16. The end points of a line segment are **P(1, 6)** and **Q(7, –2)**. Find the:*

(i)** coordinates of **M,** the midpoint of **PQ

(ii)** gradient of the line perpendicular to line segment **PQ

(iii)** equation of the line through **M** and perpendicular to line segment **PQ

*17. A line l_1 passes through the points **A(2, 1)** and **B(6, 5)**. Another line l_2 is perpendicular to line l_1 and passes through point **P(–1, 8)**. Find the:*

***(i)** equation of line l_1 and l_2*

***(ii)** coordinates of the point of intersection of the lines l_1 and l_2*

*18. Line l_1 is parallel to the line **$2y - 6x = 8$** and passes through point **P(4,***

5). *Another line l_2 is perpendicular to the line* ***5y + x – 10 = 0*** *and passes through point* ***Q(0, –9).*** *Find the:*

(i) *equation of line l_1 and l_2*

(ii) *coordinates of the point of intersection of the lines l_1 and l_2*

(iii) *coordinates of the point of intersection of line l_2 and the curve*

$y = x^2 - 3$

19. *Line l_1 is perpendicular to the line* ***2x + 5y – 15 = 0*** *and passes through point* ***P(6, 10).*** *Find the coordinates of the point of intersection of line l_1 and the line* ***y – x = 1***

20. *The line* ***y = x + 1*** *is perpendicular to the line passing through* ***(3, 0).*** *Find the coordinates of their point of intersection*

21. (i) *Show that the points* ***A(–4, –2), B(4, 2)*** *and* ***C(2, 6)*** *are vertices of a right triangle*

(ii) *Find the area of the triangle*

22. *Show that the points* ***A(4, –1), B(5, 6)*** *and* ***C(1, 3)*** *are vertices of an isosceles right triangle.*

24. (i) *The vertices of a triangle are* ***A(0, 6), B(–6, 2)*** *and* ***C(2, –10).*** *Show that angle* ***ABC*** *is* ***90°***

(ii) *Find the area of the triangle*

A PERPENDICULAR BISECTOR

Summary:

A perpendicular bisector is a line that cuts the line segment at its midpoint and at right angles

EXAMPLES:

1. *Find the equation of the perpendicular bisector of the line segment with endpoints* ***P(6, 4)*** *and* ***Q(2, 8).***

2. *Find the equation of the perpendicular bisector of the line segment with endpoints* ***P(0, 4)*** *and* ***Q(6, 0).***

3. *A perpendicular bisector of the line segment with endpoints* ***P(–7, 3)*** *and* ***Q(–3, –1)*** *cuts the* ***x–axis*** *and* ***y–axis*** *at points* ***P*** *and* ***Q*** *respectively. Find the coordinates of* ***P*** *and* ***Q***

4. *Show that point* ***P(7, 1)*** *lies on a perpendicular bisector of the line segment with endpoints* ***A(4, 6)*** *and* ***B(2, 4)***

EER:

1. *Find the equation of the perpendicular bisector of the line segment with endpoints* ***P(–7, 3)*** *and* ***Q(–3, –1)***

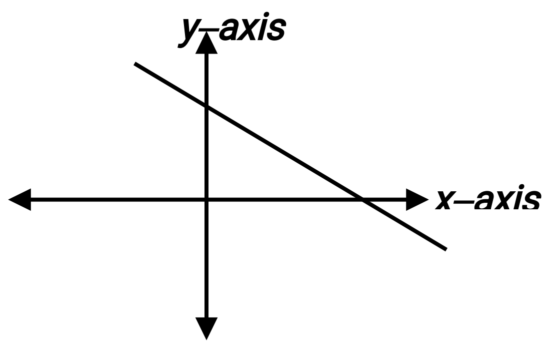
2. *A line segment has endpoints* ***P(2, 3)*** *and* ***Q(1, 1).*** *Find the equation of its perpendicular bisector*

3. *A perpendicular bisector of the line segment with endpoints* ***P(–7, 3)*** *and* ***Q(–3, –1)*** *cuts the* ***x–axis*** *and* ***y–axis*** *at points* ***P*** *and* ***Q*** *respectively. Find the coordinates of* ***P*** *and* ***Q***

4. *Find the equation of the perpendicular bisector of the line segment with* *endpoints **P(1, 2)** and **Q(3, 6).***

5.** Find the equation of the perpendicular bisector of the line segment with endpoints **P(4, 6)** and **Q(12, 10)

6.** Show that point **P(–4, 1)** lies on a perpendicular bisector of the line segment with endpoints **A(1, 2)** and **B(3, 6)



NUMBER BASES

Summary:

1. *Number bases are different ways of writing down numbers.*

2. *The most common base system is base **10.***

3. *The digits of a number in any base are less than the base itself*

4. *The digits **10** and **11** are represented by **t** and **e** respectively in number bases*

NOTE:

(i) *Base 10 is called **decimal base***

(ii) *Base 2 is called **binary base***

(iii) *Base 3 is called **trinary base***

(iv) *Base 8 is called **octal base***

EXAMPLES:

1. *Convert the following to base ten*

(i) 1011_{two} (ii) 346_{seven} (iii) 2210_{three}

(iv) $2et_{twelve}$ (v) $312 \cdot 21_{four}$ (vi) $0 \cdot 12_{six}$

solution

(i) $$1011_{two} = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$= (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1)$$

$$= 11_{ten}$$

(iv) $312 \cdot 21_{four} = (3 \times 4^2) + (1 \times 4^1) + (2 \times 4^0) + (2 \times 4^{-1}) + (1 \times 4^{-2})$

$$= (3 \times 16) + (1 \times 4) + (2 \times 1) + (2 \times \frac{1}{4}) + (2 \times \frac{1}{16})$$

$$= 54 + \frac{1}{2} + \frac{1}{16}$$

$$= 54 \frac{9}{16}_{ten} \quad \text{or} \quad 54 \cdot 5625_{ten}$$

CONVERTING FROM BASE TEN TO OTHER BASES

Summary:

(i) *Divide the number repeatedly by the required bases*

(ii) *The remainder in reverse order gives the required number*

EXAMPLES:

1. *Convert* 64_{ten} *to base three*

3	64	R
3	21	1
3	7	0
	2	1

$\therefore \; 64_{ten} = 2101_{three}$

2. *Convert* 246_{ten} *to base five*

3. *Convert* 2101_{three} *to base seven*

Hint: *First convert* 2101_{three} *to base ten*

$$2101_{three} = (2 \times 3^3) + (1 \times 3^2) + (0 \times 3^1) + (1 \times 3^0)$$
$$= 64_{ten}$$

7	64	R
7	9	1
	1	2

$$\therefore \ 2101_{three} = 121_{seven}$$

5. *Find the value of* **n** *in the following equations:*

(i) $45_n = 1112_{three}$ (ii) $21_n = 19_{ten}$ (iii) $303_n = 410_{six}$

(iv) $202_n = 37_{nine}$ (v) $112_n + 304_n = 421_n$

OPERATIONS WITH ANY BASE OTHER THAN 10

ADDITION:

If the sum of the digits exceeds the base, divide that sum by the base then write down the remainder and carry the whole number.

EXAMPLES:

1. *Workout the following leaving your answer in the base indicated*

(i) $136_{seven} + 254_{seven}$

(ii) $232_{five} + 344_{five}$

(iii) $28 \cdot 57_{nine} + 6 \cdot 34_{nine}$

Solution:

(i)
$$\begin{array}{r} 136_{seven} \\ +\,254_{seven} \\ \hline 423_{seven} \end{array}$$

(ii)
$$\begin{array}{r} 232_{five} \\ +\,344_{five} \\ \hline 1131_{five} \end{array}$$

(iii)
$$\begin{array}{r} 28 \cdot 57_{nine} \\ +\;\; 6 \cdot 34_{nine} \\ \hline 36 \cdot 02_{nine} \end{array}$$

2. Workout $122_{three} + 461_{seven}$ giving your answer in base five

Hint: First convert 122_{three} and 461_{seven} to base ten and then finally express the answer in the required base

$$122_{three} = (1 \times 3^2) + (2 \times 3^1) + (2 \times 3^0) = 17_{ten}$$

$461_{seven} = (4 \times 7^2) + (6 \times 7^1) + (1 \times 7^0) = 239_{ten}$

$\Rightarrow \quad 122_{three} + 461_{seven} = 17_{ten} + 239_{ten} = 256_{ten}$

5	256	R
5	51	1
5	10	1
	2	0

$\therefore \quad 122_{three} + 461_{seven} = 2011_{five}$

SUBTRACTION:

In case of borrowing the new value is the sum of the base and the digit which was small.

EXAMPLES:

1. Workout the following leaving your answer in the base indicated

(i) $72_{eight} - 43_{eight}$

(ii) $254_{eight} - 217_{eight}$

(iii) $30 \cdot 241_{five} - 14 \cdot 143_{five}$

Solution:

(i)
$$\begin{array}{r} 72_{eight} \\ -\,43_{eight} \\ \hline 27_{eight} \end{array}$$

(ii) $\begin{array}{r} 254_{\,eight} \\ -\,217_{\,eight} \\ \hline 35_{\,eight} \end{array}$

(iii) $\begin{array}{r} 30\cdot 241_{\,five} \\ +\;14\cdot 143_{\,five} \\ \hline 14\cdot 043_{\,five} \end{array}$

2. Workout $221_{three} - 101_{two}$ giving your answer in base four

Hint: First convert 221_{three} and 101_{two} to base ten and then finally express the answer in the required base

$$221_{three} = (2 \times 3^2) + (2 \times 3^1) + (1 \times 3^0) = 25_{ten}$$

$$101_{two} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 5_{ten}$$

$$\Rightarrow 221_{three} - 101_{two} = 25_{ten} - 5_{ten} = 20_{ten}$$

	20	R
4		
4	4	0
	1	1

$$\therefore \ 221_{\text{three}} - 101_{\text{two}} = 110_{\text{four}}$$

MULTIPLICATION AND DIVISION

EXAMPLES:

1. *Workout the following leaving your answer in the base indicated*

(i) $152_{\text{eight}} \times 43_{\text{eight}}$

(ii) $et5_{\text{twelve}} \times 8t_{\text{twelve}}$

(iii) $124_{\text{five}} \times 32_{\text{five}}$

Solution:

(i)
$$\begin{array}{r} 152_{\text{eight}} \\ \times\ 43_{\text{eight}} \\ \hline 476 \\ +650 \\ \hline 7176_{\text{eight}} \end{array}$$

(ii)
$$\begin{array}{r} et5_{\text{twelve}} \\ \times\ 8t_{\text{twelve}} \\ \hline 9t82 \\ +7te4 \\ \hline 88t02_{\text{twelve}} \end{array}$$

(iii) $\quad 124_{five}$

$$\begin{array}{r} 124_{five} \\ \times\ 32_{five} \\ \hline 303 \\ +\,432 \\ \hline 10123_{five} \end{array}$$

2. *Workout* $1011_{two} \times 12_{three}$ *giving your answer in binary base*

Hint: *First convert* 1011_{two} *and* 12_{three} *to base ten and then finally express the answer in the required base*

$$1011_{two} = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = \mathbf{11}_{ten}$$

$$12_{three} = (1 \times 3^1) + (2 \times 3^0) = \mathbf{5}_{ten}$$

$$\Rightarrow \quad 1011_{two} \times 12_{three} = 11_{ten} \times 5_{ten} = \mathbf{55}_{ten}$$

2	55	R

2	27	1
2	13	1
	6	1
2		
2	3	0
	1	1

$$\therefore \quad 1011_{two} \times 12_{three} = 110111_{two}$$

3. *Workout the following leaving your answer in the base indicated*

(i) $2001_{three} \div 12_{three}$

(ii) $110111_{two} \div 101_{two}$

Solution:

(i) Hint: *First convert* 2001_{three} *and* 12_{three} *to base ten and then finally express the answer in the required base*

$$2001_{three} = (2 \times 3^3) + (0 \times 3^2) + (0 \times 3^1) + (1 \times 3^0) = \mathbf{55}_{ten}$$

$$12_{three} = (1 \times 3^1) + (2 \times 3^0) = \mathbf{5}_{ten}$$

$$\Rightarrow \quad 2001_{three} \div 12_{three} = 55_{ten} \div 5_{ten} = 11_{ten}$$

	11	R
3		
3	3	2
	1	0

$$\therefore \quad 2001_{three} \div 12_{three} = 102_{three}$$

(ii) $110111_{two} = (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= 55_{ten}$

$101_{two} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 5_{ten}$

$$\Rightarrow \quad 110111_{two} \div 101_{two} = 55_{ten} \div 5_{ten} = 11_{ten}$$

2	11	R
	5	1
2		
2	2	1
	1	0

$$\therefore \quad 110111_{two} \div 101_{two} = 1011_{two}$$

EER:

1. *Convert the following to base ten*

(i) 2212_{three} (ii) 1011_{two} (iii) 234_{five}

2. *Express* $0 \cdot 24_{six}$ *as a fraction in base ten*

3. *Express* $45 \cdot 3_{six}$ *in base ten using point notation*

4. *Find the value of* ***n*** *if* $45_{n} = 100001_{two}$

5. *Find the value of* ***n*** *if* $103_{n} + 26_{n} = 131_{n}$

6. *Convert* 102_{three} *to binary base*

7. *Workout the following leaving your answer in the base indicated*

(i) $152_{eight} \times 43_{eight}$

(ii) $et5_{twelve} \times 8t_{twelve}$

(iii) $2212_{three} \div 21_{three}$

8. *Arrange the following numbers* 36_{eight}, 302_{four} *and* 202_{three} *in ascending order*

PROFIT AND LOSS

Summary:

1. (i) *Cost price is the price at which an item is purchased*

(ii) *Selling price is the price at which an item is sold*

2. *Selling above the cost price is a gain. Thus* ***profit =*** *selling price − cost price*

3. *Selling below the cost price is a loss. Thus* ***loss =*** *cost price − selling price*

4. (i) Percentage profit = $\frac{Profit}{Cost\ \ price} \times 100$

(ii) Percentage loss = $\frac{Loss}{Cost\ \ price} \times 100$

4. (i) *A profit of* ***20%*** *means the selling price is* ***120%*** *of the cost price*

(ii) *A loss of* ***20%*** *means the selling price is* ***80%*** *of the cost price*

EXAMPLES:

1. *A trader bought a radio at* ***Shs 16,000*** *and sold it at* ***Shs 20,000.*** *Find his:* ***(i)*** *profits*

(ii) *percentage profit*

Soln:

(i) *profits = SP− CP*

(ii) *percentage profit =* $\frac{Profit}{Cost\ \ price} \times 100$

2. *A book is bought at* ***Shs 1,500*** *and sold at a profit of* ***40%.*** *Find the selling price* ***Soln:***

SP = $\frac{140}{100} \times CP$

3. *A trader sold an item at* ***Shs 54,000*** *and made a profit of* ***20%.*** *Find the cost price*

Soln:

SP = $\frac{120}{100} \times CP$

4. *A trader sold an item at a profit of* ***5%.*** *If his profit is* ***Shs 1500,*** *find his:*

(i) *cost price*

(ii) *selling price*

Soln:

(i) *percentage profit =* $\frac{Profit}{Cost \quad price} \times 100$

$5 = \frac{1500}{CP} \times 100$

CP = 30,000

(ii) SP = *30,000 + 1,500* ***= 31,500***

OR SP = $\frac{105}{100} \times CP$

5. *The selling price of* ***12*** *eggs is equal to the cost price of* ***15*** *eggs. Find the percentage profit.*

Soln:

If 12SP = 15CP

$\Rightarrow$ ***SP =*** *1·25CP*

Percentage profit = $\frac{(1 \cdot 25CP \ - \ CP)}{CP} \times 100 \ = \ \mathbf{25}$

5. *By selling a book at* ***Shs 4,800,*** *a trader would gain* ***20%.*** *Find how much he must sell it in order to gain* ***30%.***

Soln:

Soln:

SP = $\frac{120}{100} \times CP$

4,800 = $\frac{120}{100} \times CP$

CP = 4,000

SP = $\frac{130}{100} \times CP$

SP = $\frac{130}{100} \times 4{,}000$

SP = 5,200

6. *A trader bought a car at* ***Shs 6 million*** *and sold it at* ***Shs 4·8 million.*** *Find his:* ***(i)*** *loss*

(ii) *percentage loss*

Soln:

(i) *loss = CP– SP*

(ii) *percentage loss =* $\frac{Loss}{Cost \quad price} \times 100$

7. *A plot of land is bought at* ***Shs 20 million*** *and sold at a loss of* ***20%.*** *Find the selling price*

Soln:

SP = $\frac{80}{100} \times CP$

8. *A trader sold an item at* ***Shs 7,000*** *and made a loss of* $12\frac{1}{2}\%$. *Find the cost price*

Soln:

SP = $\frac{87 \cdot 5}{100} \times CP$

9. *A trader sold an item at* ***Shs 3·6 million*** *and made a loss of* ***25%.*** *Find the profit he would make if he had sold it at* ***Shs 6 million***

Soln:

SP = $\frac{75}{100} \times CP$

3·6 = $\frac{75}{100} \times CP$

CP = 4·8

Profit = 6 $-$ *4·8* ***= 1·2m***

10. *By selling items* ***P*** *and* ***Q*** *each at* ***Shs 3,300,*** *a profit of* ***10%*** *and a loss* ***34%*** *were respectively made on the items. Find the percentage loss on both items*

Soln:

Item P:

SP **=** $\frac{110}{100} \times CP$

$3{,}300 = \frac{110}{100} \times CP$

CP = 3,000

Item Q:

SP **=** $\frac{66}{100} \times CP$

$3{,}300 = \frac{66}{100} \times CP$

CP = 5,000

Total CP *= 3000 + 5000 =* ***8,000***

Total SP *= 3,300 + 3,300 =* ***6,600***

Total percentage loss = $\frac{Total\ \ loss}{Total\ \ cost\ \ price} \times 100$

Total percentage loss = $\frac{(8{,}000\ -\ 6{,}600)}{8{,}000} \times 100 = 17 \cdot \mathbf{5}$

EER:

1. *A radio is bought at* ***Shs 15,000*** *and sold at* ***Shs 16,200.*** *Find the percentage profit*

2. *A trader sold an item at* ***Shs 4·8 million*** *and made a profit of* ***15%.*** *Find the cost price of the item*

3. *A trader buys an old radio at* ***Shs 65,000*** *and spends* ***Shs 15,000*** *on its repair. If he sells it at Shs* ***100,000,*** *find his percentage profit.*

4. *A carpenter incurs a loss of* ***20%*** *by selling a chair at* ***Shs 240,000.*** *Find* *the percentage he would gain by selling it at **Shs 360,000.***

5.** By selling an item at **Shs 2400,** a trader would gain **4%.** Find how much he must sell it in order to gain **12%.

***6.** The selling price of **5** eggs is equal to the cost price of **7** eggs. Find the percentage profit.*

***7.** The cost price of **17** books is equal to the selling price of **20** books. Find the percentage loss.*

8.** The selling price of **n** eggs is equal to the cost price of **27** eggs. If there is a profit of **35%,** find the value of **n.

9.** The selling price of **n** pens is equal to the cost price of **12** pens. If there is a loss of **20%,** find the value of **n.

10.** A trader incurs a loss of **15%** by selling an item at **Shs 18,700.** Find how much he must sell it in order to make a profit of **15%.

***11.** A trader sold two items at **Shs 18,000** each. On one he gained **20%** and the other he lost **20%.** Find the percentage loss on both items*

***12.** A trader bought two plates at **Shs 2,500** each. If he sells one at a profit of **5%,** then find how much he should sell the other so that he make a profit of **20%** on both plates.*

***13.** A trader bought **120** books at a rate of **Shs 2,000** per book. He sold **72** of them at a rate of **Shs 2,500** per book and the remaining at a rate of **Shs 2,000** per book. Find his percentage profit*

***14.** An item is usually sold at a profit of **80%.** If its cost price later went up by **20%** and its selling price remained the same, find by how much is the percentage profit reduced*

***15.** A car dealer buys a car at **Shs 1,250,000** and hire it for **25** weeks at a charge of **Shs 3,500** per day. Insurance costs **Shs 33,700** during the entire period, at the end of which he sells it at **Shs 750,000.** Calculate the profit that he makes on the transaction*

16. *A trader bought an item at **Shs 5,500** and sold it at **30%** more than the buying price. Find the trader's**:***
(i) *selling price*
(ii) *profit*

DISCOUNTS

Summary:

1. (i) *Discount is a reduction in the selling price of an item*

(ii) Sale price = *original price – discount*

4. (i) Percentage discount = $\frac{Discount}{Original \quad price} \times 100$

(ii) *A discount of **20%** means the sale price is **80%** of the original price*

EXAMPLES:

1. *An item costing **Shs 16,000** was sold at **Shs 12,000.** Find the**:***
(i) *cash value of the discount*

(ii) *percentage discount*

Soln:

(i) *Discount cash value = 16,000– 12,000 **= 4,000***

(ii) *percentage discount =* $\frac{discount}{Original \quad price} \times 100 = \frac{4{,}000}{16{,}000} \times 100 = \mathbf{25}$

2. *An item costing **Shs 60,000** was sold at a discount of **20% .** Find**:***

(i) *how much was paid for it*

(ii) *cash value of the discount*

Soln:

(i) *Actual cost price =* $\frac{80}{100} \times 60{,}000 = \mathbf{48{,}000}$

(ii) *Discount cash value =* $\frac{20}{100} \times 60{,}000 \;= \mathbf{12{,}000}$

3. *The cost of an item is* ***Shs 4,500*** *after a discount of* ***10%.*** *Find its original cost price*

Soln:

$\frac{90}{100} \times CP \;= 4{,}500$

$\therefore$ ***CP = 5,000***

4. *An item costing* ***Shs 50,000*** *was sold at two successive discounts of* ***20%*** *and* ***5%.*** *Find:*

(i) *how much was paid for it*

(ii) *the total percentage actual discount*

Soln:

(i) 1st *cost price =* $\frac{80}{100} \times 50{,}000 \;= \mathbf{40{,}000}$

$\therefore$ *Final cost price =* $\frac{95}{100} \times 40{,}000 \;= \mathbf{38{,}000}$

(ii) *Actual percentage discount =* $\frac{(50{,}000 \;-\; 38{,}000)}{50{,}000} \times\; 100 \;=\; \mathbf{24}$

5. *Find a single percentage discount equivalent to successive discounts of* ***20%, 10%*** *and* ***5%.***

Soln:

1st *selling price =* $\frac{80}{100} \times MP \;= \mathbf{0 \cdot 8MP}$

2nd *selling price =* $\frac{90}{100} \times 0 \cdot 8MP \;= \mathbf{0 \cdot 72MP}$

$\therefore$ *Final selling price =* $\frac{95}{100} \times 0 \cdot 72MP \ = \mathbf{0 \cdot 684MP}$

Actual percentage discount = $\frac{(MP \ - \ 0 \cdot 684MP)}{MP} \times 100 \ = \ \mathbf{31 \cdot 6}$

6. *The marked price of an item is **Shhs 240,000.** A trader sold it to his customer*

*at a discount of **10%** and still made a profit of **20%.** Find the:*

(i) *price at which the trader had bought the item*

(ii) *percentage profit the trader would have made if no discount was allowed*

Soln:

(i) *If discount is **10%***

$\Rightarrow \mathbf{SP} = \frac{90}{100} \times 240{,}000 \ = \mathbf{216{,}000}$

*If profit is **20%***

$\Rightarrow \frac{120}{100} \times \mathbf{CP} \ = \ 216{,}000$

$\therefore$ ***CP = 180,000***

(ii) *Percentage profit =* $\frac{(240{,}000 \ - \ 18{,}000)}{180{,}000} \times 100 \ = \ \mathbf{33 \cdot 33}$

7. *A trader buys a bag at **Shs 60,000** and marks it for sale at a price that gives a profit of **20%** when he allows a discount of **10%** on its purchase. Find the marked price of the bag*

Soln:

*If profit is **20%***

$\Rightarrow$ ***SP*** = $\frac{120}{100} \times 60{,}000 \;\; = \mathbf{72{,}000}$

*If discount is **10%***

$\Rightarrow \frac{90}{100} \times \mathbf{MP} \;=\; 72{,}000$

$\therefore$ ***MP = 80,000***

EER:

1. *The cost of an item is **Shs 4,800** after a discount of **20%.** Find its original cost price*

2. *A bicycle priced **Shs 200,000** was sold at a discount of **15%.** Find:*

(i) *how much was paid for it*

(ii) *cash value of the discount*

3. *Find the percentage discount allowed when an item costing **Shs 60,000** is sold at **Shs 48,000***

4. *Find a single percentage discount equivalent to successive discounts of **20%** and **15%.***

5. *An item costing **Shs 80,000** was sold at successive discounts of **10%** and **5%.** Find its final cost price after the discounts*

6. *The marked price of a radio is **Sh. 950,000**. A trader allows a discount of*

10% *but still makes a profit of **12%.** Find the:*

(i) *price at which the trader had bought the radio*

(ii) *percentage profit the trader would have made if no discount was allowed*

7. *A trader buys a bag at **Shs 40,000** and marks it for sale at a price that gives a profit of **20%** when he allows a discount of **4%** on its purchase. Find the marked price of the bag*

8. *A man bought a shirt at* ***20%*** *discount. If he paid* ***Shs 20,000,*** *find the original price of the shirt*

9. *The price of an article is* ***Shs 240,000.*** *If a discount of* ***12%*** *is given, find the selling price of the article*

HIRE PURCHASE

Summary:

1. *Hire purchase is the instalment payment for an item over time*

2. *The hire purchase price includes interest charges. Thus it is higher than the cash price of an item*

EXAMPLES:

1. *A radio whose cash price is* ***Shs 180,000*** *can also be bought on hire purchase by paying a deposit of* ***35%*** *of the cash value followed by* ***6*** *equal monthly instalments of* ***Shs 24,500.*** *Find the:*

(i) *hire purchase price of the radio*

(ii) *extra amount paid over the cash price using hire purchase*

Soln:

(i) Hire purchase price = $\left(\frac{35}{100} \times 180{,}000\right) + (6 \times 24{,}500) = \mathbf{210{,}000}$

(ii) Extra payment = $210{,}000 - 180{,}000 = \mathbf{30{,}000}$

2. *The marked price of an item is* ***Shs 640,000.*** *A* ***5%*** *discount is offered on cash purchase. It can also be bought on hire purchase by paying a deposit of* ***40%*** *followed by* ***18*** *equal monthly instalments of* ***Shs 23,000.*** *Find how much is saved by paying cash than using hire purchase*

Soln:

Cash price = $\frac{95}{100} \times 640{,}000 \quad = \mathbf{608{,}000}$

Hire purchase price = $\left(\frac{40}{100} \times 640{,}000\right) + (18 \times 23{,}000) \quad = \mathbf{670{,}000}$

$\therefore$ **Savings =** 670,000 – 608,000 **= 62,000**

3. A motor company had the following advertisement**:**

GET YOURSELF A CAR CHEAPLY	
CASH VALUE:	Shs 24·5 MILLION
CASH DISCOUNT:	8% OF THE CASH VALUE
HIRE PURCHASE:	DEPOSIT 60% OF THE CASH VALUE AND PAY 4 MILLION MONTHLY FOR 3 MONTHS

Calculate**:**

(i) how much is saved by paying cash than using hire purchase

(ii) the percentage profit the trader made on hire purchase price if he

had bought it at **20%** below the cash price

Soln:

(i) Cash price = $\frac{92}{100} \times 24 \cdot 5 = \mathbf{22 \cdot 54 \ million}$

Hire purchase price = $\left(\frac{60}{100} \times 24 \cdot 5\right) + (4 \times 3) = \mathbf{26 \cdot 7 \ million}$

$\therefore$ **Savings =** 26·7– 22·54 **= 4·16 million**

(ii) Cost price = $\frac{80}{100} \times 24 \cdot 5 = \mathbf{19 \cdot 6 \ million}$

$\therefore$ ***Percentage profit*** = $\frac{(26 \cdot 7 - 19 \cdot 6)}{19 \cdot 6} \times 100 = \mathbf{36 \cdot 2}$

4. *The cash price of a radio is* ***Shs 200,000.*** *It can also be bought on hire purchase by paying a deposit of* ***45%*** *followed by* ***5*** *equal monthly instalments. If the total hire purchase price is* ***Shs 240,000,*** *find the amount of each monthly instalment*

Soln:

Hire purchase price = deposit + total instalments

$$\left(\frac{45}{100} \times 200{,}000\right) + (5 \times \mathbf{P}) = \mathbf{240{,}000}$$

$\therefore$ ***P = 30,000***

EER:

1. *An item whose cash price is* ***Shs 60,000*** *can also be bought on hire purchase by paying a deposit of* ***Shs 20,000*** *followed by* ***4*** *equal weekly instalments of* ***Shs 13,000.*** *Find how much is saved by paying cash than using hire purchase*

2. *The marked price of a car is* ***Shs 6·4 million.*** *A* ***7·5%*** *discount is offered on cash purchase. It can also be bought on hire purchase by paying* ***4*** *equal monthly instalments of* ***Shs 1·65 million.*** *Find how much is saved by paying cash than using hire purchase*

3. *A radio whose cash price is* ***Shs 300,000*** *can also be bought on hire purchase by paying an extra charge of* ***Shs 60,000.*** *If* ***9*** *equal monthly instalments are to be made, find the amount of each monthly instalment*

4. *A printer whose cash price* ***Sh. 800,000*** *can also be bought on hire purchase by paying a deposit of* ***Sh.260,000*** *and* ***18*** *equal monthly installments . If the hire purchase price is* ***45%*** *more than the cash price, find the amount of each monthly instalment*

5. *The marked price of a car is* ***Shs 4·5 million.*** *A company bought it at a discount of* ***20%*** *for its employee, who was then to pay a deposit of* ***Shs 1·8***

million *and* ***12*** *equal monthly instalments of* ***Shs 0·27 million.*** *Find the percentage profit the company got from the employee on the car*

6. *A motor company had the following advertisement:*

GET YOURSELF A CAR CHEAPLY	
CASH VALUE:	***Shs 48·5 MILLION***
CASH DISCOUNT:	***8% OF THE CASH VALUE***
HIRE PURCHASE:	***DEPOSIT 60% OF THE CASH VALUE AND PAY 7 MILLION MONTHLY FOR 3 MONTHS***

(a) *Calculate how much Tom would save by paying cash than using hire purchase*

(b) *Tom bought the car by hire purchase and then sold it at* ***35*** *million. Find the percentage loss he made*

7. *A Smartphone company had the following advertisement:*

BUY A PHONE NOW WHILE STOCK LASTS	
CASH VALUE:	***Shs 320, 000 LESS 15% DICOUNT***
HIRE PURCHASE:	***DEPOSIT 15% OF THE CASH VALUE AND PAY EITHER Shs 82, 000 MONTHLY FOR 4MONTHS OR Shs 25, 000 WEEKLY FOR 12WEEKS***

(a) *Calculate the:*

(i) *saving made by paying cash than using monthly hire purchase.*

(ii) *saving made by using weekly hire purchase than monthly hire purchase*

(b)*If the trader had bought the phone at* ***20%*** *below cash value and sold it on monthly hire purchase, find the percentage profit he made*

COMMISSION

Summary:

1. *Commission is a reward to the sales agent based on the level of sales*

2. *Commission is usually a percentage of the value of goods sold*

EXAMPLES:

1. *A sales agent gets a commission of* ***15%*** *for selling goods. Find his commission for sales worth* ***Shs 600,000***

Soln:

Commission = $\frac{15}{100} \times 600{,}000 = \mathbf{90{,}000}$

2. *A salesman gets a commission of* ***15%*** *on the first* ***Shs 120,000*** *of his total sales and* ***20%*** *on the rest. Find his commission for sales worth* ***Shs 370,000***

Soln:

Commission = $\left(\frac{15}{100} \times 120{,}000\right) + \frac{20}{100} \times (370{,}000 - 120{,}000) = \mathbf{68{,}000}$

3. *A salesman earns a basic salary of* ***Shs 100,000*** *per month and a commission of* ***8%*** *for sales above* ***Shs 150,000.*** *In a certain month, he allowed a discount of* ***5%*** *on the sold goods worth* ***Shs 400,000.*** *Calculate his income for that month*

Soln:

Total sales *=* $\frac{85}{100} \times 400{,}000 \ = \mathbf{340{,}000}$

Earned Income *=* $100{,}000 \ + \frac{8}{100} \times (340{,}000 \ - 150{,}000) \ = \mathbf{115{,}200}$

4. *A salesman earned* ***Shs 180,000*** *as commission for selling* ***10*** *shirts at* ***Shs 25,000*** *each****,*** ***15*** *skirts at* ***Shs 10,000*** *each and* ***7*** *trousers at* ***Shs 50,000*** *each. Find his percentage rate of commission*
Soln:

Total sales *=* $(10 \times 25{,}000) + (15 \times 10{,}000) + (7 \times 50{,}000) = 750{,}000$

Percentage commission *=* $\frac{180{,}000}{750{,}000} \times 100 \ = \mathbf{24}$

5. *A salesman gets a commission of* ***8%*** *for selling goods****.*** *Find his sales when he receives* ***Shs 40,000*** *as commission*
Soln:

If required sales ***= y***

$\Rightarrow \ \frac{8}{100} \times y = \mathbf{40{,}000}$

$\therefore$ ***y = 500,000***

6. *A salesman gets a commission of* ***12%*** *on the first* ***Shs 150,000*** *of his total sales and* ***20%*** *on the rest. Find his sales when he receives* ***Shs 108,000*** *as commission*

Soln:

If required sales ***= y***

$\Rightarrow \ \left(\frac{12}{100} \times 150{,}000 \ \right) + \frac{20}{100} \times (y - 150{,}000) \ = 108{,}000$

$\therefore$ ***y = 600,000***

__7.__ A sales agent earns a salary of __Shs 200,000__ per month and a commission of __15%__ for the sales in excess of __Shs 100,000.__ In the first month, his total earning amounted to __Shs 320,000.__
__(a)__ Find his sales for that month
__(b)__ If the total sales in the second month increased by __20%,__ find the commission he received in that month.
__(c)__ If the total sales in the third month decreased by __25%,__ find his income for that month

Soln:

__(i)__ If required sales __= y__

$$\Rightarrow \ 200{,}000 + \frac{15}{100} \times (y - 100{,}000) = 320{,}000$$

$$\therefore \ \mathbf{y = 900{,}000}$$

(ii) Total sales = $\frac{120}{100} \times 900{,}000 = \mathbf{1{,}080{,}000}$

Commission = $\frac{15}{100} \times (1{,}080{,}000 - 100{,}000) = \mathbf{147{,}000}$

(iii) Total sales = $\frac{75}{100} \times 1{,}080{,}000 = \mathbf{810{,}000}$

Earned Income = $200{,}000 + \frac{15}{100} \times (810{,}000 - 100{,}000) = \mathbf{306{,}500}$

EER:

__1.__ A salesman earns a basic salary of __Shs 120,000__ per month and a commission of __8%__ of the month's total sales. In a certain month, he sold goods worth __Shs 1,350,000.__ Calculate his income for that month

__2.__ A salesman gets a commission of __4%__ on the first __Shs 800,000__ of his

*total sales and **5%** on the rest. Find his:*
(i)** commission for sales worth **Shs 1,500,000
***(ii)** sales when he receives **Shs 172,000** as commission*

***3.** A bookshop employs two salesmen Bob and Tom. Bob earns a basic salary of **Shs 18,500** per week and a commission of **2·5%** on each book sold. Tom earns a basic salary of **Shs 24,000** per week and a commission of **5%** for sales in excess of **1,800** books. In a certain year, each sold a total of **15,000** books at **Shs 4,000** each.*

***(i)** Find who earned more money and by how much (Assume **52** weeks in a year)*

***(ii)** In another year, Bob earned a total of **Shs 2,962,000.** Find the number of books he sold that year*

***4.** A salesman gets commission as follows:*

10%** on the first **Shs 100,000
12%** on the next **Shs 400,000
***15%** on the remainder*

Find his:

(i)** commission for sales worth **Shs 800,000
***(ii)** sales when he receives **Shs 52,000** as commission*

***5.** A hawker sells cups at **Shs 500** each. He sold **50** cups in the first week. In the second week he sold **20%** more than in the first week. In the third week he sold **10%** more than in the second week. Each week he receives a commission of **8%** of the price of the first **20** cups sold, and 12% for any cups sold in excess of **20.** **(a)** Express the number of cups sold in the third week as a percentage of the number sold in the first week*
***(b)** Calculate the commission he received in the third week*
***(c)** If in the fourth week the hawker received **Shs 2,000** as commission, calculate the number of cups he sold in that week*

CURRENCY EXCHANGE

Summary:

***1.** The price of one currency relative to another is called currency exchange* *rate*

2. *The rate at which the bank buys your foreign currency is called buying rate*

3. *The rate at which the bank sells you foreign currency is called selling rate*

4. *Currency conversion problems can be solved using a summary table and ratio theorem*

EXAMPLES:

1. *A radio costs **£30.** Find its cost in dollars if **£1 = $1·6.***

Soln:

Pounds	*1*	*30*
Dollars	*1·6*	*x*

$\frac{1}{1\cdot6} = \frac{30}{x}$

$\therefore$ ***x = $48***

2. *Find how many dollars are worth **Ug Shs 21,000,** if **$1 = Ug Shs 3,500.***

Soln:

Dollars	*1*	*x*
Ug Shs	*3,500*	*21,000*

$\frac{1}{3500} = \frac{x}{21{,}000}$

$\therefore$ ***x = $6***

3. *Find how many pounds are worth* ***$600,*** *if* ***$1 = Ug Shs 1,900*** *and* ***£1= Ug Shs 5,700.***

Soln:

Dollars	*1*	*600*
Ug Shs	*1,900*	*x*

Pounds	*1*	***y***
Ug Shs	*5,700*	*x*

$\frac{1}{1,900} = \frac{600}{x}$

$\therefore$ ***x = Ug Shs 1,140,000***

Also $\frac{1}{5700} = \frac{y}{1,140,000}$

$\therefore$ ***y = £200***

4. *Find how many dollars are worth* ***K Shs 9,750,*** *if* ***1K Shs = 24 Ug Shs*** *and* ***$1= Ug Shs 1,950.***

Soln:

K Shs	*1*	*9,750*
Ug Shs	*24*	*x*

Dollars	*1*	***y***
Ug Shs	*1,950*	*x*

$\frac{1}{24} = \frac{9,750}{x}$

$\therefore$ ***x = Ug Shs 234,000***

Also $\frac{1}{1,950} = \frac{y}{234,000}$

$\therefore$ ***y = $120***

__5__. Bank of Uganda buys and sells foreign currencies as follows:

	Exchange rate	
Foreign currency	***Buying***	***Selling***
*1 Euro **(€)***	*Shs 3,400*	*Shs 3,500*
*1 Pound sterling **(£)***	*Shs 4,000*	*Shs 4,200*

*A tourist arrived in Uganda with **1,300** Euros which he exchanged for Uganda shillings. During his stay he spent **Ug Shs 2,320,000** and converted the remainder in to pound sterling before he left. Calculate the amount he received on his departure*

Soln:

*The bank buys **1 Euro (€)** at **Shs 3,400***

Euros	*1*	*1,300*
Ug Shs	*3,400*	*x*

$$\frac{1}{3{,}400} = \frac{1{,}300}{x}$$

$\therefore$ ***x = Ug Shs 4,420,000***

*Remainder = 4,420,000 − 2,320,000 = **2,100,000***

*The bank sells **1 pound (£)** at **Shs 4,200***

Pound	*1*	***y***
Ug Shs	*4,200*	*2,100,000*

$$\frac{1}{4{,}200} = \frac{y}{2{,}100{,}000}$$

$\therefore$ ***y = £500***

EER:

1.** Convert **£10** to Euros, if **£1 = € 1·12

2.** A Smartphone costs **24,000** Indian rupees. Find its cost in pound sterling if **£1 = 96 rupees

***3.** A tourist exchanged **$1200** for Uganda shillings at a rate of **$1 = Ug Shs 3500.** Find the amount he received in Uganda currency*

4.** A radio costs **Ug Shs 787,500.** Find its cost in pounds sterling, if **US $1 = Ug Shs 2500** and **£25 = US $ 35.

***5.** A tourist exchanged **$900** for pound sterling at a rate of **£1 = $1·5.** Find the amount he received in pound sterling, if the bank charged him **5%** commission*

6.** A tourist exchanged **$750** for Uganda shillings. He spent **Ug Shs 1,260,000** and converted the remainder in to pound sterling before he left. Find the amount he received on his departure, if **$1 = Ug Shs 3600** and **£1 =

Ug Shs 4,800.

7. *Bank of Uganda buys and sells foreign currencies as follows:*

	Exchange rate	
Foreign currency	***Buying***	***Selling***
1 US Dollar ***($)***	*Shs 2,900*	*Shs 3,000*
1 Pound sterling ***(£)***	*Shs 4,650*	*Shs 4,700*

A tourist arrived in Uganda with ***$4,500*** *which he exchanged for Uganda shillings. During his stay he spent* ***Ug Shs 10,230,000*** *and converted the remainder in to pound sterling before he left. Calculate the amount he received on his departure*

8. *A tourist changed* ***$900*** *into Euros when the exchange rate was* ***€1 = $1·5.*** *He spent* ***€450*** *and received* ***$210*** *in exchange of the remainder. Find the exchange rate of this second transaction*

9. *A tourist had* ***£ 8,000*** *when he visited Uganda. He changed all of it to Uganda shillings (Ug Shs) at a rate of* ***Shs 3200 per £ 1*** *while in Uganda, he spent* ***Ug Shs 7,200,000*** *and later exchanged the balance to pounds at a rate of* ***Ug Shs 4000 per £1****. How much did he get.*

SIMPLE INTEREST

Summary:

1. *The amount borrowed or lent is called the principal*

2. *The reward to the lender is called interest*

3. *In solving simple interest problems, the following relations apply:*

(i) Simple interest = $\dfrac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

(ii) Amount = principal + interest

EXAMPLES:

1. *Find the simple interest on* ***Shs 25,000*** *for* ***3*** *years at a rate of* ***8%*** *per annum*

Soln:

$$\boldsymbol{I} = \frac{PRT}{100} = 2{,}5000 \times \frac{8}{100} \times 3 = \boldsymbol{6{,}000}$$

2.*Find the amount to which* ***Shs 80,000*** *accumulates in* ***9*** *months at a simple interest rate of* ***15%*** *per annum*

Soln:

If ***A =*** *P + I****,***

$$\Rightarrow A = 80{,}000 + \left(80{,}000 \times \frac{15}{100} \times \frac{9}{12}\right) = \boldsymbol{89{,}000}$$

3. *Find the simple interest on* ***Shs 100,000*** *at a rate of* ***6%*** *per annum from* ***9^{th} oct 2016*** *to* ***21^{st} Dec 2016***

Soln:

Hint: *In calculating interest on days, do not count the day when the money is lent out, but count the day when it is returned*

No of days *= 22 (in Oct) + 30 (in Nov) + 21 (in Dec) =* ***73***

$$\boldsymbol{I} = \frac{PRT}{100} = 100{,}000 \times \frac{6}{100} \times \frac{73}{365} = \boldsymbol{1{,}200}$$

4. *Find how long will it take for a sum of* ***Shs 80,000*** *to yield an interest of* ***Shs 12,000*** *at a rate of* ***5%*** *per annum simple interest*

Soln:

If $I = \frac{PRT}{100}$*,*

$$\Rightarrow 80{,}000 \times \frac{5}{100} \times T = 12{,}000$$

$\therefore$ ***T = 3 years***

5. *Find the principal that yields a simple interest of **Shs 27,000** in **9** years at a rate of **6%** per annum*

Soln:

If $I = \frac{PRT}{100}$,

$$\Rightarrow P \times \frac{6}{100} \times 9 = 27{,}000$$

$\therefore$ ***P = Shs 50,000***

6. *Find the principal that will amount to **Shs 147,200** in **6** years at a simple interest rate of **14%** per annum*

Soln:

*If **A = P + I,***

$$\Rightarrow P + \left(P \times \frac{14}{100} \times 6\right) = 147{,}200$$

$\therefore$ ***P = Shs 80,000***

7. *A sum of **Shs 12,500** amounts to **Shs 15,500** in **4** years at the rate of simple interest. Find the rate of interest per annum*

Soln:

*If **A = P + I,***

$$\Rightarrow 12{,}500 + \left(12{,}500 \times \frac{R}{100} \times 4\right) = 15{,}500$$

$\therefore$ ***R = 6%***

8. *A man borrowed* ***Shs 15·6*** *million from a bank at a simple interest rate of* ***15%*** *per annum. He has to repay the loan within* ***2*** *years in equal weekly instalments.   Calculate the:*
(i) *interest he paid to the bank*
(ii) *total amount to be paid*
(iii) *amount he paid per week*

Soln:

(i) $I = \frac{PRT}{100} = 15 \cdot 6 \times \frac{15}{100} \times 2 = \mathbf{4 \cdot 68 \ million}$

(ii) A = P + I = *15·6 + 4·68* ***= 20·28 million***

(iii) *Weekly payment* ***=*** $\frac{Amount}{No\ of\ weeks\ in\ 2\ years} = \frac{20 \cdot 28}{2 \times 52} = \mathbf{0 \cdot 195\ million}$

EER:

1. *Find the simple interest on* ***Shs 96,000*** *for* ***10*** *months at a rate of* $8\frac{1}{3}\%$ *per annum*

2. *A sum of* ***Shs 25,000*** *is invested for* ***3*** *years at a simple interest rate of* ***6%*** *per annum. Find the interest and amount received in that period*

3.*Find the amount to which* ***Shs 60,000*** *accumulates in* ***4*** *years at a simple interest rate of* ***15%*** *per annum*

4. *Tom deposited* ***Shs 8,000*** *which amounted to* ***Shs 9,200*** *after* ***3*** *years at simple interest. Find the rate of interest per annum*

5. *Tom deposited* ***Shs 40,000*** *in a bank which offers a simple interest rate of* ***5%*** *per annum. Find how much interest he earned after* ***8*** *years*

6. *A sum of money lent out at simple interest amounts to* ***Shs 84,000*** *in* ***10*** *years at a rate of* ***3%*** *per annum. Find the sum lent out*

7. *A bank lent out* ***Shs 500,000*** *to Tom for* ***2*** *years and* ***Shs 300,000*** *to Bob* *for **4** years at the same rate of simple interest. If the total interest from both men amounted to **Shs 220,000,** find the rate of interest per annum*

8. *Tom and Bob borrowed **Shs 30,000** and **Shs 35,000** respectively at the same rate of simple interest for **3** years. If Bob paid **Shs 1,500** more interest than Tom, find the rate of interest per annum*

9. *Tom wants to buy a car which is priced at **Shs 6 million**. He deposits **Shs 2·4 million** and **15** months later his required to pay the rest on which his charged a simple interest rate of **20%** per annum. Find the total amount of money Tom will have to pay for the car*

10. *A sum of money lent out at simple interest amounts to **Shs 62,000** after **4** years and to **Shs 77,000** after **9** years. Find the sum lent out and the rate of interest per annum*

COMPOUND INTEREST

Summary:

1. *Compound interest is the interest earned on both the principal and on the interest previously earned*

2. *In solving compound interest problems, the following relations apply:*

(i) $A = P\left(1 + \frac{r}{100}\right)^n$

(ii) Compound Interest = Amount − principal

(iii) The principal for the next period *= Interest earned + principal*

EXAMPLES:

1. *Find the amount and interest received on* ***Shs 600,000*** *invested for* ***2*** *years at a compound interest rate of* ***15%*** *per annum.*

Soln:

If $A = P\left(1+\frac{r}{100}\right)^n$

$\Rightarrow A = 600{,}000 \times \left(1+\frac{15}{100}\right)^2 = \mathbf{793{,}500}$

If $C \cdot I = A - P$

$\Rightarrow C \cdot I = 793{,}500 - 600{,}000 = \mathbf{193{,}500}$

METHOD 2: (STEP BY STEP METHOD)

First year amount $A = \frac{115}{100} \times 600{,}000 = \mathbf{690{,}0000}$

Second year Amount $A = \frac{115}{100} \times 690{,}000 = \mathbf{793{,}5000}$

If $C \cdot I = A - P$

$\Rightarrow C \cdot I = 793{,}500 - 600{,}000 = \mathbf{193{,}500}$

METHOD 3: (STEP BY STEP METHOD)

First year interest $I = \frac{PRT}{100} = 600{,}000 \times \frac{15}{100} \times 1 = \mathbf{90{,}0000}$

First year amount ***A*** *= 600,000 + 90,000 = 690,000*

Second year interest $I = 690{,}000 \times \frac{15}{100} \times 1 = \mathbf{103{,}500}$

Second year Amount ***A =*** *690,000 + 103,500* ***= 793,500***

If ***C·I = A − P***

$\Rightarrow$ ***C·I =*** *793,500 − 600,000* ***= 193,500***

2. *Tom deposited* ***Shs 750,000*** *in a bank which offers a compound interest rate of* ***20%*** *per annum. Find how much interest he earned after* ***3*** *years*

Soln:

If ***C·I = A − P***

$$\Rightarrow C \cdot I = 750{,}000 \times \left(1 + \frac{20}{100}\right)^3 - 750{,}000 = \mathbf{546{,}000}$$

3. *Find the difference between the simple interest and compound interest on* ***Shs 50,000*** *for* ***2*** *years at a rate of* ***5%*** *per annum*

Soln:

$$I = \frac{PRT}{100} = 50{,}000 \times \frac{5}{100} \times 2 = \mathbf{5{,}000}$$

If ***C·I = A − P***

$$\Rightarrow C \cdot I = 50{,}000 \times \left(1 + \frac{5}{100}\right)^2 - 50{,}000 = \mathbf{5{,}125}$$

Required difference = *5,125− 50,000* ***= 125***

4. *A sum of money amounts to* ***Shs 1,296,000*** *in* ***3*** *years at a compound interest rate of* ***20%*** *per annum. Find the sum invested*

Soln:

$$\text{If } A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow P\left(1 + \frac{20}{100}\right)^3 = 1{,}296{,}000$$

$\therefore$ ***P = 750,000***

5. *A sum of money lent out at compound interest yielded an interest of* ***Shs 36,400*** *in* ***3*** *years at a rate of* ***20%*** *per annum. Find the sum lent out*

Soln:

If ***C· I = A − P***

$$\Rightarrow P\left(1+\frac{20}{100}\right)^{3} - P = 36{,}400$$

$\therefore$ ***P = 50,000***

6. *Tom deposited* ***Shs 750,000*** *which amounted to* ***Shs 1,296,000*** *after* ***3*** *years at compound interest. Find the rate of interest per annum*

Soln:

$$\text{If } A = P\left(1+\frac{r}{100}\right)^{n}$$

$$\Rightarrow 750{,}000 \times \left(1+\frac{r}{100}\right)^{3} = 1{,}296{,}000$$

$$\Rightarrow 1+\frac{r}{100} = \sqrt[3]{\frac{1{,}296{,}000}{750{,}000}}$$

$\therefore$ ***r = 20%***

7. *A man invests* ***Shs 200,000*** *at the beginning of each year in a bank which offers a compound interest rate of* ***5%*** *per annum. Calculate the amount of money the man had in the bank immediately after payment of the third investment.*

STEP BY STEP METHOD

First year amount $\boldsymbol{A} = \frac{105}{100} \times 200{,}000 \quad = \mathbf{210{,}0000}$

Second year principal = 210,000 + 200,000 = 410,000

Second year Amount $\boldsymbol{A} = \frac{105}{100} \times 410{,}000 \quad = \mathbf{430{,}500}$

Third year principal = 430,500 + 200,000 ***= 630,500***

$\therefore$ ***Required amount = 630,500***

8. *A man borrows* ***Shs 250,000*** *at* ***10%*** *per annum compound interest. He pays back* ***Shs 100,000*** *at the end of each year. Calculate his debt at the beginning of the third year.*

STEP BY STEP METHOD

First year amount $\boldsymbol{A} = \frac{110}{100} \times 250{,}000 \quad = \mathbf{275{,}0000}$

Second year principal = 275,000 – 250,000 = 175,000

Second year Amount $\boldsymbol{A} = \frac{110}{100} \times 175{,}000 \quad = \mathbf{192{,}500}$

Third year principal = 192,500 – 100,000 ***= 92,500***

$\therefore$ ***Remaining debt = 92,500***

8. *A sum of money lent out at compound interest amounts to* ***Shs 60,000*** *after one year and to* ***Shs 72,000*** *after* ***2*** *years. Find the sum lent out and the rate of interest per annum*

9. *Tom deposited* ***Shs 800,000*** *in a bank which offers a compound interest rate of* ***8·5%*** *per annum. Find how long will it take for his money to accumulate to* ***Shs 941,780***

PERIODIC COMPOUNDING

Summary:

1. *An investment may be compounded more than once a year. It can be compounded daily, weekly, monthly, quarterly and semiannually.*

2. *If interest is compounded more than once a year, then:*

$A = P\left(1 + \frac{r}{100k}\right)^{nk}$, *where **n** = time frame in years, **k =** number of compoundings within a year,* ***nk =** number of compoundings over **n** year,* $\frac{r}{k}$ *= interest rate for one period*

3. (i) *Interest compounded quarterly means that its compounded four times a year. Thus **k = 4***
(ii) *Interest compounded semiannually means that its compounded two times a year. Thus **k = 2***
(iii) *Interest compounded monthly means that its compounded **12** times a year. Thus **k = 12***
(iv) *Interest compounded weekly means that its compounded **52** times a year. Thus **k = 52***
(v) *Interest compounded daily means that its compounded **365** times a year. Thus **k = 365***

4. *The term* $\frac{r}{k}$ *means that the interest rate per year is converted into interest rate for one period*

EXAMPLES:

1. *Tom invested **Shs 60,000** for* $1\frac{1}{2}$ *years at a rate of **12%** per annum compounded quarterly. Find the amount received in that period*

Soln:

If $A = P\left(1+\frac{r}{100k}\right)^{nk}$

$$\Rightarrow A = 60{,}000 \times \left(1+\frac{12}{100(4)}\right)^{1\cdot 5(4)} = 71{,}643$$

2. *Find the interest received on* ***Shs 800,000*** *invested at a rate of* ***20%*** *per annum, compounded semiannually for* $2\frac{1}{2}$ *years*

Soln:

If ***C· I = A – P,*** *where* $A = P\left(1+\frac{r}{100k}\right)^{nk}$

$$\Rightarrow C\cdot I = 800{,}000 \times \left(1+\frac{20}{100(2)}\right)^{2\cdot 5(2)} - 800{,}000 = 488{,}408$$

3. *Find the amount received on* ***Shs 400,000*** *invested at a rate of* ***15%*** *per annum, compounded monthly for* ***2*** *years*

Soln:

If $A = P\left(1+\frac{r}{100k}\right)^{nk}$

$$\Rightarrow A = 400{,}000 \times \left(1+\frac{15}{100(12)}\right)^{2(12)} = 538{,}940 \cdot 42$$

EER:

1. *Find the amount and interest received on* ***Shs 750,000*** *invested for* ***3*** *years at a compound interest rate of* ***20%*** *per annum.*

__2.__ Tom invested __Shs 60,000__ for $1\frac{1}{2}$ *years at a rate of __12%__ per annum compounded quarterly. Find the interest received in that period*

__3.__ Find the amount received on __Shs 800,000__ invested at a rate of __20%__ per annum, compounded semiannually for $2\frac{1}{2}$ *years*

__4.__ Tom and Bob were each given Uganda shilling __980,000__ at the beginning of __2015.__ Tom exchanged his money to US dollars and then banked it on his foreign currency account at a compound interest rate of __2%__ per annum, while Bob banked his money without exchanging it, at a compound interest rate of __12%__ per annum. The exchange rate in __2015__ and __2016__ were __Ug Shs 1,250__ and __Ug Shs 1,500__ to a dollar respectively. If Bob withdrew __Shs 120,000__ at the end of __2016,__ __(i)__ Calculate the amount of money __(in UG Shs)__ each man had in the bank at the end of __2016__

__(ii)__ who had more money and by how much?

__5.__ Jane and Joan invested __Shs 600,000__ each in a savings society for __2__ years. Jane opted for simple interest while Joan opted for compound interest. Both interest rates were at __12%__ per annum.

__(i)__ Find the interest earned by each of them

__(ii)__ who had more money and by how much?

__6.__ Tom wants to buy a house which is priced at __Shs 56,000,000.__ A deposit of __25%__ of the value of the house is required. A bank will lend him the rest of the money at a compound interest of __15%__ per annum and payable after two years. Calculate the:

__(i)__ deposit Tom must make

__(ii)__ amount of money Tom will have to pay the bank after two years

__(iii)__ total money which Tom will spend to buy the house

__7.__ At the beginning of the year __2015,__ Bob deposited __Shs 1,900,000__ in a bank which offers a compound interest rate of __2·75%__ per four months. Find how much interest he earned at the end of the year

__8.__ Tom borrows __Shs 750,000__ at a simple interest rate of 2__0%__ per annum and then immediately lends it to Bob at the same rate but at a compound interest. Find how much will Tom gain by this transaction after __3__ years

***9**. A man borrowed **Shs 14·85** million from a bank at a compound interest rate of **12%** per annum. He has to repay the loan within **2** years in **6** equal instalments. Calculate the:*

***(i)** total amount he paid to the bank*

***(ii)** interest he paid to the bank*

(iii) amount he paid per instalment

***10**. A man borrowed **Shs 39·6** million from a bank at a compound interest rate of **10·5%** per annum. He has to repay the loan within **2** years in **8** equal instalments. Calculate the:*

***(i)** total amount he paid to the bank*

***(ii)** interest he paid to the bank*

(iii) amount he paid per instalment

11.** At the beginning of each year starting in **2011,** a man invests **Shs 800,000** in a bank which offers a compound interest rate of **5%** per annum. Calculate the amount of money the man had in the bank at the end of **2013

APPRECIATION AND DEPRECIATION

Summary:

***1. (i)** The gradual gain in the value of an asset is called appreciation*

***(ii)** If an asset appreciates in value, then* $A = P\left(1 + \frac{r}{100}\right)^n$

***2. (i)** The gradual loss in the value of an asset is called depreciation*

***(i)** If an asset depreciates in value, then* $A = P\left(1 - \frac{r}{100}\right)^n$

EXAMPLES:

1. *A plot of land valued* ***Shs 400,000*** *appreciates in value at* ***15%*** *per annum. Find its value after* ***2*** *years*

Soln:

$$If\ A = P\left(1 + \frac{r}{100}\right)^{n}$$

$$\Rightarrow A = 400{,}000 \times \left(1 + \frac{15}{100}\right)^{2} = 529{,}000$$

2. *A house valued* ***Shs 600,000*** *appreciates in value at* ***10%*** *per annum. Find its value after* ***3*** *years*

Soln:

$$If\ A = P\left(1 + \frac{r}{100}\right)^{n}$$

$$\Rightarrow A = 600{,}000 \times \left(1 + \frac{10}{100}\right)^{3} = 798{,}600$$

3. *A bicycle valued* ***Shs 250,000*** *depreciates in value at* ***20%*** *per annum. Find its value after* ***3*** *years*

Soln:

$$If\ A = P\left(1 - \frac{r}{100}\right)^{n}$$

$$\Rightarrow A = 250{,}000 \times \left(1 - \frac{20}{100}\right)^{3} = 128{,}000$$

4. *A laptop valued* ***Shs 500,000*** *depreciates in value at* ***10%*** *per annum. Find its value after* ***3*** *years*

$$If\ A = P\left(1 - \frac{r}{100}\right)^{n}$$

$$\Rightarrow A = 500{,}000 \times \left(1 - \frac{10}{100}\right)^3 = \mathbf{256{,}000}$$

5. *A car was bought at **Shs 5 million.** In the first year**,** its value depreciated by **20%,** in the second year by **15%** and in the third year by **10%.** Find its value after **3** years*

STEP BY STEP METHOD

*First year amount **A*** $= \frac{80}{100} \times 5 = \mathbf{4\ million}$

*Second year Amount **A*** $= \frac{85}{100} \times 4 = \mathbf{3 \cdot 4\ million}$

*Third year Amount **A*** $= \frac{90}{100} \times 3 \cdot 4 = \mathbf{3 \cdot 06\ million}$

$\therefore$ ***Required amount = 3·06 million***

EER:

1. *A house was bought at **Shs 150,000.** In the first year, its value appreciated by **25%,** in the second year by **10%** but dropped by **20%** in the third year. Find its value after **3** years*

2. *A new factory machine depreciates in value at **25%** per annum. If its value after **3** years is **Shs 135,000,** find its value when new*

TRADE PARTNERSHIP

Summary:

1. *Partnership bussiness is a bussiness owned by two or more people*
2. *Partners must negotiate and agree upon a fair profit–sharing ratio*

EXAMPLES:

1. *Tom**,** Bob and Ben form a trade partnership**.** Tom contributes **Shs 750,000,** Bob **Shs 500,000** and Ben **Shs 900,000. 20%** of the annual gross* *profits are reinvested and a monthly taxation of **Shs 10,000** is to be paid by each shareholder. The net profit is to be shared in the ratio of their capital contributions. If at the end of the year their bussiness made a gross profit of **Shs 3,160,000 ,** find how much did each member get as his net profit*

Soln:

Amount re–invested =

Annual tax $= 3 \times 10{,}000 \times 12 = \mathbf{360{,}000}$

Net profit = 3,160,000 – (632,000 + 360,000) ***= 2,168.000***

Profit–sharing ratio ***=*** *750,000 : 500,000 : 900,000* ***= 15 : 10 : 18***

Tom's share =

Bob's share =

Ben's share =

2. *Tom, Bob and Ben decided to buy a bus. The bus owner offered the bus at*

Shs 280 *million but agreed to be paid **60%** of the value as initial deposit in the*

*ratio **3:2:5** respectively and the remaining amount to be paid after **1** year. The*

total balance was to be paid from the profits of the bus in the same ratio as the

deposits. They agreed to share the net profits in the ratio of their deposits. If at

*the end of the year the bus realized **Shs 208** million, find:*

(i) *how much deposit did each contribute*

(ii) *how much of the balance did Ben contribute.*

(iii) *how much money was left with Tom after paying the balance*

Soln:

(i) *Amount deposited = 0·6 × 280 = **168 million***

*Total ratio **=** 3 +2 + 5 **= 10***

Tom's contribution =** 0·3 × 168 = **50·4 million

Bob's contribution =** 0·2 × 168 = **33·6 million

Ben's contribution =** 0·5 × 168 = **84 million

(ii) *Total balance = 280 – 168 = **112 million***

Ben's balance contribution =** 0·5 × 112 = **56 million

(iii) *Net profit = 208 – 112 = **96 million***

Tom's balance contribution =** 0·3 × 96 = **28·8 million

EER:

1. *Tom and Bob decided to buy a plot. The plot owner offered the plot at **Shs 20** million but agreed to be paid **75%** of the value as initial deposit in the*

*ratio **5:3** respectively and the remaining amount to be paid after **2** years*

*including an additional **5%** of the initial value for processing the plot*

documents. The total balance was to be paid in the same ratio as the deposit.

Find how much of the balance did Bob contribute.

2. *Tom and Bob have shares in a company. Tom contributed **Shs 400** million and Bob **Shs 600** million as share capital. The company expenses that year were electricity **Shs 12** million, salaries **Shs 55** million and* *transport **Shs 13** million. The net profit is to be shared in the ratio of their share capital. If at the end of the year their company made a gross profit of **Shs 200** million*

***(a)** find the:*

***(i)** total expenditure for the company*

***(ii)** percentage of the company's expenses to the net profit*

***(b)** Find how much money each share holder got that year if **10%** of this was paid as income tax*

***3.** Tom and Bob contributed **Shs 112,000** and **Shs 128,000** respectively to start a partnership business. They agreed to share their profits as follows.*

***35%** to be shared equally*

***25%** to be shared in the ratio of their respective contributions and **40%** to*

*be retained for the running of the business. If their profit was **Shs 864,000,** calculate the:*

***(i)** amount Bob received*

***(ii)** amount retained for the business*

4.** Five members of a self supporting enterprise Tom, Bob, Ben, Sam and Tim were given a certain amount of money to share amongst themselves. Tom got **3/8** of the total amount while Bob got **2/5** of the remainder. The remaining amount was shared equally among Ben, Sam and Tim each of which received **Shs 600,000

***(i)** How much was shared among the five business men?*

***(ii)** How much did Bob get?*

(iii)** Tom, Bob and Tim invested their money and earned a profit of **Shs

1,200,000.
A third of the profit was reinvested and the rest was shared in the ratio of their investments. Find how much each got.

TAXATION

(b) *A man earns a gross monthly salary of* ***sh 630,000.*** *His allowance in a given month amounted to* ***sh 128,000*** *and tax was levied on taxable income as follows:*

5% *for the first* ***sh 200,000***

35% *for the rest*

Find the:

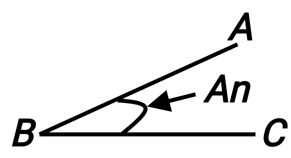
(i) *taxable income*

(ii) *income tax paid by the man*

ANGLES

Summary:

1. (i) *The space enclosed between the lines **AB** and **BC** is referred to as an angle*

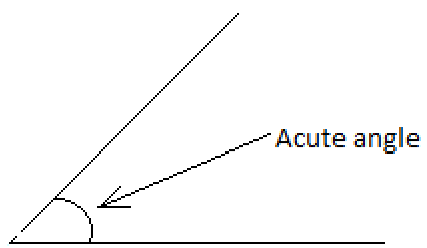


(ii) *The above angle is described as **∠ABC** or **∠CBA** or **∠B***

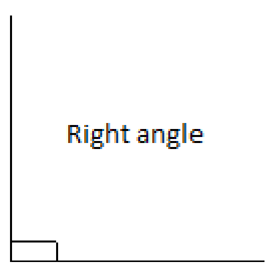
(iii) *In geometry, angles are measured in degrees using a protractor*

TYPES OF ANGLES.

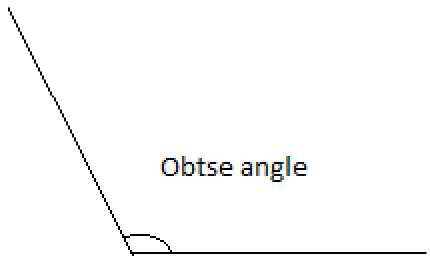
1. *An **acute angle** is an angle that is less than 90^0.*



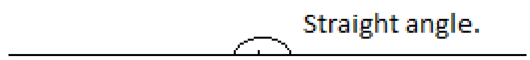
2. *A **right angle** is an angle that is equal to 90^0.*



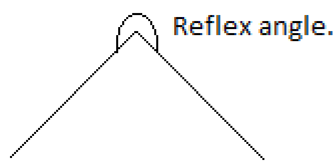
3. *An **obtuse angle** is an angle between 90^0 and 180^0.*



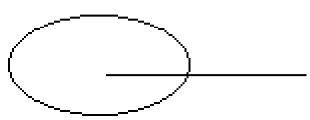
4. A **straight angle** is an angle that is equal to 180^0.



5. *A **reflex angle** is an angle that lies between 180^0 and 360^0.*



6. *A **full angle** is an angle that is equal to 360^0.*

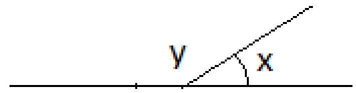


complete angle.

ANGLE RELATIONSHIPS.

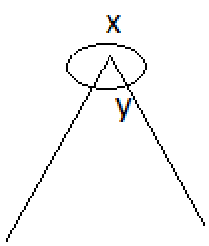
1. ***Complementary angles** are two angles that add up to 90^0. Thus 40^0 and 50^0 are complementary angles since they add up to 90^0.*
2. *Two angles are **supplementary** when they add up to 180^0. Thus 30^0 and 150^0 are supplementary angles since they add up to 180^0.*

3. *Angles on a line add up to 180^0.*



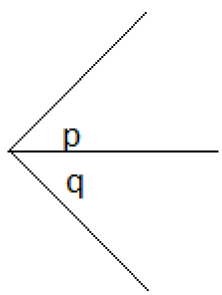
Thus ***$x + y = 180^0$****.*

4. *Angles around a point add up to 360^0.*

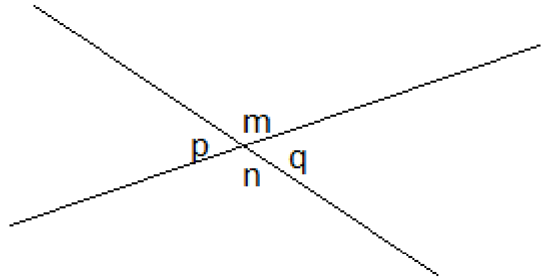


Thus, ***$x+y=360^0$****.*

5. *Two angles next to each other are called* ***adjacent angles****. Thus, in this example, p and q are adjacent angles.*



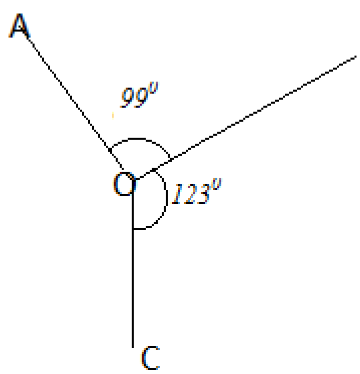
6. ***Vertically opposite angles*** *are angles opposite to each other when two lines* cross. In this example, p and q are vertically opposite angles.



Vertically opposite angles are equal. Thus, p=q and m=n.

Examples:

*1. In the figure below, find the size of angle **AOC**.*



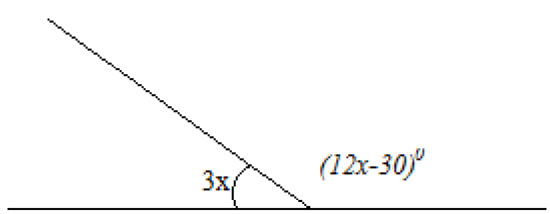
Solution

If <AOC =x

$x+99^0+123^0=360^0$. *(Angles around a point.)*

$x = 138^0$.

2. Find the size of each angle in the table below.



Solution

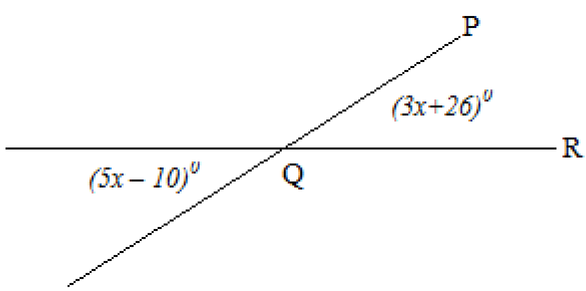
$3x + (12x-30)^0 = 180^0$.

$x = 14^0$.

Angle $3x = 3(14) = 42^0$.

Angle $(12x-30) = 12(14)-30 = 138^0$.

3. *The figure below shows two intersecting lines.*



*Find the size of angle **PQR**.*

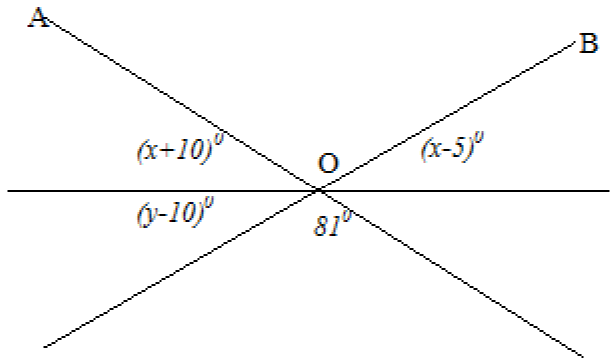
Solution:

$(5x - 10)^0 = (3x+26)^0$. (Vertically opposite angles)

$x = 18^0$.

Angle PQR $= 3(18) + 26 = 80^0$.

4. *The figure below shows three intersecting lines.*



Find the values of x and y.

Solution:

$<AOC=81^0$. *(Vertically opposite angles)*

$(x+10)^0+81^0+(x-5)^0=180^0$ *(linear angles)*

$\therefore x=47^0$.

$(x+10)^0+(y-10)^0+81^0=180^0$ *(linear angles)*

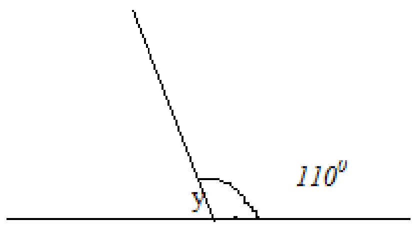
$(47+10)^0+y-10+81=180^0$

$\therefore y=52^0$.

ERR.

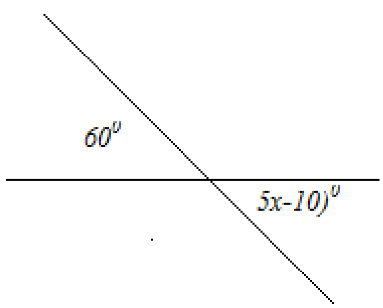
1. *Find the size of two complementary angles that are such that the size of one of them is four times the size of the other.*

2. *The ratio of two complementary angles is 1:5. Find the size of each of them.*
3. *Find the size of an angle that is such that when added on to one – sixth of its complement, the result is 40^0.*
4. *Find the size of angle marked y in the figure below.*



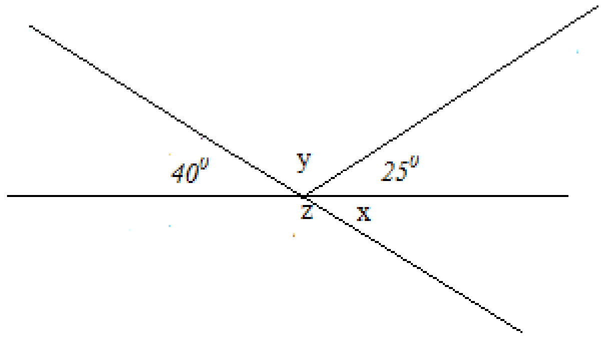
5. *Find the value of x for which the angles $(2x+10)^0$ and $(130\text{-}x)^0$ are vertically opposite.*

6. *The figure below shows two intersecting lines.*



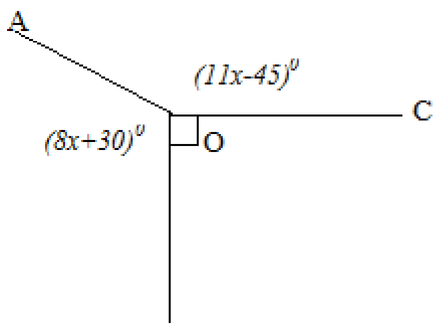
Find the values of x.

7. *The figure below shows three intersecting lines.*



Find the values of x, y and z.

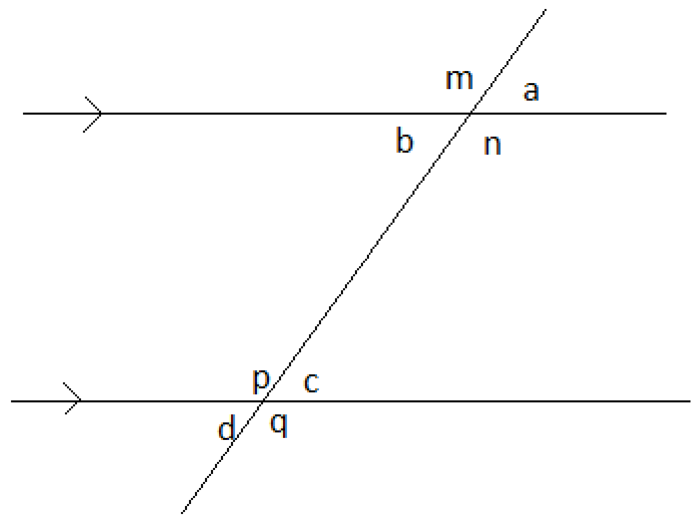
8. *In the figure below, find the size of reflex angle AOC.*



ANGLES ON A TRANSVERSAL

Summary:

1. *A line cutting across parallel lines is called a* ***transversal.***
2. *(i) Below is an illustration of the different angle formed on a transversal.*



*(ii) The angles in matching corners are called **corresponding angles**.*

*(iii).The interior angles on the opposite sides of a transversal are called **co-interior angles**.*

3. The following are the transversal angle properties

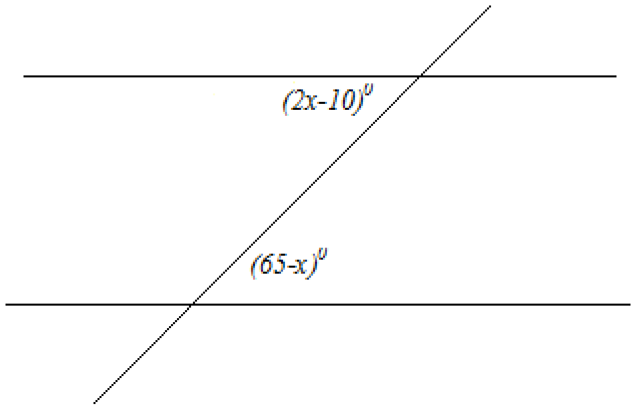
(i) Corresponding angles are equal. Thus <a=<c, <b=<d, <m=<p and <n=<q

(ii) Alternate angles are equal. Thus <b=<c and <n=<p.

(iii) Co-interior angles add up to $\mathbf{180^0}$ *,thus* $\mathbf{b + p = 180^0}$

EXAMPLES:

1. The figure below shows parallel lines cut by a transversal.



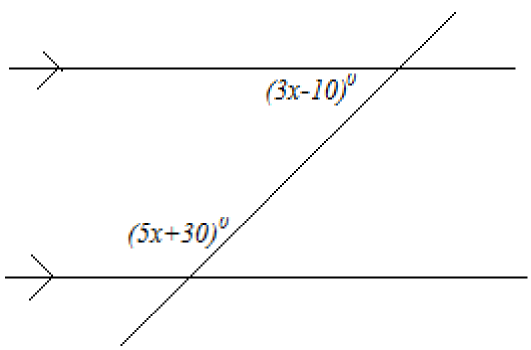
Find the value of x.

Solution

$(2x-10)^0 = (65-x)^0$ (alternate angles)

$\therefore x=25^0$.

2. *The figure below shows parallel lines cut by a transversal.*



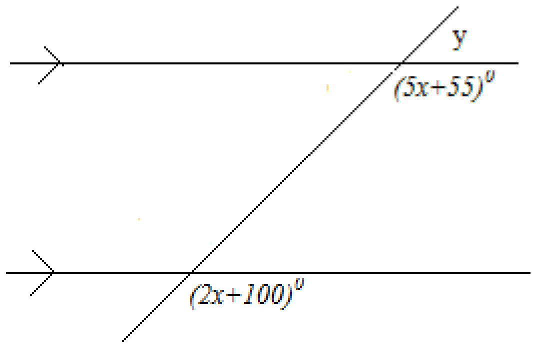
Find the value of x

Solution

$(3x-10)^0 + (5x+30)^0 = 180^0$ (co-interior angles)

$x=20^0$

3. The below shows parallel lines cut by transversal



Find the size of the angle marked y

Solution

$(5x+55)^0 = (2x+100)^0$ *(corresponding angles)*

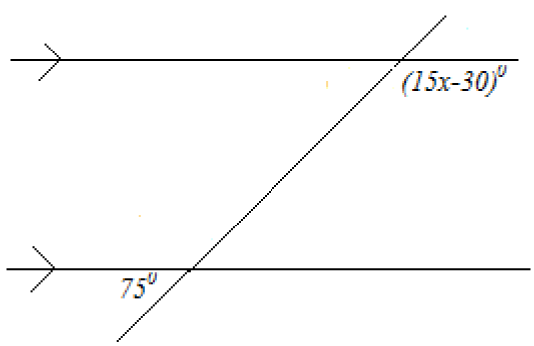
$\therefore x=15^0$

$y+ (5x+55) =180$

$y+5(15) +55=180$

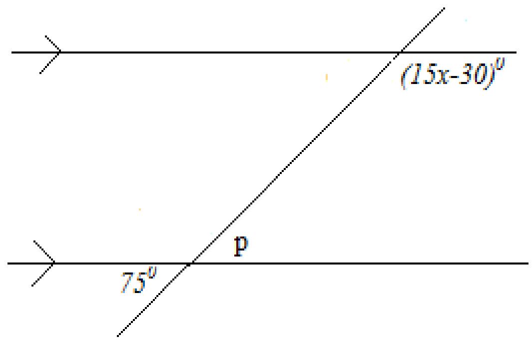
$\therefore y=50^0$

4. *The figure below shows parallel lines cut by a transversal*



Find the value of x

Solution

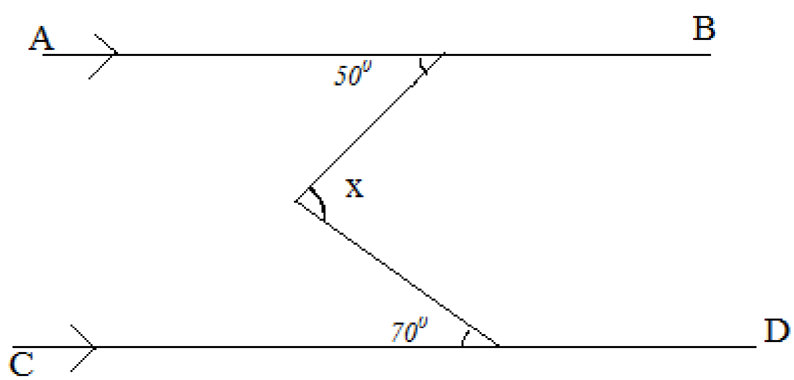


<p=75^0 (vertically opposite angles)

$75^0 + (15x-30)^0 = 180^0$

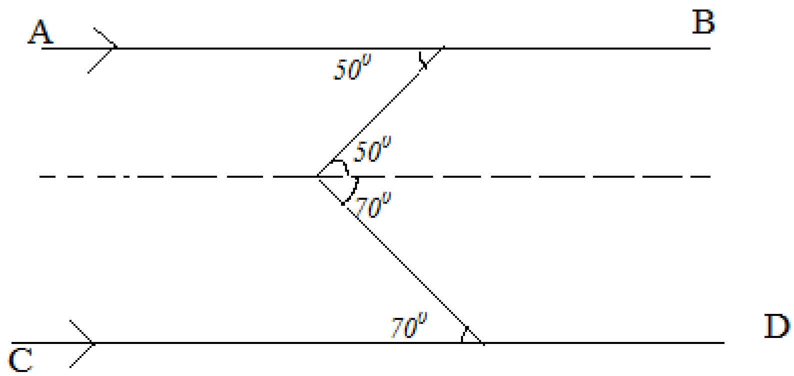
$\therefore x = 9^0$.

5. The figure below shows parallel lines AB and CD.



Find the size of angle marked x.

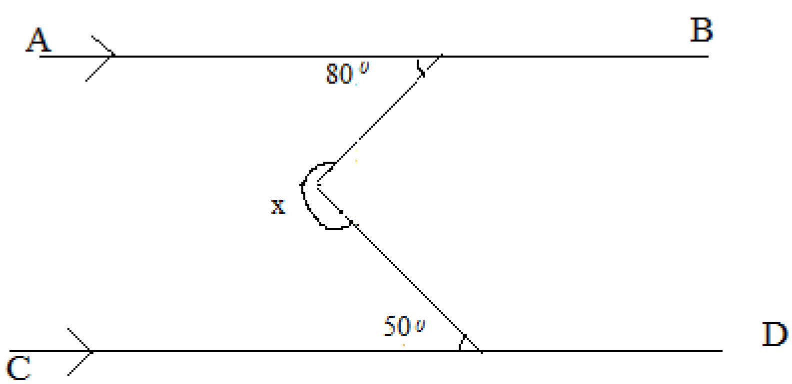
Solution



$X=50^0+70^0$

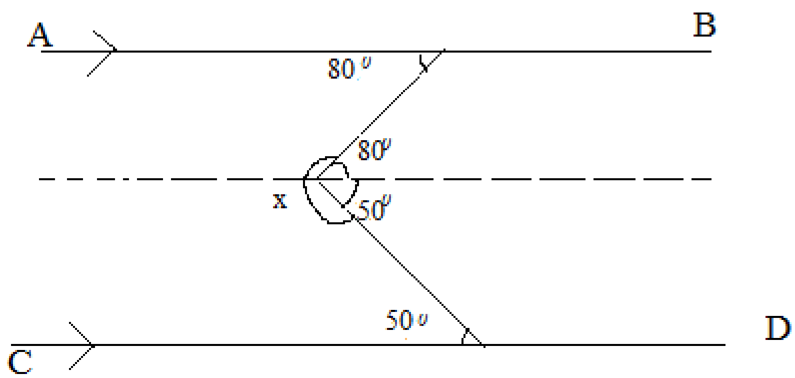
$X=120^0$.

6. The figure below shows parallel lines AB and CD.



Find the size of the angle marked x.

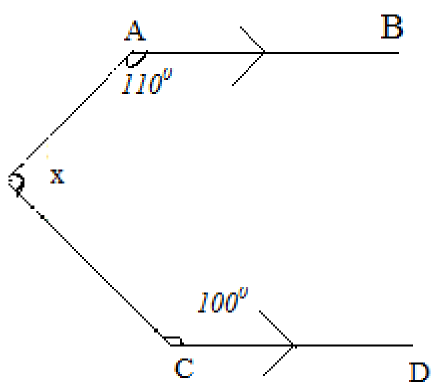
Solution



$x+80^0+50^0=360^0$ (angles around a point)

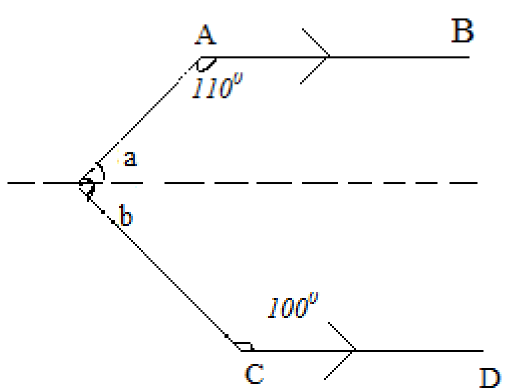
$\therefore x = 230^0$.

7. *The figure below shows parallel lines AB and CD.*



Find the size of the angle marked x.

Solution



$a+110^0=180^0$ (co-interior angles)

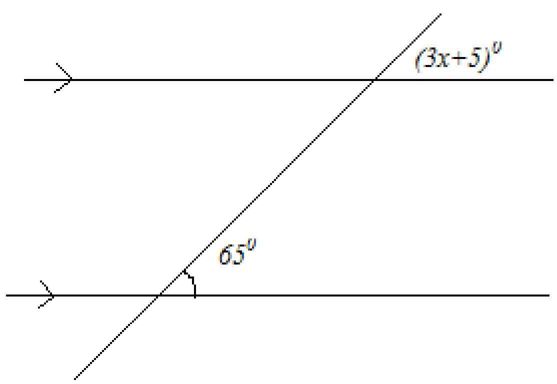
$\therefore$ $a=70^0$

$b+100^0=180^0$

$\therefore b=80^0$

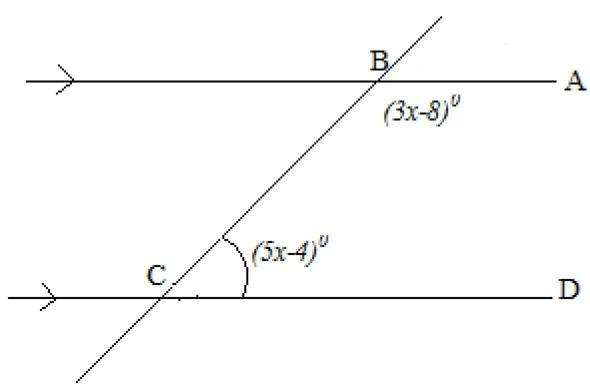
EER

1. *The figure below shows parallel lines cut by a transversal.*



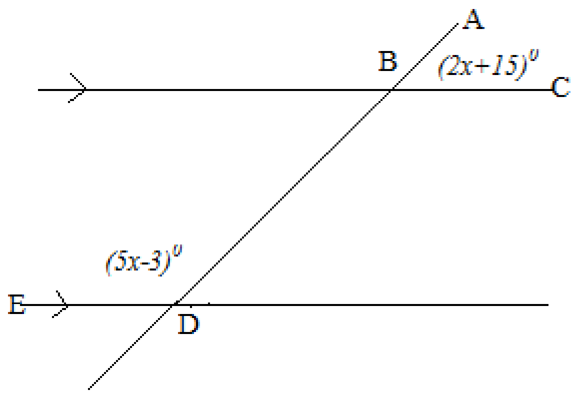
Find the value of x.

2. *The figure below shows parallel lines cut by a transversal.*



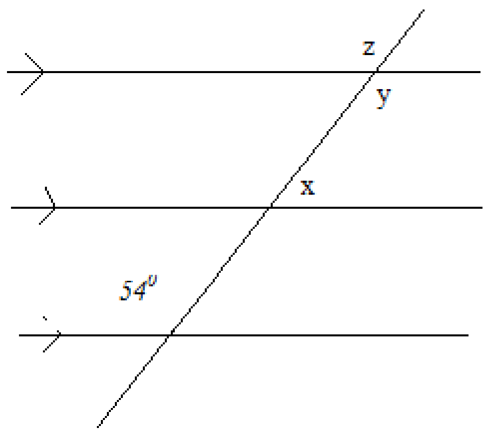
Find the size of angle ABC and angle BCD.

3. *The figure below shows parallel lines cut by a transversal.*



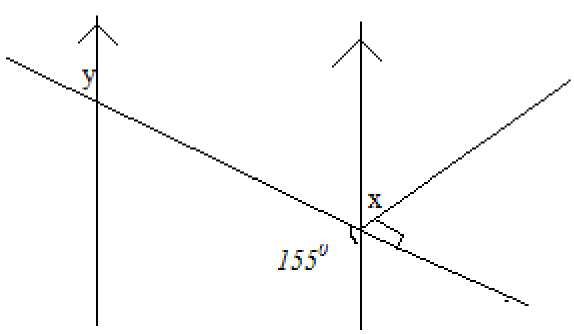
Find the size of angle ABC and angle BDE.

4. *The figure below shows parallel lines cut by a transversal.*



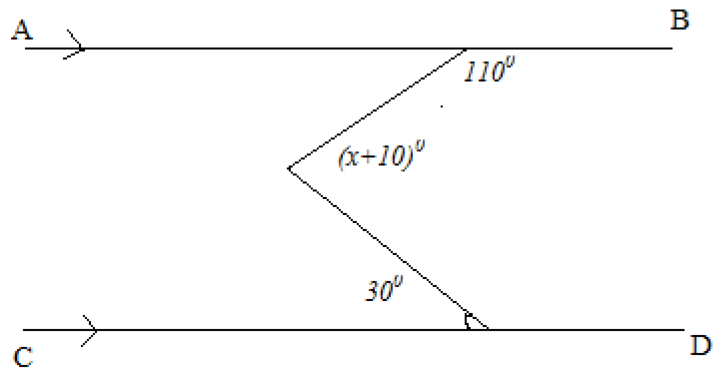
Find the values of x, y and z.

5. *The figure below shows parallel lines cut by a transversal.*



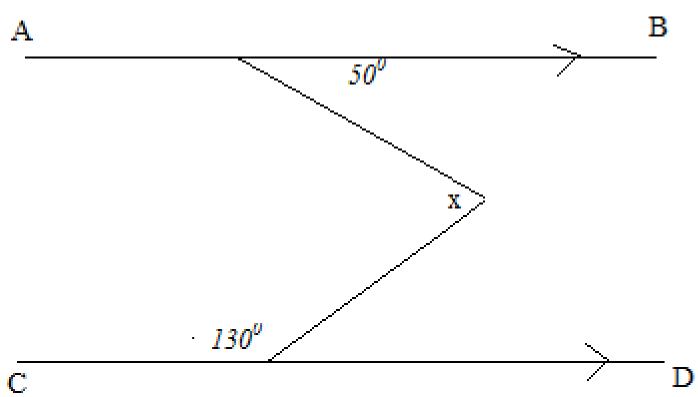
Find the values of x and y.

6. *The figure below shows parallel lines AB and CD.*



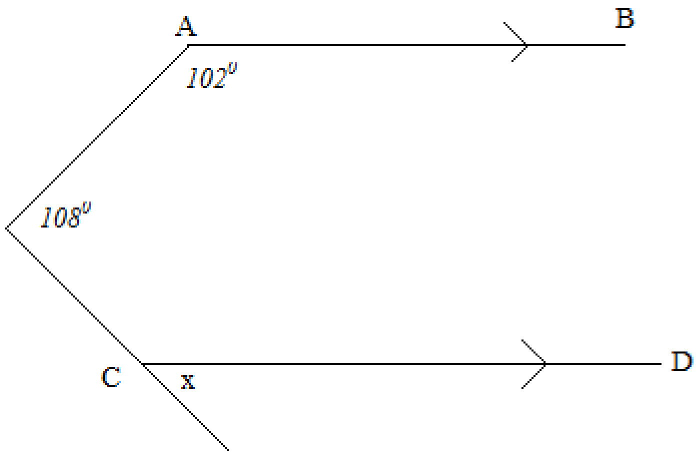
Find the value of x.

7. *The figure below shows parallel lines AB and CD.*



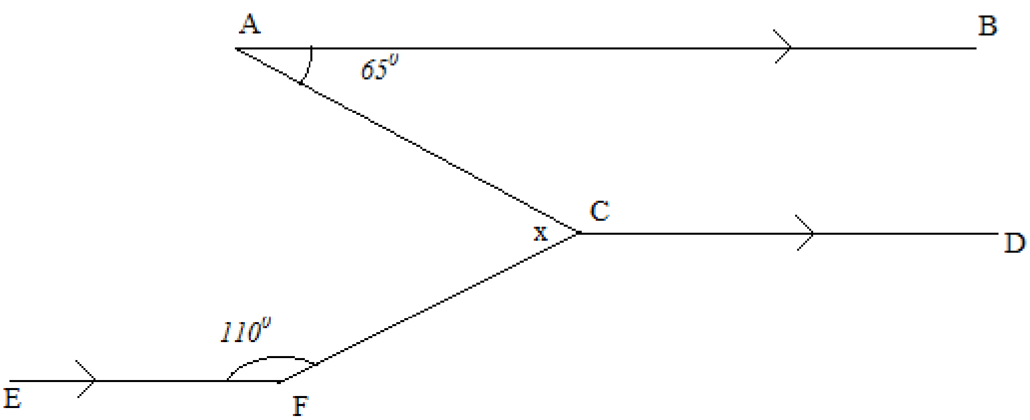
Find the value of x.

8. *The figure below shows parallel lines AB and CD.*



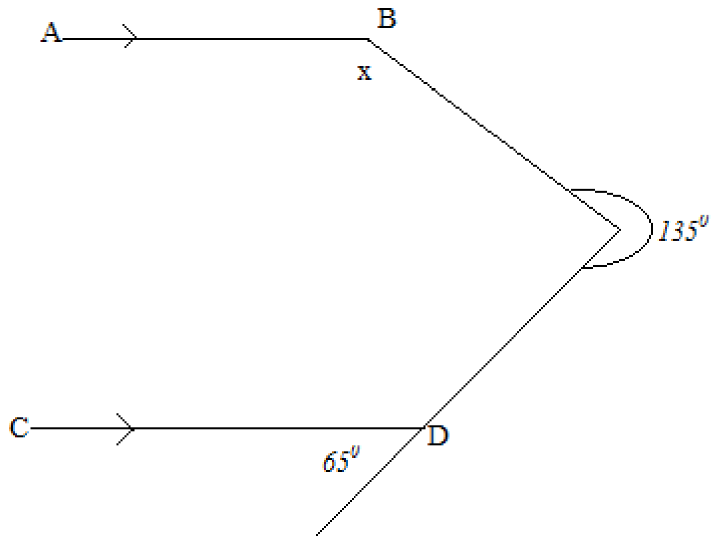
Find the value of x.

9. *The figure below shows parallel lines AB, CD and EF.*



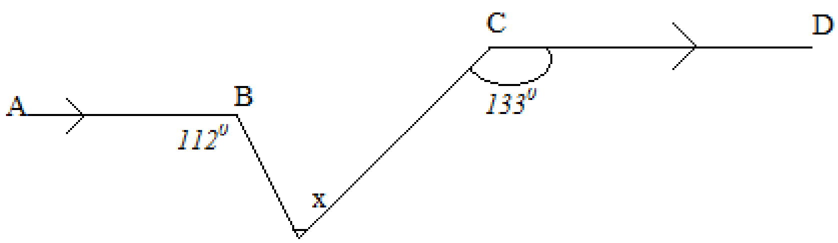
Find the value of x.

10. *The figure below shows parallel lines AB and CD.*



Find the value of x.

11. *The figure below shows parallel lines AB and CD.*



Find the value of x.

DIRECTION AND BEARINGS

Summary:

(i) *Bearings are used to show the direction of one point relative to another*

(ii) *The four main directions of a compass are North* ***(N),*** *East* ***(E),*** *South* ***(S)*** *and West* ***(W)***

(iii) *A directional compass is as follows:*

(iv) *The bearing to a point is the angle measured in a clockwise direction from the north line.*

(v) *Bearings are stated using three digits. Thus* ***5°*** *is written as* ***005°***

(vi) *The north line represents a bearing of* ***000°***

(vii) *The bearing of* ***N60°E*** *means an angle of* ***60°*** *measured from* ***N*** *towards* ***E***

EXAMPLES:

1. *The bearing of point* ***P*** *from point* ***Q*** *is* ***060°****. Find the bearing of* ***Q*** *from* ***P***

2. *The bearing of point* ***N*** *from point* ***M*** *is* ***310°.*** *Find the bearing of* ***M*** *from* ***N***

3. *Find the angle between the direction* ***N70°E*** *and* ***S70°W***

4. *A boat sails* ***15km*** *on a bearing of* ***000°.*** *It then sails* ***8km*** *due East. Calculate how far it is from the starting point*

5. *Two ships* ***P*** *and* ***Q*** *leave port* ***K*** *at the same time.* ***P*** *sails* ***9km*** *on a bearing of* ***030°*** *and* ***Q*** *sails* ***12km*** *on a bearing of* ***120° .*** *Calculate how far apart are the ships*

6. *An observer at point* ***P*** *sees an object on a bearing of* ***100°.*** *Another observer at point* ***Q*** *sees the same object on a bearing of* ***150°.*** *Given that the distances of the object from* ***P*** *and* ***Q*** *are equal, determine the bearing of* ***P*** *from* ***Q***

Soln:

View from P ***View from Q***

Combined View

If $x + x + 50° = 180°$ *"Isosceles triangle"*

$\Rightarrow \; x = 65°$

$\therefore$ ***Required bearing*** $= 150° + x = 150° + 65° = 215°$

7. *A plane flies* ***300km*** *from airport* ***A*** *to airport* ***B*** *on a bearing of* 060^0. *It then flies* ***450km*** *to airport* ***C*** *on a bearing of* 150^0 .

(a) *Use a scale of **1cm** to represent **50km,** make a scale drawing to*

show the route of the plane.

(b) *Find the distance and bearing of airport **A** from **C.***

(c) *If the plane flies directly back to **A** at a speed of* $200kmh^{-1}$*, determine how*

*long it takes to fly back to **A.***

8. *Town **Q** is on a bearing of* $\mathbf{060^0}$ *from Town **P** and **120 km** away. Town **R** is on a*

bearing of $\mathbf{130^0}$ *from **P** and* $\mathbf{220^0}$ *from **Q.***

(a) *By scale drawing show the relative positions of **P, Q** and **R.***

*[Use a scale of **1cm** to represent **20km**]*

(b) *Find the distance between:*

(i) P** and **R

(ii) Q** and **R

(c) *A plane flies from town **R** on a bearing of* $\mathbf{210^0}$ *at a speed of* $150kmh^{-1}$.

*After **40** minutes of flying, the pilot decides to fly directly to town **P.** Find the*

*time it would take to reach **P** and the bearing to which it would fly*

9. *A plane flies on a bearing of* $\mathbf{060^0}$ *from airport **A** to airport **B** at a steady speed*

of $\mathbf{200kmh^{-1}}$ *for **2hours.** It then flies on a bearing of* $\mathbf{150^0}$ *to air strip **C** at the* *same original speed for $2\frac{1}{2}$ hours.*

*(a) Use a scale of **1cm** to represent **50km,** construct a scale drawing*

to show the route of the plane.

*(b) Find the distance and bearing of **A** from **C.***

*(c) If the plane flies directly back to **A** at a speed of $200kmh^{-1}$, determine how*

*long it takes to fly back to **A.***

EER:

***1.** A ship sails **10km** due north and then **24km** due east. Calculate how far it is from the starting point*

2.** A man walks from town **P 9 km** due north then **12km** due east to town **Q.** Calculate the distance of **P** from **Q

3.** Find the angle between the direction **N450°E** and **S25°W

***4.** A ship sails equal distances due South–East and then due South–West to end up **14km** due South of its starting point. Calculate how long is each part of its journey*

***5.** The bearing of **P** and **Q** from **A** are **200°** and **290°** respectively. Given that distance **AP = 5·6km** and **AQ = 4·7km,** find by scale drawing the:*

*(i) distance **PQ***

*(ii) bearing of **P** from **Q***

6. *A plane flies from airport **K** due North for **350km** to airport **R.** It then flies on a*

*bearing of **295°** for **250km** to air strip **N.** From there it flies on a bearing of*

090^0 *for **500km** to air strip **M.***

(a) *Use a scale of **1cm** to represent **50km,** draw an accurate diagram*

to show the route of the plane.

(b) *Find the distance and bearing of **K** from **M.***

(c) *If the plane flies directly back to **K** at a speed of $250kmh^{-1}$, determine how*

*long it takes to fly back to **K.***

7. *Town **B** is **100km** away from town **A** on a bearing of **135°.** Town **D** is **124km***

*away from town **B** on a bearing of **090°.** Town **C** is **160km** away from town **D***

*on a bearing of **030°.***

(a) *Use a scale of **1cm** to represent **20km,** draw an accurate diagram to show the*

relative positions of the towns.

(b) *Find the:*

(i) *distance and bearing of **C** from **A***

(ii) *distance and bearing of* ***B*** *from* ***C***

8. *A plane flies due west from airport* ***A*** *to airport* ***B*** *at a steady speed*

of $280kmh^{-1}.^{1}$ *for* $\frac{3}{4}$ *hours. It then alters its course and flies North–West to air*

strip ***C*** *at* $220kmh^{-1}$. *From there it flies on a bearing of* $\mathbf{060^0}$ *to air strip* ***D*** *at*

$240kmh^{-1}$ *for* $1\frac{1}{2}$ *hours. The total time of flight between the four air strips is*

$4\frac{1}{2}$ *hours.*

(a) *By scale drawing, determine the distance and bearing of* ***A*** *from* ***D***

[Use a scale of ***1cm*** *to represent* ***20km*** *]*

(b) *Find the average speed for the journey from* ***A*** *to* ***D.***

(c) *If the plane flies directly back to* ***A*** *at a speed of* $200kmh^{-1}$, *determine how*

long it takes to fly back to ***A.***

9. *A boat sails* ***450km*** *from island* ***M*** *to island* ***X*** *on a bearing of* $\mathbf{080^0}$ *at a speed*

of $150kmh^{-1}$. *It then sails on a bearing of* $\mathbf{200^0}$ *to island* ***Y*** *at the same original*

speed for ***3*** *hours. From there it sails at a speed of* $200kmh^{-1}$ *to island* ***Q*** *which is west of **M** and **400km** away from it.*

***(a)** Use a scale of **1cm** to represent **50km,** draw an accurate diagram*

to show the route of the boat.

(b)** Find the distance and bearing of **Q** from **Y.

***(b)** Find the:*

***(i)** total time the boat takes to cover the whole journey*

(ii)** average speed of the boat for the whole journey **.

***10.** In a sports field, four points **A, B, C** and **D** are such that **B** is due south of A*

*and due west of **D. AB = 10·8m, BD = 18·8m, CD = 16·6m, ∠BDA = 60°,***

∠CDB = 40°** and **∠BCD = 80°.

***(a)** Use a scale of **1cm** to represent **2m,** draw an accurate diagram to show the*

the relative positions of the points.

***(b)** Find the:*

(i)** distances **BC** and **AD

(ii)** bearing of **B** from **C.

(c)** If an athlete runs from point **A** through **B, C, D** and back to **A** in **16 seconds,

find the athlete's average speed

***11.** The bearing of tower **A** from point **O** is **060°** and that of tower **B** from **O,** is*

200°. *Given that distance **OA = 24km, OB = 33km** and tower C is exactly half*

*way between towers **A** and **B,***

(a) *Use a scale of **1cm** to represent **5km,** draw an accurate diagram to show the*

relative positions of the towers.

(b) *Find the:*

(i) *distances **AB** and **OC***

(ii) *bearing of **B** from **A***

(iii) *bearing of **C** from **O***

(c) *Find:*

(i) *the average speed of the cyclist who takes $2\frac{1}{4}$ hours to travel*

*directly from **A** to **O***

(ii) *how long it takes another cyclist to travel from **A** to **B** via **O** at a*

*speed of $4 \cdot 5kmh^{-1}$ faster than that of the cyclist in **(c) (i)** above*

12. *A plane left airport **K** at **0600** hours and flew on a bearing of **090°** at a speed*

*of $300kmh^{-1}$. It landed at airport **R** at **0830** hours. At exactly **0900** hours, it*

*left **R** and flew on a bearing of **340°,** at the same original speed. It then landed*

*at airport **M** at **1200** hours*

*(a) Use a scale of **1cm** to represent **100km,** draw an accurate diagram to show the route of the plane.*

(b) Find the:

*(i) distance of **M** from **K*** *(ii) bearing of **K** from **M***

13.** A rally car travels from point **R** to point **S** which is **260km** away on a bearing of **060°** from **R.** The car is then set off from **S** at **9:30 am** towards **T** at an average speed of $150kmh^{-1}$ where it is expected to reach at **11:30 am.** After travelling for one hour and twenty minutes, it broke down at **P**. The bearing of **T** and **P** from **S** is **300°.

*(a) Using a scale of **1cm:40km,** show positions of points **R, S, P** and **T***

(b) Determine the:

*(i) distance from **R** to **P***

*(ii) bearing of **P** from **R***

*(c) Given that the repair took ten minutes and later the car is set off to complete the journey to **T**. Find the speed at which the car must be driven to reach **T** on time.*

EER: S·3 WORK

***1.** A boat sails **15km** on a bearing of **000°**. It then sails **8km** due East. Calculate the distance and bearing of the ship from its starting point*

2. *Two ships set off from port* ***P*** *at the same time. One ship sails* ***8km*** *on a*

bearing of ***030°*** *to reach point* ***Q*** *and the other ship sails* ***15km*** *on a bearing*

of ***120°*** *to reach point* ***R****. Calculate the:*

(i) *distance and bearing of* ***R*** *from* ***Q***

(ii) *area of the figure bounded by* ***P QR***

3. *Two ships set off from port* ***P*** *at the same time. One ship sails* ***70km*** *on a*

bearing of ***050°*** *to reach point* ***Q*** *and the other ship sails* ***150km*** *on a bearing*

of ***110°*** *to reach point* ***R.***

(a) *Calculate the:*

(i) *distance and bearing of* ***R*** *from* ***Q***

(ii) *area of the figure bounded by* ***P QR***

(b) *If both ships take* ***t*** *hours to reach their destination and the speed of the*

faster ship is $60kmh^{-1}$*, find the:*

(i) *value of* ***t***

(ii) *speed of the slower ship*

4. *A man walks from town* ***P 9 km*** *due north then* ***12km*** *due east to town* ***Q.*** *Calculate the distance and bearing of* ***P*** *from* ***Q***

5. *Port* ***B*** *is* ***25 km*** *east of port* ***C****. A navigator observes that the bearing of* ***C***

from

*his ship is **310º** and that of **B** is **018º.***

(a)** Calculate the**:

(i)** distance and bearing of the ship from **B

(ii)** distance and bearing of the ship from **C

***(b)** If the ship begins to sail at a speed of **10 kmh^{-1}** on the bearing of **240º**,*

*determine the distance and bearing of the ship from **C** after **48 minutes.***

TRIANGLES

Summary:

1. *In any triangle:*

(i) *The three angles add up to 180^0.*

(ii) *The exterior angle is equal to the sum of the two opposite interior angles.*

(iii) *The largest angle is always opposite to the longest side.*

(iv) *The smallest angle is always opposite to the shortest side.*

2. *In an equilateral triangle:*

(i) *All the three sides are equal in length.*

(ii) *The size of each angle is 60^0.*

(iii) *There are three lines of symmetry.*

3. *In an isosceles triangle:*

(i) *Two of its sides are equal.*

(ii) *The angles opposite to the equal sides are equal.*

(iii) *There is one line of symmetry.*

4. *In a scalene triangle:*

(i) *All the three sides are un-equal.*

(ii) *All the three angles are un-equal.*

(iii) *There is no line of symmetry.*

5. *In a right angled triangle:*

(i) *One of the angles is 90^0.*

(ii) *The three sides are related by Pythagoras property* $\boldsymbol{a^2+b^2=c^2}$**.**

6. *In an acute angled triangle, all the angles acute.*

7. *In an obtuse angled triangle, one of its angles is obtuse.*

EXAMPLES:

1. *If two angles of a triangle are 45^0 and 75^0, find the size of third angle.*

2. *The angles of a triangle are $(x+37^0)$, $(2x+15^0)$ and $(3x+8^0)$. Find the:*

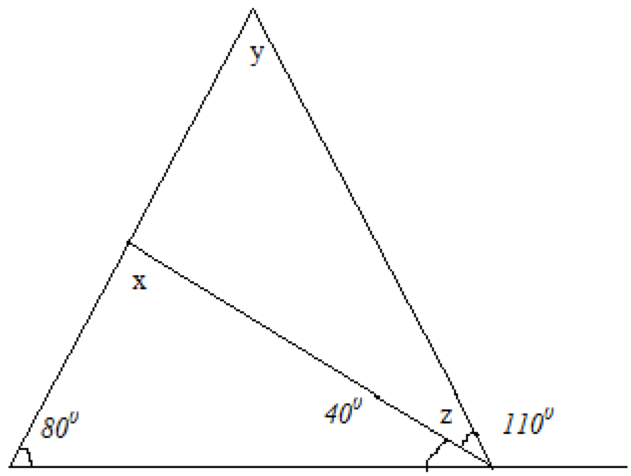
(i) *value of x.*

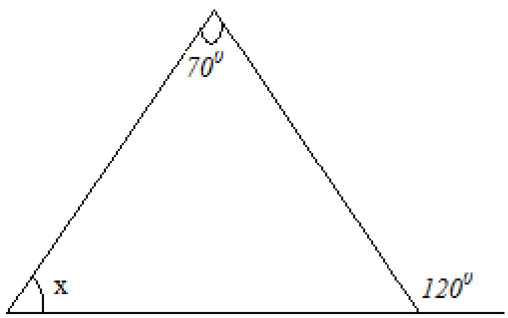
(ii) *size of each angle.*

3. *If the angles of a triangle are in the ratio 3:4:5, find all the angles.*

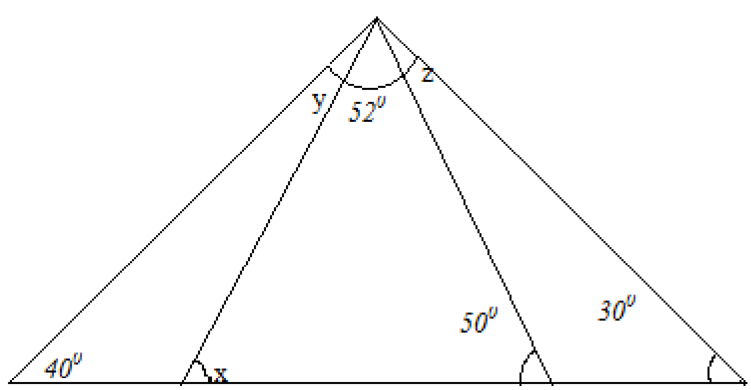
4. *Find the angles marked with letters in the diagrams below:*

(i)

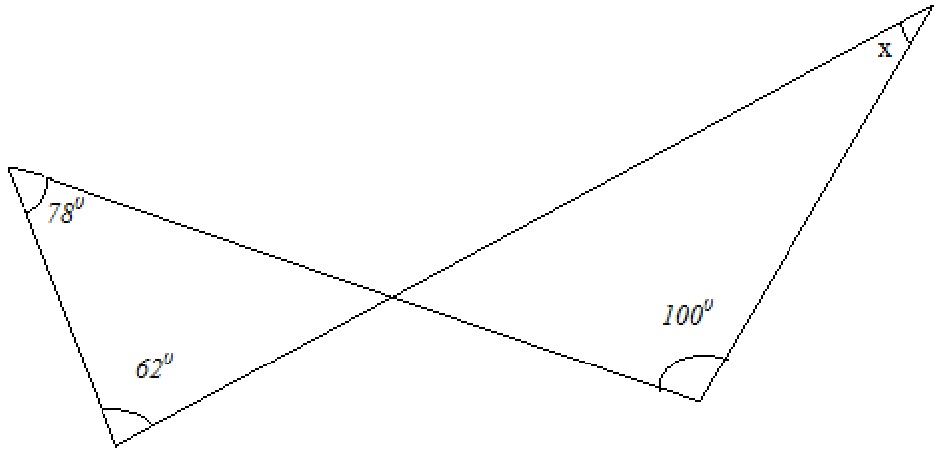




(ii)



(iii)



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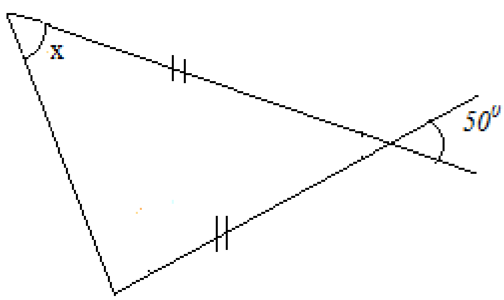
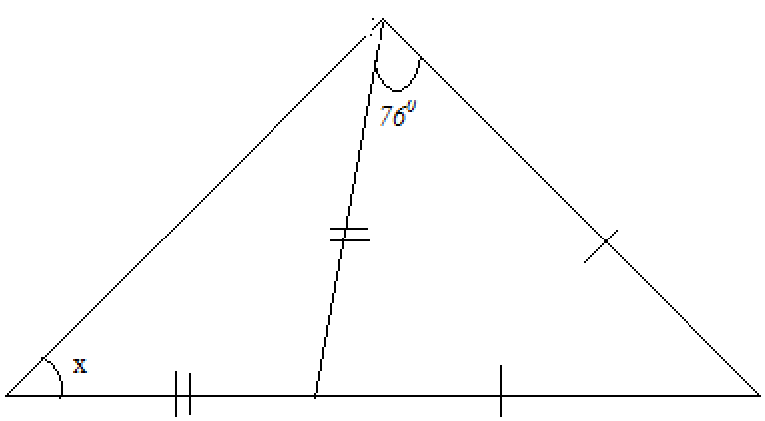


(v)

(vi)



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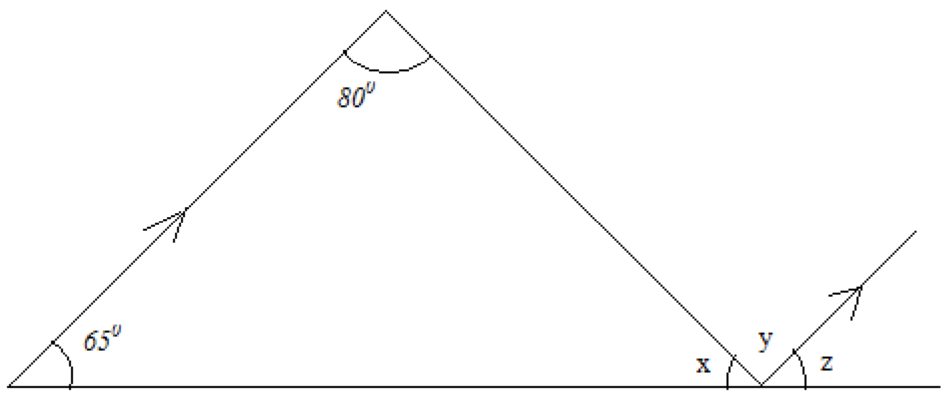
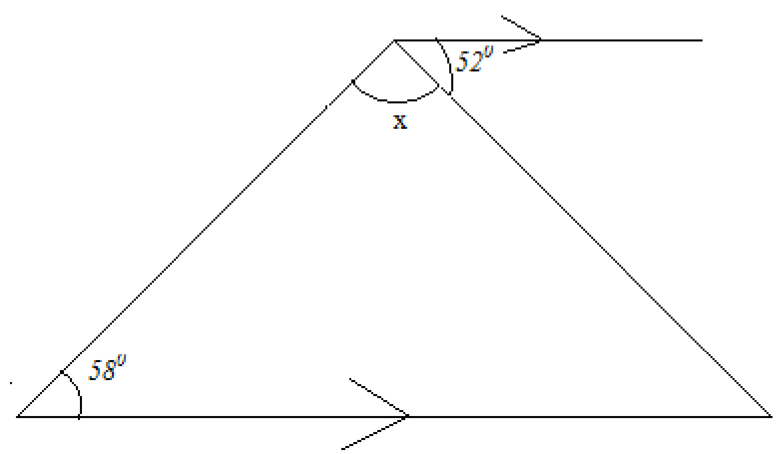
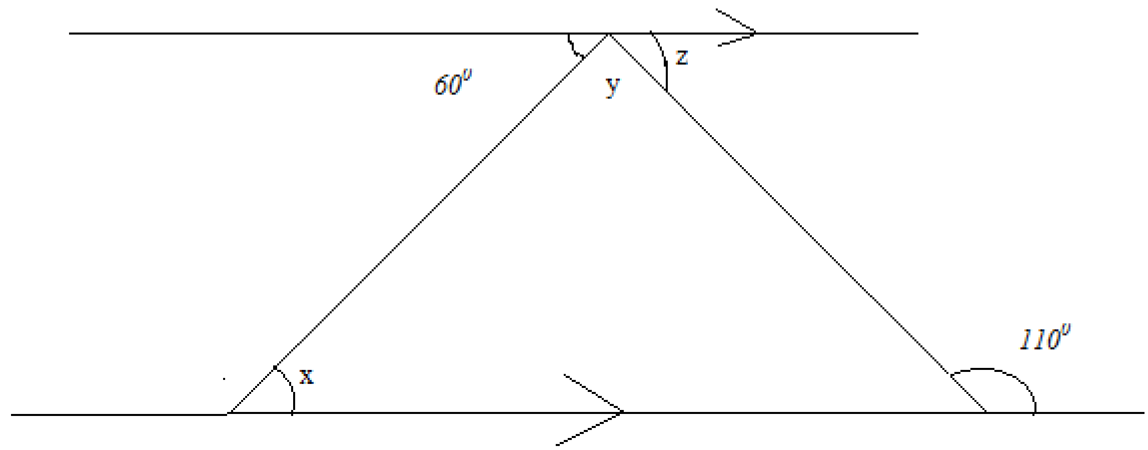
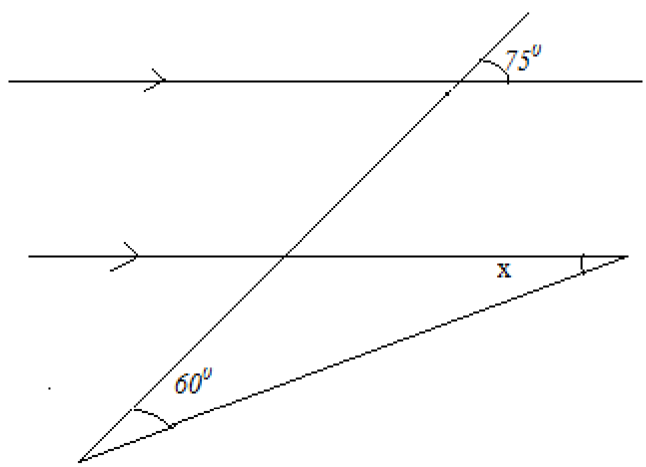




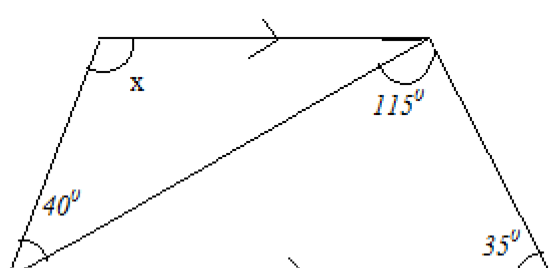
(viii)

(ix)





(x)



(xi)

5. A ladder of length 13m rests against a vertical wall with its floor 5m away from the wall. Find how far up the wall does the ladder reach.
6. Find the diagonal of a rectangle of length 8cm and width 6cm.
7. A square has diagonals of length 10cm. Find the sides of a square.
8. A cone has base radius 8cm and slant height 17cm. Find its vertical height.
9. Find the length of each side of an equilateral triangle whose height is 15cm.
10. Two buildings 24m apart are 39m and 32m tall. Find the distance between their tops.
11. Find the length of a diagonal of a rectangular box of length 12cm, width 9cm and height 8cm.
12. Find the area of a triangle whose sides are 13cm, 24cm and 13cm.

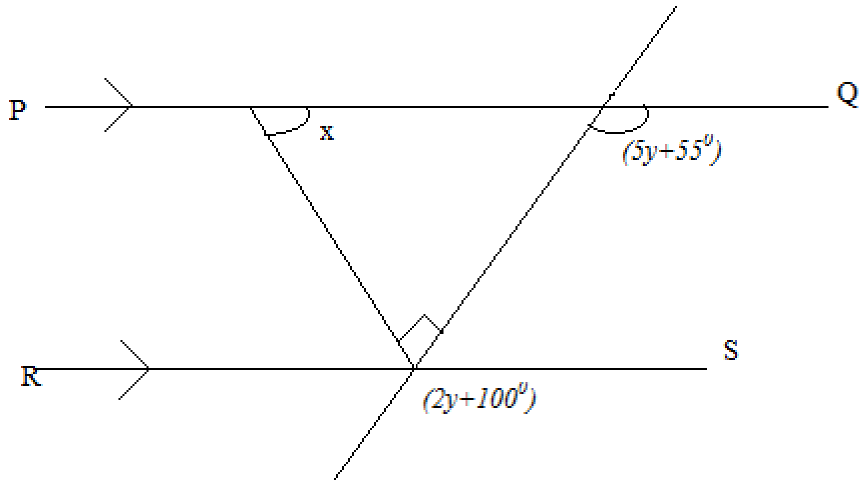
EER

1. The angles of a triangle are $(5x-17^0)$, $(3x+20^0)$ and $(2x-13^0)$. Find the size of each angle.

2. The vertical angle of an isosceles triangle is 58^0. Find the base angles.

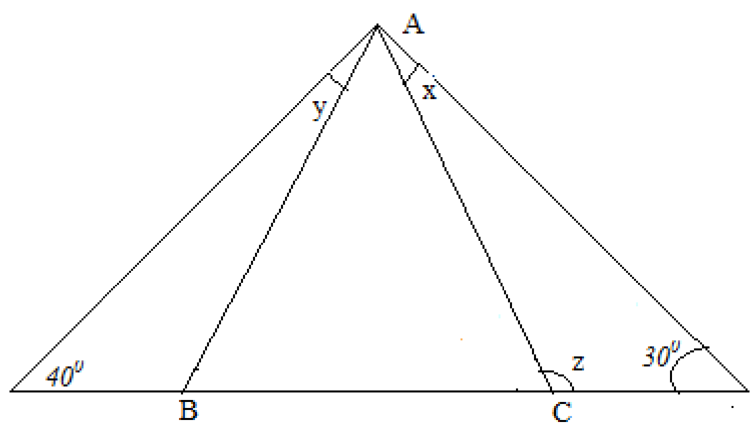
3. Find the angles of an isosceles triangle if the vertical angle is thrice the base angles.

4. In the diagram below, PQ is parallel to RS.



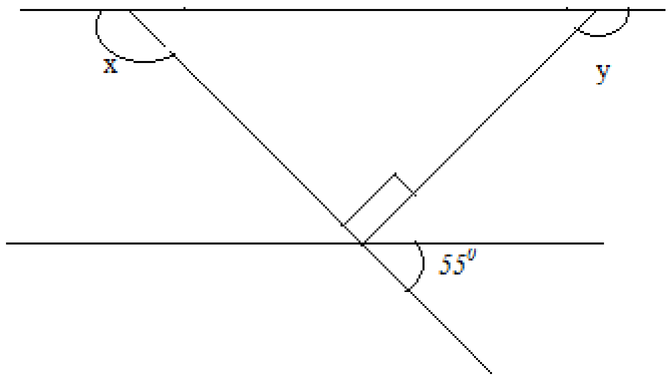
Find the angle marked x.

5. In the diagram, ABC is an equilateral triangle.

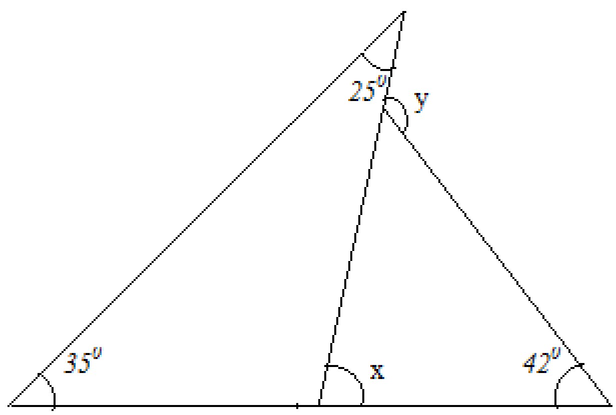


Find the size of angles marked x, y and z.

6. In the figure below, PQ is parallel to RS.

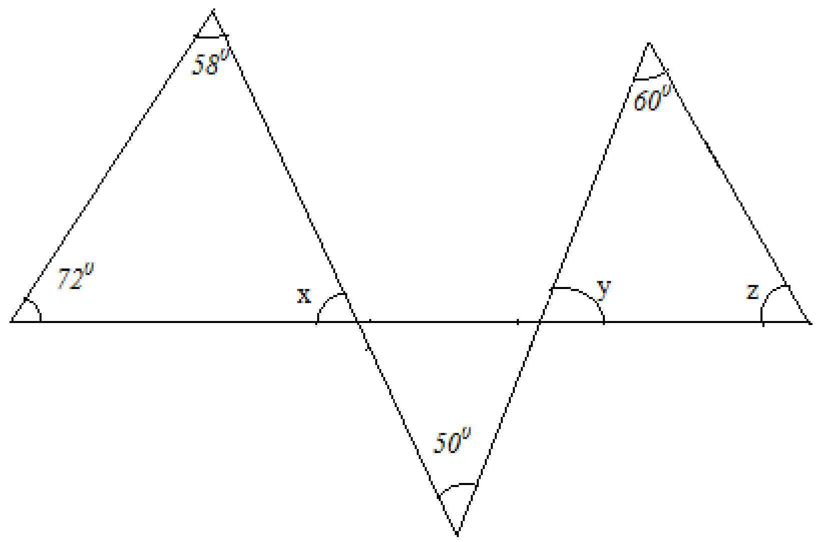


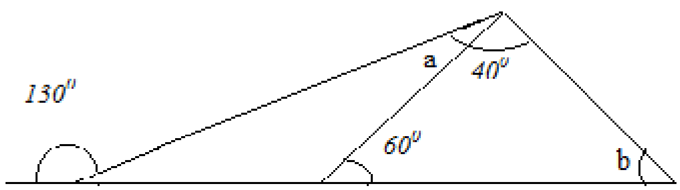
7. *Find the angles marked with letters in the figures below:*



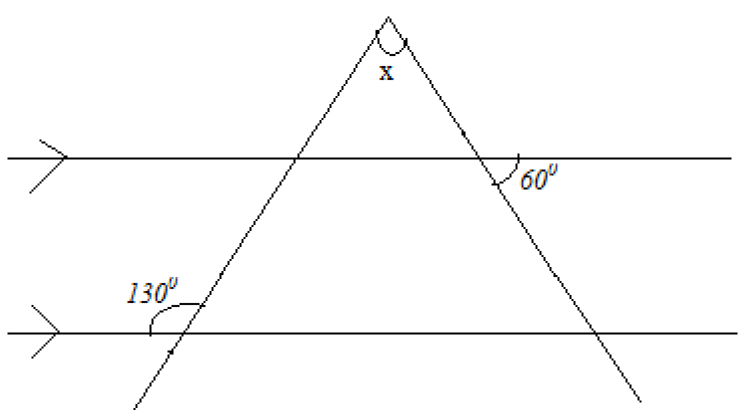
(i)

(ii)





(iii)



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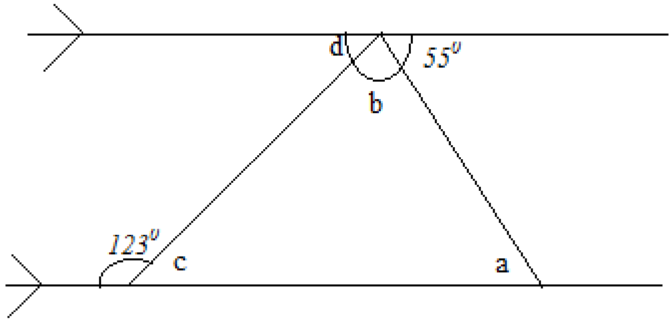
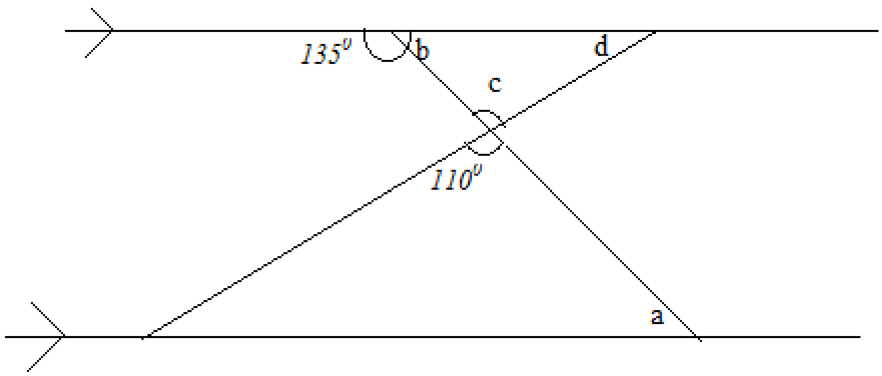
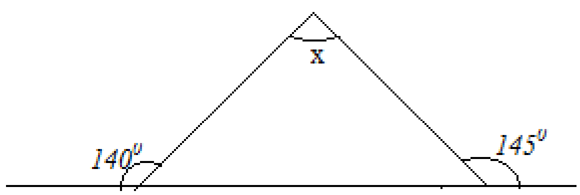
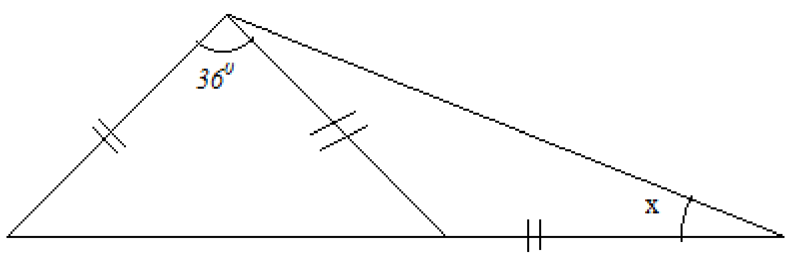


(v)



(vi)





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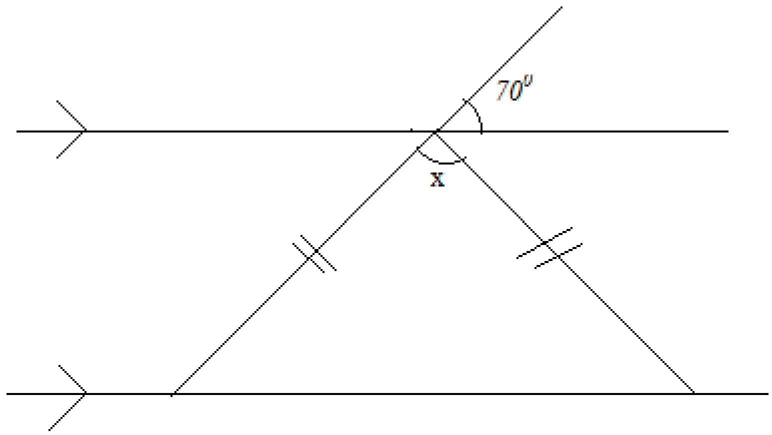
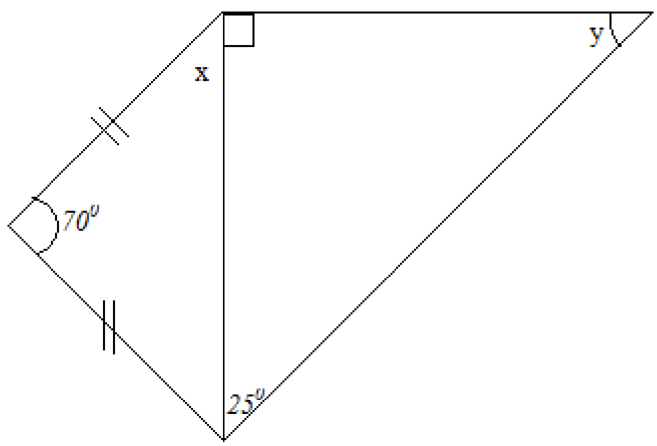
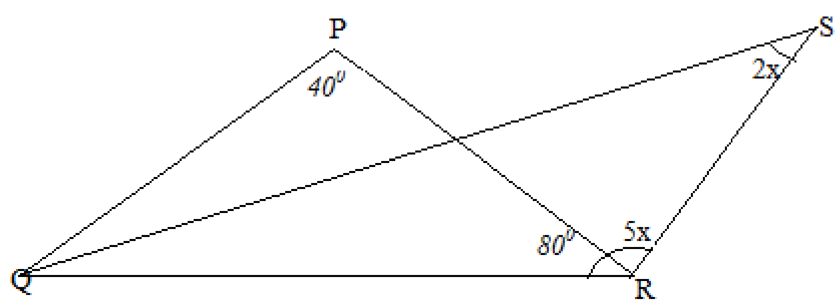


(viii)

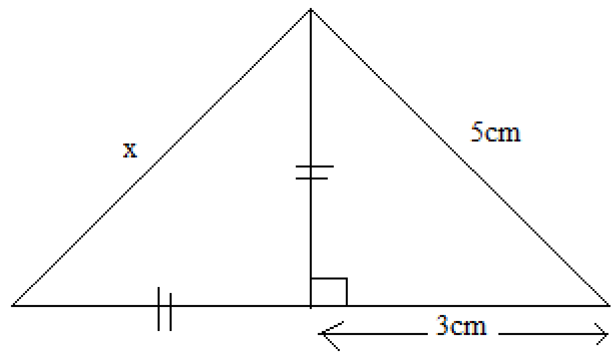


8.In the figure below, QS is the bisector of angle PQR.



Find the value of x, hence find the angles 2x and 5x.

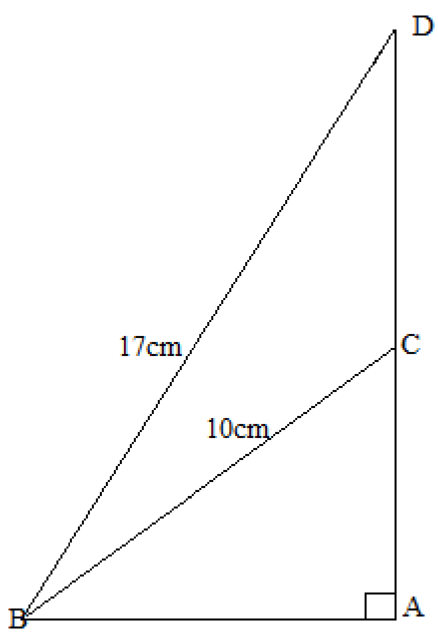
9.In the figure below, find the length of the side marked x.



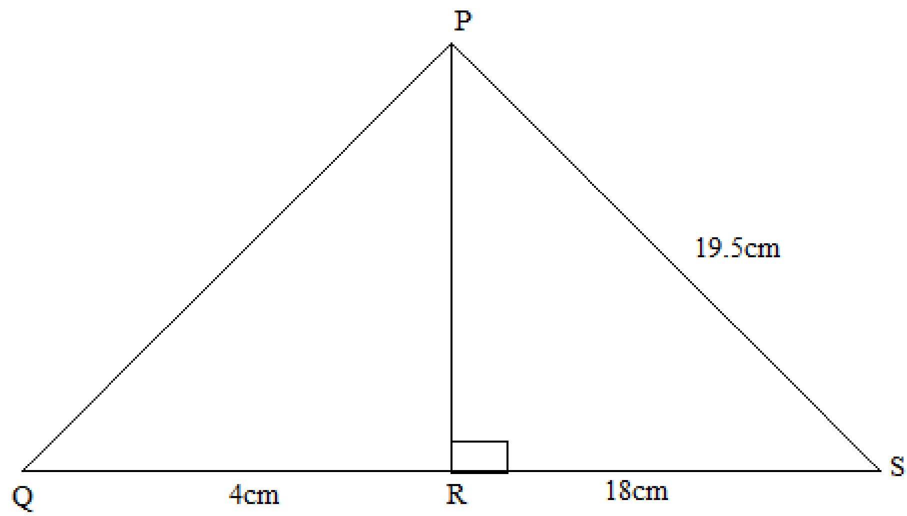
10. *Find the perimeter of a rectangle whose length is 150cm and its diagonal is 170cm long.*

11. *Find the perimeter of an isosceles triangle whose base is 16cm and its area is 240cm^2.*

12. *In the figure below, find the length of CD.*



13. *In the figure below, find the lengths PR and PQ.*

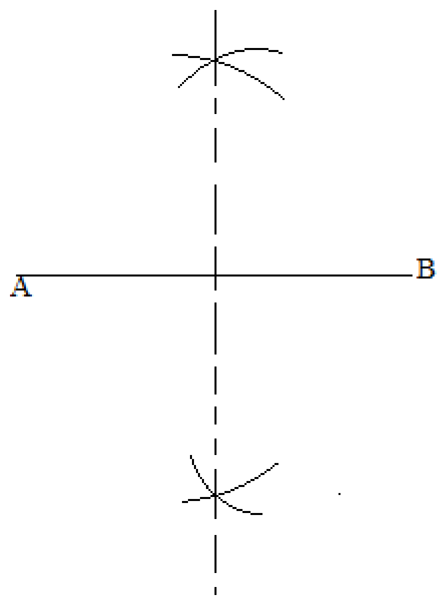


GEOMETRIC CONSTRUCTION

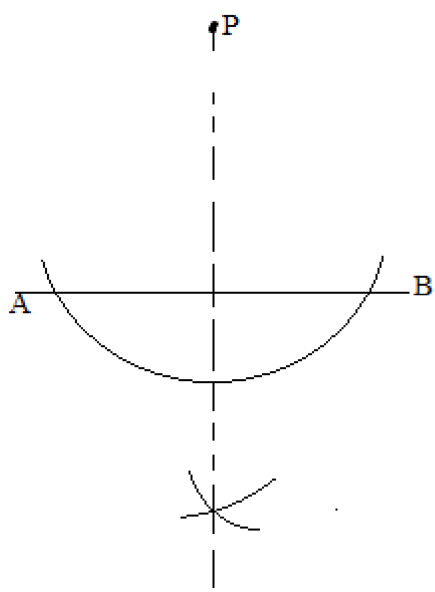
Summary

1. *In construction, we draw angles, lines and shapes using a ruler, pencil and pair of compasses only.*
2. *The angle bisector method can be used to create other angles. Thus, an angle of 45^0 is obtained by bisecting an angle of 90^0.*
3. *The supplementary angle construction method can be used to get obtuse angles. Thus, an angle of 120^0 is obtained by constructing an angle of 60^0.*

4. *The construction below shows how to draw a perpendicular bisector of a given line segment AB.*



5. *The construction below shows how to draw a perpendicular to the line AB from a given external point P.*



6. *The steps for constructing a circle inscribed in a triangle are as follows:*

(i) Construct angle bisectors of a triangle to meet at the centre of the

circle.

(ii) Construct a perpendicular from the centre point to one side of the triangle.

(iii) Place the compass at the centre point and adjust its length up to where the perpendicular crosses the triangle, and then draw the inscribed circle.

7. The steps for circumscribing a circle on a triangle are as follows:

(i) Construct the perpendicular bisectors of the two sides of the triangle to meet at the centre point of the circle.

(ii) Place the compass at the centre point and adjust its length up to any vertex of the triangle, the draw the circumscribed circle.

EXAMPLES:

1. Using a pencil, ruler and pair of compasses only, construct the following angles:

(i) 90^0 *(ii)* 45^0 *(iii)* $22\frac{1}{2}^0$ *(iv)* 135^0 *(v)* 60^0

(vi) 30^0 *(vii)* 15^0 *(viii)* $7\frac{1}{2}^0$ *(ix)* 150^0 *(x)* 165^0.

(xi) 75^0 *(xii)* 82.5^0 *(xiii)* 105^0.

1. (a) *Using a ruler and a pair of compasses only, construct a triangle **ABC** in*

*which **AB = 6·8cm, AC = 5·5cm** and **BC = 4·8cm.** Measure angle **ABC***

(b) *Draw a perpendicular from **C** onto **AB** to meet it at **D.** Measure length **CD***

(c) *Draw an inscribed circle of triangle **ABC.** Measure the radius of the* *circle*

(d) *Calculate the area enclosed between the inscribed circle and the sides of the*

*triangle **ABC.***

2. (a) *Using a ruler, pencil and a pair of compasses only, construct a triangle **ABC** such that **AB = 8·8cm,** angle **BAC = 75°** and angle **ABC = 45°.** Measure length **AC***

(b) *Draw a perpendicular from **C** onto **AB** to meet it at **D.** Measure length **CD***

(c) *Draw an inscribed circle of triangle **ABC.** Measure the radius of the circle*

(d) *Calculate the area enclosed between the inscribed circle and the sides of the*

*triangle **ABC.***

3. (a) *Using a ruler and a pair of compasses only, construct a triangle **ABC** in*

*which **BC = 7·2cm, AC = 8·4cm** and angle **ABC = 75°.** Measure length **AB***

*and angle **ACB***

(b) *Draw a perpendicular from **A** onto **BC** to meet it at **D.** Measure length **AD***

(c) *Draw a circle circumscribing triangle **ABC.** Measure the radius of the* *circle*

(d) *Calculate the area of the segments cut off by triangle* ***ABC.***

4. (a) *Using a ruler and a pair of compasses only****,*** *construct a triangle* ***ABC*** *in*

which ***AB = 5·6cm, BC = 6·2cm*** *and angle* ***ABC=135°.*** *Measure length* ***AC*** *and angle* ***BCA.***

(b) *Draw a perpendicular from* ***C*** *to meet* ***AB*** *produced at* ***D.***

(c) *Construct a circle circumscribing triangle* ***BCD*** *and state its radius.*

(d) *Calculate the area of the segments cut off by triangle* ***BCD.***

5. (a) *Using a ruler, pencil and a pair of compasses only****,*** *construct a triangle* ***ABC*** *such that* ***AB = 8·6cm,*** *angle* ***BAC = 60°*** *and angle* ***ABC = 45°.***

(b) D *is a point on the opposite side of* ***AB*** *as* ***C*** *such that* ***AD = BD*** *and*

CD = 11cm. *Draw a circle through the points* ***A, B*** *and* ***D.*** *Measure the:*

(i) *length of* ***AC*** *and angle* ***ABD***

(i) *radius of the circle*

Soln

(b) HINT: *Triangle* ***ABD*** *must be isosceles. Thus point* ***D*** *lies on the perpendicular bisector of* ***AB***

6. *Using a ruler and pair of compasses only****;***

(i) *Construct a parallelogram* ***ABCD*** *such that* ***AB=6cm, BC=4·8cm*** *and*

angle ***ABC = 150°.***

(ii) *Draw a perpendicular from* ***D*** *onto* ***AB*** *to meet it at* ***M.*** *Measure the length* ***DM.*** *Hence find the area of the parallelogram* ***ABCD.***

(iii) *Draw a circle through the points* ***M, A*** *and* ***D.*** *Measure the radius of the circle*

7. *Using a ruler and pair of compasses only, construct:*

(i) *a quadrilateral* ***PQRS*** *such that* ***QR = 4·5cm, RS =6cm, SP = 7·5cm, PQ = 10·5cm*** *and angle* ***QRS = 45°.***

(ii) *point* ***T*** *on* ***RQ*** *produced such that* ***PT = ST.*** *Join the points* ***P, S*** *and* ***T.*** *Measure length* ***PT*** *and angle* ***PTS.***

(iii) *a circle through the points* ***P, T*** *and* ***R.*** *Measure the radius of the circle*

8. (a) *Using a ruler, pencil and a pair of compasses only, construct a triangle* ***ABC*** *such that* ***AB = 6cm, AC = 8cm*** *and angle* ***BAC = 30°.***

(b) S *is a point on the opposite side of* ***AC*** *as* ***B*** *such that* ***AS = SC*** *and* ***BS = 8cm.*** *Measure length* ***AS*** *and angle* ***ABS***

(c) (i) *On the same side of* ***BS*** *as* ***C,*** *construct the locus of a point* ***K*** *such that its distance from* ***BS*** *is the same as the distance of* ***C*** *from* ***BS***

(ii) *Given further that angle* ***BKS = 90°,*** *find by construction two possible positions* k_1 *and* k_2 *of point* ***K.*** *Measure length* k_1k_2

Soln

(b) HINT: *Triangle* ***ACS*** *must be isosceles. Thus point* ***S*** *lies on the* *perpendicular bisector of **AC***

(c) (i) HINT:** The locus of **K** is a line through **C** and parallel to **BS.

***(ii)** Since **∠BKS = 90°, K** must lie on the semi–circle with **BS** as diameter*

EER:

1. (a)** Using a ruler, pencil and pair of compasses only, construct a triangle **ABC

*such that **AB = 7·5cm** and **AC = 11·4cm** and angle **ABC = 120°.***

(b)** Construct a perpendicular from **C** to meet **AB** produced at **D.

***(c)** Draw a circle circumscribing triangle **BCD.** Hence calculate the area of the*

*circle to **2** decimal places.*

2. (a)** Using a ruler, pencil and a pair of compasses only, construct a triangle **ABC** such that **AB = 9·7cm, BC = 8·6cm** and angle **BAC = 60°.

(b) D** is a point on the opposite side of **AB** as **C** such that angle **ABD = 45°

*and **CD = 10·4cm.** Draw a circle through the points **B, C** and **D.** Measure*

*length **AC** and the radius of the circle*

3. (a)** Using a ruler and a pair of compasses only, construct a triangle **ABC

in

*which **AB = BC = 6·8cm** and angle **ABC=120°.** Measure length **AC** and angle **BCA.***

(b)** Draw a perpendicular from **C** to meet **AB** produced at **O.

***(c)** Construct a circle circumscribing triangle **BOC** and state its radius.*

(d)** Calculate the area of the segments cut off by triangle **BOC.

***44. (a)** Using a ruler and a pair of compasses only, construct a quadrilateral **ABCD** in*

*which **AB = 5cm, BC = 6cm, CD = 9cm** and angle **BCD = 135^0***

(b)** Construct a perpendicular from **D** to meet **BC** produced at **M.

***(c)** Construct a circumcircle of triangle **CDM** and determine the:*

(i)** length of **AD

***(i)** radius of the circle*

***45. (a)** Using a ruler and a pair of compasses only, construct a quadrilateral*

***ABCD** in which **AB = 7cm, BC = 6cm, AD = 5cm,** angle **BAD = 105°** and*

ABC = 60°**. Join **A** to **C** to form triangle **ABC.

***(b)** Construct an inscribing circle of triangle **ABC** and determine the:*

(i) *length of* ***AC***

(i) *radius of the circle*

7. (a) (i) *Find the area of a triangle with vertices* ***P(-2, -2) Q(2 , 4)*** *and* ***R(5 , 0).***

(ii) *Construct a circle circumscribing triangle* ***PQ R.*** *Hence calculate the*

area of the segments cut off by triangle ***PQ R.***

4. (a) *Using a ruler and a pair of compasses only, construct a quadrilateral*

ABCD *in which* ***AB = 5cm, BC = 6cm, CD = 9cm*** *and angle* ***BCD = 135°***

(b) *Construct a perpendicular from* ***D*** *to meet* ***BC*** *produced at* ***M.***

(c) *Construct a circumcircle of triangle* ***CDM*** *and determine the:*

(i) *length of* ***AD***

(ii) *radius of the circle*

45. (a) *Using a ruler and a pair of compasses only, construct a quadrilateral* ***ABCD*** *in*

which ***AB = 7cm, BC = 6cm, AD = 5cm,*** *angle* ***BAD = 105^0*** *and*

ABC = 60º*. Join* ***A*** *to* ***C*** *to form triangle* ***ABC.***

(b) *Construct an inscribing circle of triangle* ***ABC*** *and determine the:*

(i) *length of* ***AC***

(ii) *radius of the circle.*

POLYGONS

Summary:

1. (i) A polygon is a closed figure with straight sides.

(ii) The table below shows the different polygons.

Number of sides	***Polygon name***
3	*Triangle*
4	*Quadrilateral*
5	*Pentagon*
6	*Hexagon*
7	*Heptagon*
8	*Octagon*
9	*Nonagon*
10	*Decagon*
11	*Hendecagon*
12	*Dodecagon*

2. In any polygon with n- sides, the following properties apply

(i) Interior angle sum $=(n-2)\times 180^0$

(ii) Exterior angle sum $= 360^0$.

(iii) Each interior angle + each exterior angle $=180^0$.

(iv) Number of diagonals $=\frac{1}{2}n(n-3)$.

3. In a regular polygon with n- sides, the following properties apply

(i) *All the sides and angles are equal.*

(ii) *Each interior angle =* $\frac{\text{interior angle sum}}{\text{Number of angles}}$

$$= \frac{(n-2)\times 180^0}{n}$$

(iii) *Each exterior angle =* $\frac{\text{Exterior angle sum}}{\text{Number of angles}}$

$$= \frac{360^0}{n}$$

EXAMPLES:

1. *Find the interior angle sum of a decagon.*

   Solution:

   HINT: *A decagon has 10 sides.*

   Interior angle sum $= (n-2)\times 180^0$

   $= (10-2)\times 180^0$

   $= 1440^0$

2. *Find the number of sides of a polygon whose interior angle sum is* 900^0.

   Solution:

   If $(n-2)\times 180^0 = 900^0$

   $\Rightarrow n=7.$

3. *The angles of a hexagon are x, $x+58^0$, $x-4^0$, 120^0, 130^0 and 140^0. Find the value of x.*

Solution

If angle sum = $(n-2)\times 180^0$

$\Rightarrow x+x+58+x-4+120+130+140 = (6-2)\times 180^0$

$3x+444=720^0$

$\therefore x=92^0$.

4. *The angles of a pentagon are in the ratio 3:7:5:4:8. Find the smallest and largest angles of the pentagon.*

Solution

Angle sum = $(n-2)\times 180^0 = (5-2)\times 180^0 = 540^0$.

Smallest angle = $\frac{3}{27}\times 540^0 = 60^0$

Largest angle = $\frac{8}{27}\times 540^0 = 160^0$

5. *Four angles of a polygon are 110^0 each and the remaining angles are 170^0 each. Find the number of sides of the polygon.*

Solution

Angle sum = $(n-2)\times 180^0$

$4(110^0) + 170^0(n-4) = (n-2)\times 180^0$

$\therefore n=12$.

6. *Find the size of each interior angle of a regular hexagon.*

   Solution

   Each interior angle= $\frac{(n-2)\times 180^{0}}{n}$

   $= \frac{(6-2)\times 180^{0}}{6}$

   $=120^{0}$.

7. *Find the number of sides of a regular polygon whose each interior angle is* 135^{0}.

   Solution

   Each interior angle= $\frac{(n-2)\times 180^{0}}{n}$

   $\frac{(n-2)\times 180^{0}}{n} = 135^{0}$

   $\therefore n=8$.

8. *Find the size of each exterior angle of a regular pentagon.*

   Solution

   Each exterior angle= $\frac{360^{0}}{n}$

   $=\frac{360^{0}}{5}$

   $=72^{0}$.

9. *Find the number of sides of a regular polygon whose each exterior angle is* 40^{0}.

   Solution

Each exterior angle= $\frac{360^0}{n}$

$$\frac{360^0}{n} = 40^0$$

$\therefore$ *n=9.*

10. *The size of each interior angle of a regular polygon is 4 times the exterior angle. Find the number of sides of the polygon.*

Solution

If I=4E.

$\Rightarrow \frac{(n-2)\times 180^0}{n} = 4(\frac{360^0}{n})$

$\therefore$ *n=10.*

11. *The size of each interior angle of a regular polygon is one and a half times the exterior angle. Find the number of sides of the polygon.*

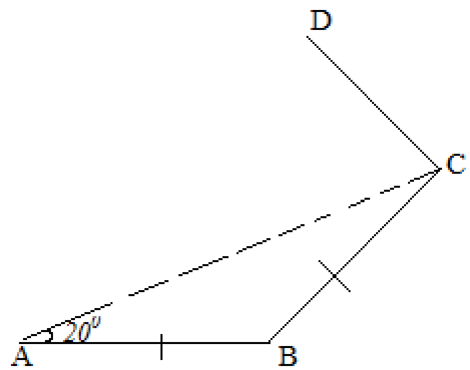
Solution

If I= $1\frac{1}{2}E$

$\Rightarrow \frac{(n-2)\times 180^0}{n} = \frac{3}{2}(\frac{360^0}{n})$

$\therefore$ *n=5.*

12. *The figure ABCD below shows part of the sides of a regular polygon.*

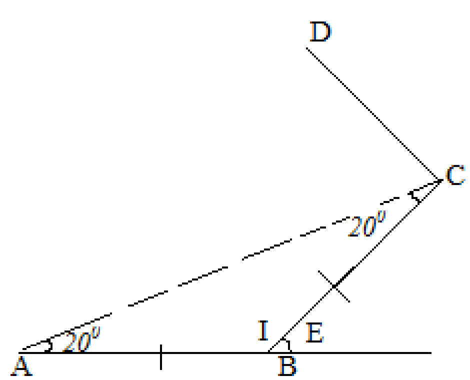


Find the:

(i) *Size of each interior and exterior angle of the polygon.*

(ii) *Number of sides of the polygon.*

Solution



(i) $I+20^0+20^0=180^0$

$I=140^0$

Also, $E=20^0+20^0=40^0$.

(ii) *Each exterior angle* $=\dfrac{360^0}{n}$

$$\frac{360^0}{n}=40^0$$

$$\therefore n=9.$$

METHOD II

Each interior angle $=\dfrac{(n-2)\times 180^0}{n}$

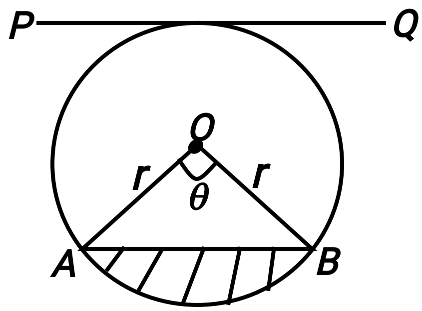
$$\frac{(n-2)\times 180^0}{n}=140^0$$

$$\therefore n=9.$$

PARTS OF A CIRCLE

Summary:

1. *The following are the main parts of a circle with centre **O***



(i) AB *is called an arc* ***(ii) OAB*** *is called a sector*

(iii) *line **AB** is called a chord* ***(iv)*** *line **PQ** is called a tangent*

(v) *The shaded part is called a segment* ***(vi)*** θ *is the angle subtended by an arc*

(vii) OA = OB = *radius of the circle*

2. Theorem: *Since **OA = OB,** then **OAB** is an isosceles triangle with its line of symmetry bisecting chord **AB***

3. *The following formulas are used in relation to the above circle:*

(i) *Circumference* $= 2\pi r$ *or* πd ***(ii)*** *Area* $= \pi r^2$

(iii) *Arc length **AB*** $= \frac{\theta}{360} \times 2\pi r$ ***(iv)*** *Sector area **OAB*** $= \frac{\theta}{360} \times \pi r^2$

(v) *Segment area **= sector area** − **triangle area***

EXAMPLES:

1. *Find the length of an arc which subtends an angle of **126°** at the centre* *of a circle of radius **14cm.** Find also the length of the major arc*

***2.** A chord is **8cm** away from the centre of a circle of radius **17cm.** Find the:*

***(i)** length of the chord*

***(ii)** size of the angle subtended by the chord at the centre*

***3.** A chord of length **10cm** is **12cm** away from the centre of the circle. Find the:*

***(i)** radius of the circle*

***(ii)** length of the minor arc*

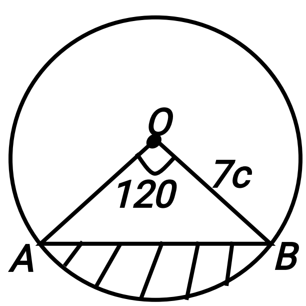
***(iii)** area of the minor sector*

***(iv)** area of the minor segment*

***4.** A sector of a circle of radius **12cm** has an angle of **150°** at the centre. The sector is folded to form a cone. Find the radius of the circular end of the cone*

5.** Find the distance between two parallel chords of lengths **32cm** and **24cm** which lie on opposite sides of the centre in a circle of radius **20cm

6.** in the circle below, sector **OAB** has a radius of **7cm** and subtends an angle of **120°** at its centre **O.



Find the:

***(i)** shortest distance of chord **AB** from the centre* *(ii) perimeter of the shaded segment*

*(iii) perimeter of the region enclosed between chord **AB** and the major arc*

(iv) area of the shaded segment

***7.** The length of the common chord of two intersecting circles of radius **10cm** and **17cm** is **6cm.** Find the:*

***(i)** angle subtended by the chord at the centre of the two circles*

***(ii)** area common to the two circles*

***8.** Two equal circles of radius **5cm** intersect at right angles. Find the:*

***(i)** distance between the centres of the two circles*

***(ii)** area common to the two circles*

***9.** A sector of a circle of radius **25cm** has an angle of **100·8°** at the centre. The sector is folded to form a cone. Find the:*

***(i)** radius of the circular end of the cone*

***(ii)** height of the cone*

***(iii)** volume of the cone*

***(iv)** total surface area of the cone*

***HINT: (iv) T·S·A** = πrl since its circular end is open*

***10.** The minor segment of a circle has a height of **4cm** and a chord of length **16cm.** Find the:*

***(i)** radius of the circle*

***(ii)** area of the segment*

11. *A dog tied on a rope* ***5m*** *long is tethered to a tree* ***3m*** *from a straight path. For what distance along the path is one in danger of being bitten by the dog?*

EER:

1. *A chord of length* ***70cm*** *subtends an angle of* ***120°*** *at the centre of the circle. Find the:*

(i) *radius of the circle*

(ii) *distance of the chord from the centre*

(iii) *area of the minor segment*

2. *A chord of length* ***6cm*** *is* ***4cm*** *away from the centre of the circle. Find the circumference of the circle*

3. *A dog tied on a rope* ***2·5m*** *long is tethered to a tree* ***2m*** *from a straight path. For what distance along the path is one in danger of being bitten by the dog?*

4.*A sector of a circle of radius* ***10cm*** *has an angle of* ***100°*** *at the centre. Find the*

(i) *perimeter of the sector*

(ii) *area of the minor segment*

5. *A chord of length* ***6cm*** *makes an angle of* ***40°*** *with the radius of the circle. Find the circumference of the circle*

6. *A chord* ***3·5cm*** *away from the centre of the circle subtends an angle of* ***120°*** *at its centre. Find the area of the major segment*

7. *The chord of a circle of radius* ***10cm*** *subtends an angle of* ***120°*** *at its* *centre. Find the perimeter of the region enclosed between the chord and the major arc*

8. *Find the distance between two parallel chords of lengths* ***32cm*** *and* ***24cm*** *which lie on the same side of the centre in a circle of radius* ***20cm***

9. *A sector of a circle of radius* ***12·5cm*** *has an angle of* ***100·8°*** *at the centre. The sector is folded to form a cone. Find the:*

(i) *radius of the circular end of the cone*

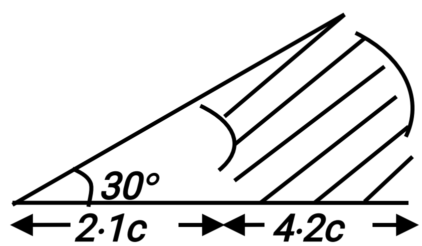
(ii) *height of the cone*

(iii) *volume of the cone*

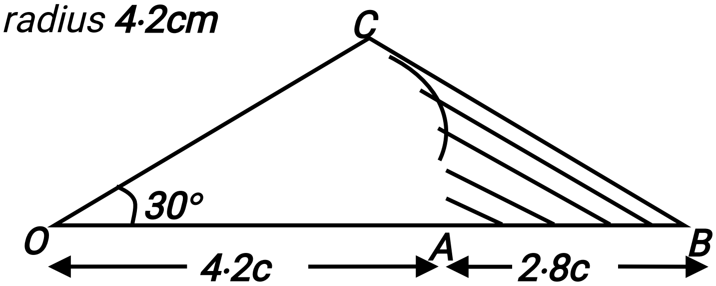
(iv) *total surface area of the cone*

10. *Find the distance between two parallel chords of lengths* ***24cm*** *and* ***10cm*** *which lie on opposite sides of the centre in a circle of radius* ***13cm***

11. *In the figure below, find the shaded area bounded by two concentric arcs*



12. *In the triangle* ***OBC*** *below, arc* ***AC*** *subtends an angle of* ***30°*** *at the centre* ***O*** *of a circle of radius* ***4·2cm***



If ***AB = 2·8cm,*** *find the area of the shaded region*

13. *The length of the common chord of two intersecting circles of radius* ***28cm*** *and* ***20cm*** *is* ***30cm.*** *Find the:*

(i) *angle subtended by the chord at the centre of the two circles*

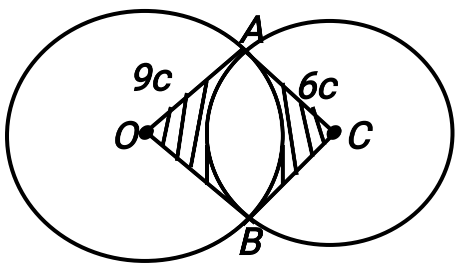
(ii) *area common to the two circles*

14. *The distance between the centres of two intersecting equal circles of radius* ***5cm*** *is* ***8cm.*** *Find the:*

(i) *length of the common chord of the two circles*

(ii) *area common to the two circles*

15. *Two circles with centres* ***O*** *and* ***C*** *and radius* ***9cm*** *and* ***6cm*** *intersect at points* ***A*** *and* ***B*** *as shown*



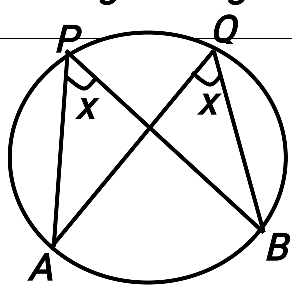
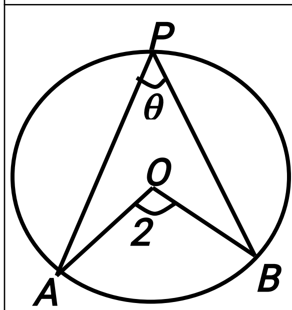
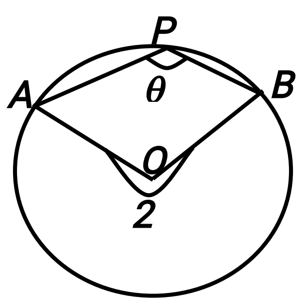
Given that the distance between ***O*** *and* ***C*** *is* ***13cm,*** *find the:*

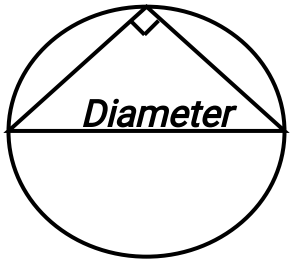
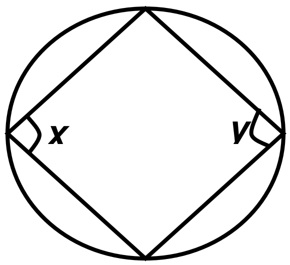
(i) *reflex angle* ***AOB***

(ii) *length of chord* ***AB***

(iii) *area of the shaded region*

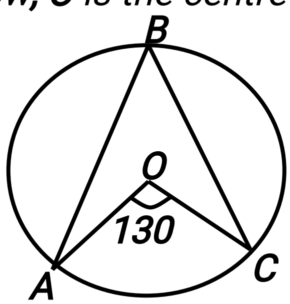
ANGLES IN A CIRCLE

Angles diagram	*Circle theorems*
	*The angles subtended at the circumference by the same arc are equal* ∴ $\angle APB = \angle AQB$ ***In short:*** *Angles in the same segment are equal*
 	*The angle subtended at the centre by an arc is twice the angle it subtends at its circumference* ∴ $\angle AOB = 2\angle APB$ ***In short:*** *Angle at the centre is twice the angle at the circumference*

	*The angles subtended at the circumference by a semi–circle is **90°***
	*The opposite angles of a cyclic quadrilateral add up to **180°*** $\therefore$ ***x + y = 180°***

EXAMPLES:

1. *In the figure below, **O** is the centre of the circle and **∠AOC = 130°**. Find **∠ABC***



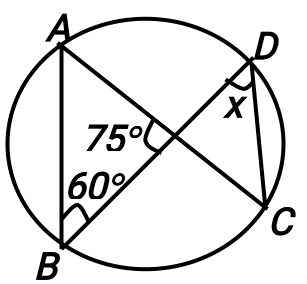
Soln:

$\angle AOC = 2\angle ABC$

$\Rightarrow 130° = 2\angle ABC$

$\therefore\ \angle ABC$ ***= 65°***

2. *In the circle below, find the size of angle marked **x***



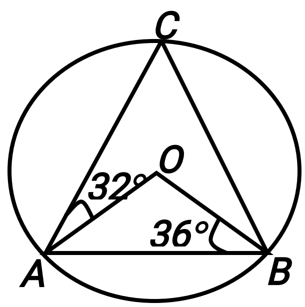
Soln:

$\angle BAC = x$ *(angles in the same segment)*

$\Rightarrow x = 180^\circ - (60^\circ + 75^\circ)$ *(angles in a triangle)*

$\therefore$ $\boldsymbol{x = 45^\circ}$

3. *In the circle below* ***O*** *is its centre* $\boldsymbol{\angle ABO = 36^\circ}$ *and* $\boldsymbol{\angle OAC = 32^\circ}$*. Find* $\boldsymbol{\angle ACB}$ *and* $\boldsymbol{\angle OBC}$



Soln:

$\boldsymbol{\nabla AOB}$ *is isosceles since OA = OB*

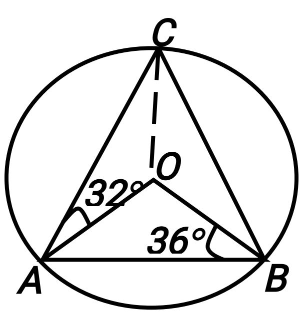
$\Rightarrow \boldsymbol{\angle AOB} = 180^\circ - (2 \times 36^\circ) = \boldsymbol{108^\circ}$ *(angles in a triangle)*

$\angle AOB = 2\angle ACB$

$\Rightarrow 108^\circ = 2\angle ACB$

$\therefore \boldsymbol{\angle ACB} = \boldsymbol{54^\circ}$

Also:



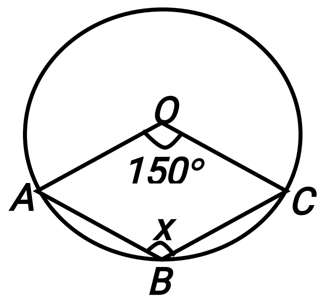
$\Rightarrow \boldsymbol{\angle AOC} = 180^\circ - (2 \times 32^\circ) = \boldsymbol{116^\circ}$ *(angles in a triangle)*

$\angle BOC = 360^\circ - (108^\circ + 116^\circ) = \boldsymbol{136^\circ}$ *(angles at a point)*

$\Rightarrow\ 2\angle OBC = 180° - 136°$ (angles in an triangle)

$\therefore\ \angle OBC = 22°$

4. In the circle below **O** is its centre and $\angle AOC = 150°$. Find the size of angle marked **x**



Soln:

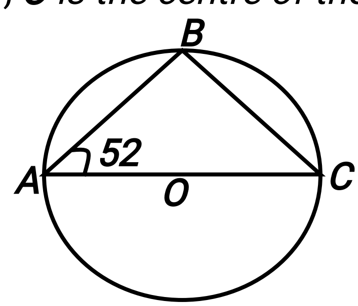
Reflex angle $\angle AOC = 360° - 150° = 210°$ (angles at a point)

$\Rightarrow\ \angle AOC = 2\angle ABC$

$210° = 2\angle ABC$

$\therefore\ \angle ABC = 105°$

5. In the figure below, **O** is the centre of the circle of radius **7cm** and $\angle BAC = 52°$.



Find the:

(i) size of angle **ACB**

(ii) lengths of **AB** and **BC**

Soln:

(i) $\angle$***ABC = 90°*** *(angle in a semi–circle)*

$\therefore$ $\angle$***ACB*** *= 180° – (90° + 52°) =* ***38°*** *(angles in a triangle)*

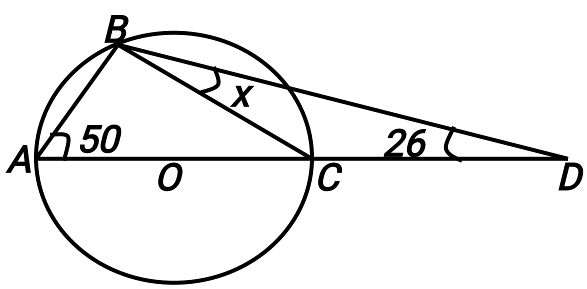
(ii) $\cos 52 = \frac{AB}{14}$

$\therefore$ *AB =* ***8·6193cm***

Also $\sin 52 = \frac{BC}{14}$

$\therefore$ *AB =* ***11·0322cm***

6. *In the circle below* **O** *is its centre. Find the size of angle marked* ***x***

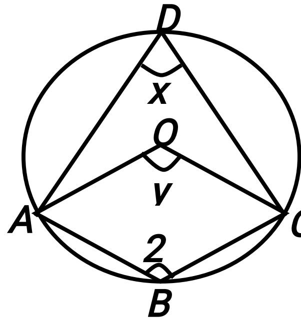


Soln:

(i) $\angle$***ABC = 90°*** *(angle in a semi–circle)*

$\therefore$ ***x*** *= 180° – (90° + 50° + 26°) =* ***14°*** *(angles in a triangle)*

7. *In the circle below* **O** *is its centre. Find the size of the angles marked* ***x*** *and* ***y***



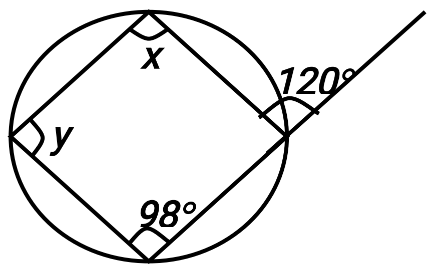
Soln:

x + 2x = 180° (angles in a cyclic quadrilateral)

$\therefore$ ***x = 60°***

Also ***y*** *= 2x = 2(60°) =* ***120°***

8. *In the circle below, find the size of the angles marked* ***x*** *and* ***y***



Soln:

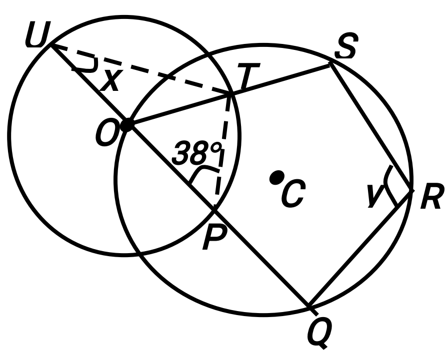
x + 98° = 180° (angles in a cyclic quadrilateral)

$\therefore$ ***x = 82°***

Also *y + 60° = 180° (angles in a cyclic quadrilateral)*

$\therefore$ ***y = 120°***

9. *In the diagram below,* ***C*** *and* ***O*** *are centres of two intersecting circles.*
$\angle$ ***CPT = 38°***



Find the size of the angles marked ***x*** *and* ***y***

Soln:

(i) *$\angle$**UTP** = **90°**   (angle in a semi–circle)*

*$\therefore$ **x** = 180° − (90° + 38°) = **52°**   (angles in a triangle)*

Also: *$\angle$**POT** = 180° − (2 × 38°) = **104°**   (angles in a triangle)*

y + 104° = 180° (angles in a cyclic quadrilateral)

*$\therefore$ **y = 76°***

10. *The points **P(–2, –1), Q(h, 7)** and **R(–3, 6)** lie on a circle with diameter **PQ.***

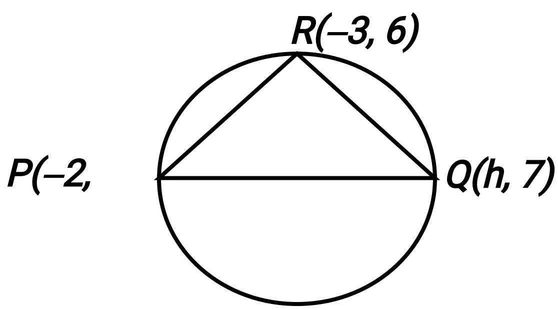
(i) *State with a reason the size of angle **PRQ***

(ii) *Show that **h = 4***

(iii) *Find the coordinates of the centre and radius of the circle*

Soln:

(i)



*$\angle$**PRQ** = **90°**   (angle in a semi–circle)*

(ii) Hint: *This could be done using gradient method*

*Gradient of **PR** =* $\frac{6 - {}^{-}1}{-3 - {}^{-}2} = -7$

*Gradient of **QR** =* $\frac{7 - 6}{h - {}^{-}3} = \frac{1}{h+3}$

For perpendicular lines, $-7 \times \left(\frac{1}{h+3}\right) = -1$

*$\therefore$ **h = 4***

(iii) ***Hint:*** *Centre is the midpoint of the diameter and radius is half the diameter*

$$\textbf{Centre} = \left(\frac{-2+4}{2}, \frac{-1+7}{2}\right) = (2, 3)$$

$$\textbf{Radius} = \sqrt{(4-2)^2 + (7-3)^2} = \textbf{5 units}$$

EER:

1. *The vertices of an equilateral triangle* ***A, B*** *and* ***C*** *lie on a circle of radius* ***8cm.*** *Find the:*

(i) *distance of any side of the triangle from the centre of the circle*

(ii) *length of the side of the triangle*

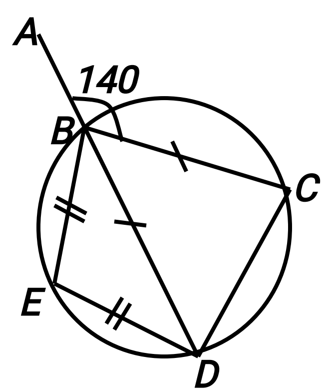
2. *The vertices of an equilateral triangle of side* **12cm** *lie on a circle. Find the:*

(i) *radius of the circle*

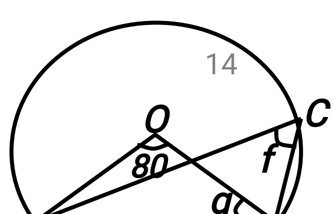
(ii) *distance of any side of the triangle from the centre of the circle*

(iii) *area of the segments cut off by the triangle*

3. *In the circle below,* ***BC = BD, BE= DE*** *and* ***∠ABC = 140°.*** *Find the size of angle* ***EDB***

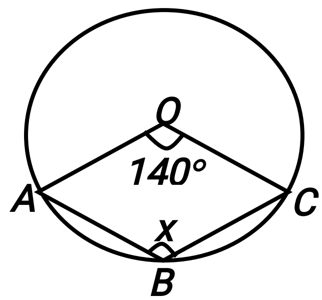


4. *In the circle below* ***O*** *is its centre and* ***∠AOB = 80°.*** *Find the size of the*

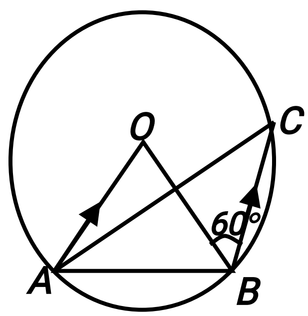


*angles marked **f** and **g***

5.** In the circle below **O** is its centre and **∠AOC = 150°.** Find the size of angle marked **x



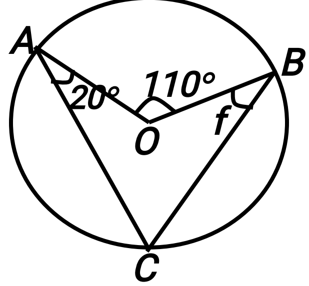
6.** In the circle below **O** is its centre**, AO** is parallel to **BC** and **∠OBC = 60°.



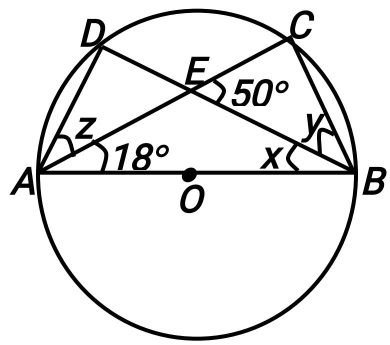
Find the size of angle:

(i) AOB ***(ii) ACB*** ***(iii) CAB***

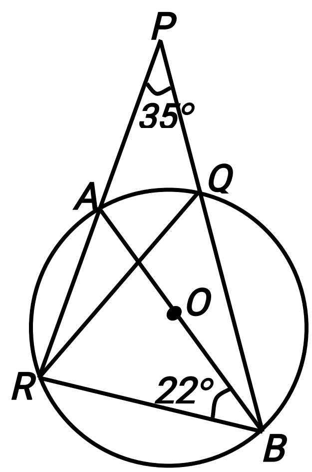
7.** In the circle below **O** is its centre**, ∠AOB = 110°** and **∠OAC = 20°.** Find the size of angle marked **f



8.** In the figure below **AB** is the diameter of a circle centre **O.** **∠BAC = 18°** and **∠BEC = 50°.** Find the size of the angles marked **x, y** and **z



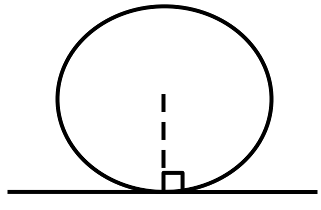
9.** In the figure below **AB** is the diameter of a circle centre **O.** **∠APQ = 35°** and **∠ABR = 22°.

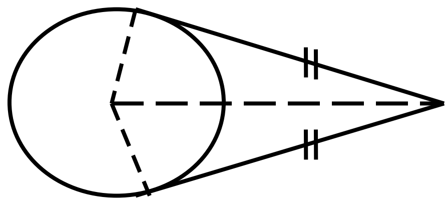
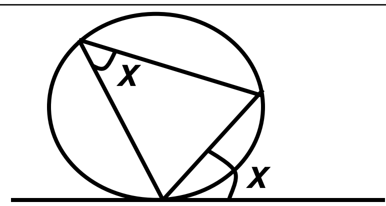


Find the size of angle:

(i) ABQ ***(ii) QRA*** ***(iii) AOR***

TANGENT PROPERTIES

Tangent diagrams	***Circle theorems***
	*The angle between a tangent and the radius is **90°***

	*Tangents from an external point are equal in length*
	*The angle between a tangent and a chord is equal to the angle the chord subtends in the opposite segment (alternate segment)*

EXAMPLES:

1. *A tangent from* ***T*** *to a circle, centre* ***O*** *and radius* ***7cm*** *touches the circle at* ***P****. If* ***OT = 25cm,*** *find the:*

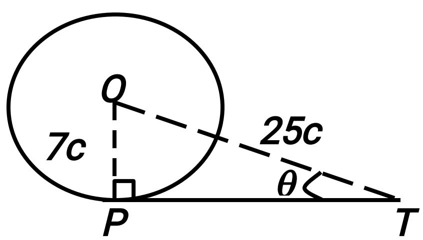
(i) *length of* ***PT***

(ii) *size of angle* ***PTO***

(iii) *area of* ***PTO*** *that lies outside the circle*

Soln:

(i)



$\angle$***OPT = 90°*** *(angle between a tangent and the radius)*

$\therefore\ PT = \sqrt{25^2 - 7^2} = \mathbf{24cm}$

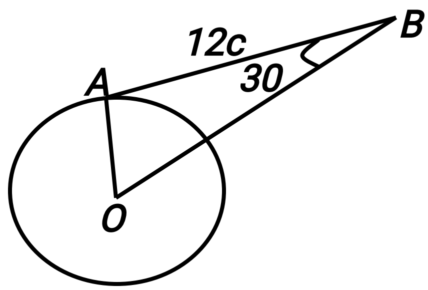
(ii) $\sin\theta = \frac{7}{25}$

$\therefore$ ***θ = 16·26°***

(iii) POT *= 180° − (90° + 16·26°) =* ***73·74°*** *(angles in a triangle)*

$\therefore$ *Required area* $= \frac{1}{2}\times 24\times 7 - \frac{73\cdot 74}{360}\times\frac{22}{7}\times 7^2 = $ $\mathbf{52\cdot 4557cm^2}$

2. *In the circle below* ***O*** *is its centre.* ***AB = 12cm*** *is a tangent to the circle at* ***A*** *and* ***∠OBA = 30°.***



Find the:

(i) *length of* ***OB***

(ii) *radius of the circle*

Soln:

(i) ∠OAB = 90° *(angle between a tangent and the radius)*

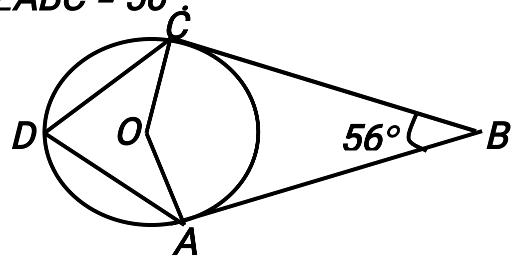
$\Rightarrow \cos 30 = \frac{12}{OB}$

$\therefore$ ***OB = 13·8564cm***

(ii) $\tan 30 = \frac{OA}{12}$

$\therefore$ ***OA = 6·9282cm***

__3.__ In the circle below __O__ is its centre. __AB__ and __CB__ are tangents to the circle and $\angle ABC = 56°$.



Find the size of angle __ADC__

Soln:

$\angle AOC + 90° + 90° + 56° = 360°$ *(angles in a quadrilateral)*

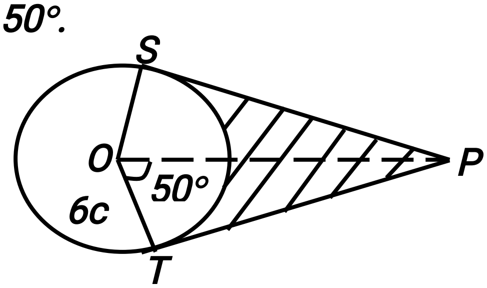
$\therefore \angle AOC = 124°$ *(angles in a triangle)*

If $\angle AOC = 2\angle ADC$

$\Rightarrow \quad 124° = 2\angle ADC$

$\therefore \angle ADC = 62°$

__4.__ In the circle below __O__ is its centre. __PT__ and __PS__ are tangents to the circle of radius __6cm__ and $\angle POT = 50°$.



Find the area of the shaded region

Soln:

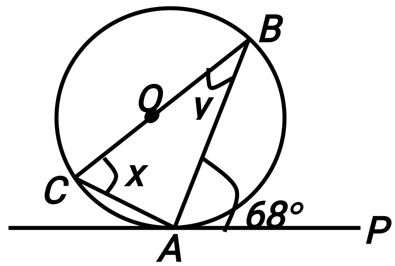
(i) $\angle OTP = 90°$ *(angle between a tangent and the radius)*

$\Rightarrow \tan 50 = \frac{PT}{6}$

$\therefore PT = 7{\cdot}1505cm$

$\therefore$ *Shaded area* $= 2\left(\frac{1}{2}\times 7\cdot 1505\ \times 6\right) - \frac{100}{360}\times\frac{22}{7}\times 6^{2} = 11\cdot 4744cm^{2}$

5. *In the circle below **O** is its centre. **AP** is a tangent to the circle and* ***∠PAB = 68°.***



*Find the size of the angles marked **x** and **y***

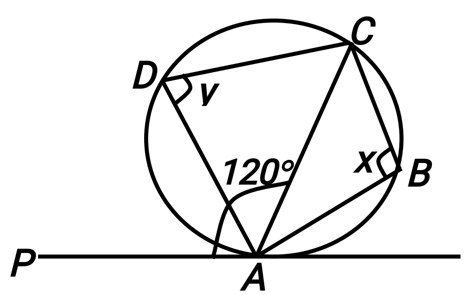
Soln:

x = 68° *(angle in alternate segments)*

∠BAC = 90° *(angle in a semi–circle)*

$\therefore$ ***y*** *= 180° − (90° + 68°) =* ***22°*** *(angles in a triangle)*

6. *In the circle below **AP** is a tangent to the circle and **∠PAC = 120°.***



*Find the size of the angles marked **x** and **y***

Soln:

x = 120° *(angle in alternate segments)*

Also: *y + 120° = 180° (angles in a cyclic quadrilateral)*

$\therefore$ ***y = 60°***

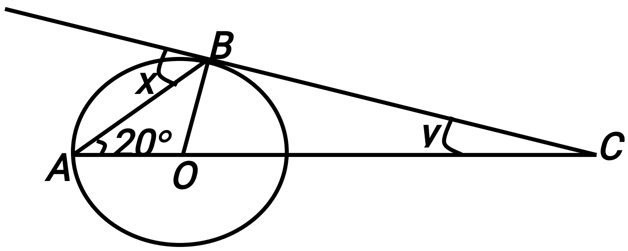
EER:

1. *The tangents at **A** and **B** on a circle of radius **7cm** intersect at **T,** and **C** is any point on the major arc **AB**. If $\angle$**ATB = 48°,** find the:*

(i) *size of angle **ACB***

(ii) *area bounded by the tangents and the minor arc **AB***

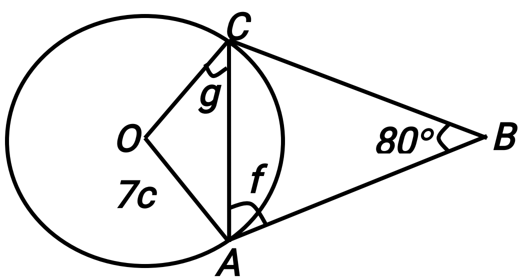
2. *In the circle below **O** is its centre. **BC** is a tangent to the circle and $\angle$**BAO = 20°.***



*Find the size of the angles marked **x** and **y***

3. *The angles of a triangle are **50°, 60°** and **70°,** and a circle touches the sides at **A, B** and **C.** Find the angles of triangle **ABC***

4. *In the circle below **O** is its centre. **AB** and **CB** are tangents to the circle of radius **7cm** and $\angle$**ABC = 80°.***



Find the:

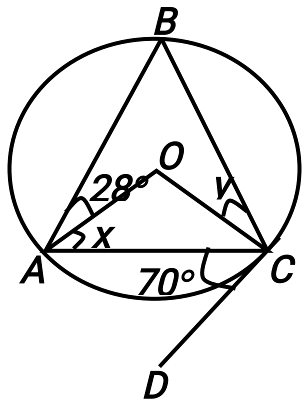
(i) *size of the angles marked **f** and **g***

(ii) *size of the reflex angle **AOC***

(iii) *area bounded by the tangents and the minor arc **AC***

5. *In the circle below* ***O*** *is its centre****.*** ***DC*** *is a tangent to the circle* ***∠BAO*** *=* ***28°***

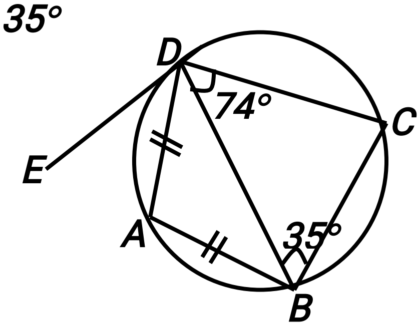
and ***∠ACD = 70°***



Find the size of the angles marked ***x*** *and* ***y***

6. *In the circle below* ***ED*** *is a tangent to the circle****, AB = AD, ∠BDC = 74°***

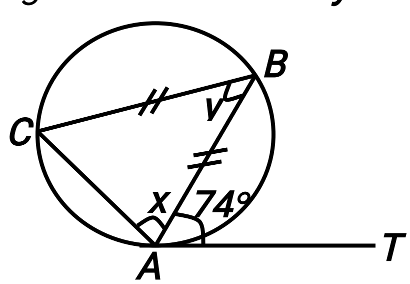
and ***∠DBC = 35°***



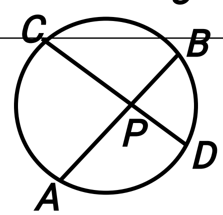
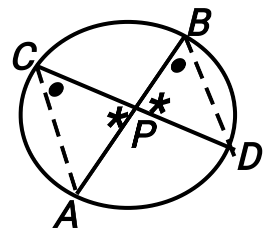
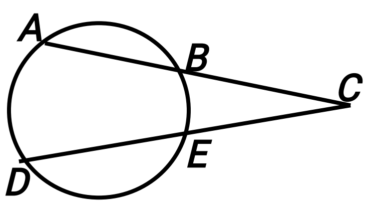
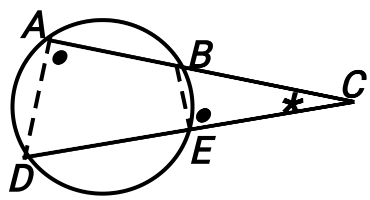
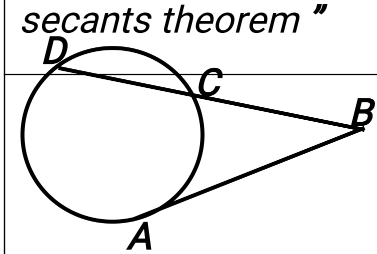
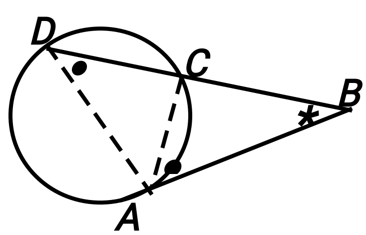
Find the size of angle:

(i) DAB ***(ii) BDE*** ***(iii) DBA*** ***(iv) EDA***

7. *In the circle below* ***AT*** *is a tangent to the circle****, AB = CB*** *and* ***∠BAT = 74°.*** *Find the size of the angles marked* ***x*** *and* ***y***

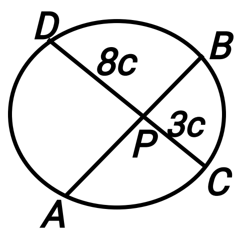


CIRCLES AND SIMILAR TRIANGLES

Circle diagrams	*Analysis diagrams*	*Conclusion*
 *“ intersecting chords theorem ”*		*Triangles* ***ACP*** *and* ***PBD*** *are similar* $\Rightarrow \frac{AP}{PD} = \frac{CP}{PB}$ $\therefore AP \cdot PB = CP \cdot PD$ *“ Each length is measured from the meeting point”*
 *“ intersecting secants theorem ”*		*Triangles* ***ACD*** *and* ***ECB*** *are similar* $\Rightarrow \frac{AC}{CE} = \frac{DC}{CB}$ $\therefore CA \cdot CB = CD \cdot CE$ *“ Each length is measured from the meeting point”*
 *“ intersecting tangent– secant theorem ”*		*Triangles* ***ABC*** *and* ***ABD*** *are similar* $\Rightarrow \frac{AB}{BC} = \frac{DB}{AB}$ $\therefore BA^2 = BD \cdot BC$ *“ Each length is measured from the meeting point”*

EXAMPLES:

1. *In the circle below chords* ***AB*** *and* ***CD*** *intersect at* ***P.*** *If* ***CP = 3cm, DP = 8cm*** *and* ***AB = 10cm,*** *find the length of* ***AP***



Soln:

If AP = x, PB = 10 – x

Using $AP \cdot PB = CP \cdot PD$ *(intersecting chords theorem)*

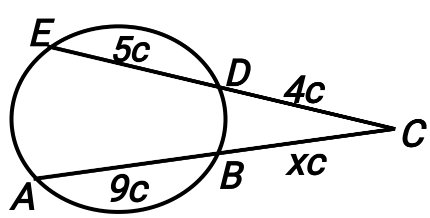
$\Rightarrow$ *x(10 – x) = 3(8)*

$x^2 - 10x + 24 = 0$

$x = \frac{10 \pm \sqrt{100 - 96}}{2}$

$\therefore$ ***x*** *= 6cm or 4cm*

2. *In the circle below chords* ***AB*** *and* ***ED*** *are produced to intersect at* ***C.*** *If* ***CD = 4cm, ED = 5cm, AB = 9cm*** *and* ***BC = xcm,***



find the:

(i) *value of* ***x***

(ii) *ratio of the areas of triangle* ***ACE*** *to that of* ***BCD***

(iii) *area of* ***ABDE*** *if the area of triangle* ***ACE*** *is* $54cm^2$

Soln:

(i) *Using* $CB \cdot CA = CD \cdot CE$ *(Each length from* ***C****)*

$\Rightarrow \quad x(x + 9) = 4(9)$

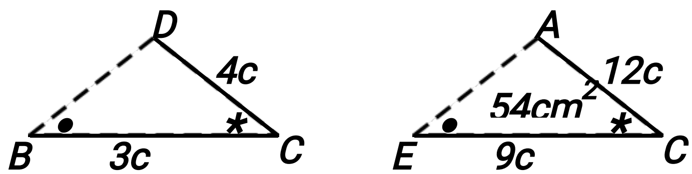
$x^2 + 9x - 36 = 0$

$x = \dfrac{-9 \pm \sqrt{81 + 144}}{2}$

$\boldsymbol{x} = 3 \quad or \quad -12$

$\therefore \quad \boldsymbol{x} = 3$

(ii) *Using similar triangles* ***ACE*** *and* ***BCD***



$\dfrac{Area \quad ACE}{Area \quad BCD} = \left(\dfrac{12}{4}\right)^2 = 9$

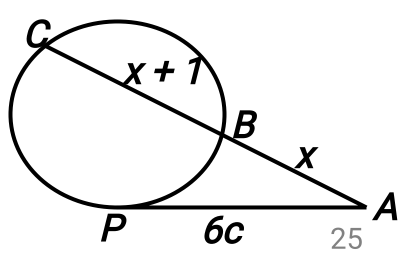
$\therefore$ *Required ratio =* ***9:1***

(iii) $\dfrac{Area \quad BCD}{54} = \left(\dfrac{4}{12}\right)^2$

$\therefore$ *Area* ***BCD*** $= 6cm^2$

$\Rightarrow$ *Required area* $= 54 - 6 = 48cm^2$

3. *In the circle below secant* ***ABC*** *intersects tangent* ***AP*** *at* ***A.*** *If* ***AP= 6cm, AB = xcm*** *and* ***BC = (x + 1)cm,*** *find the value of* ***x***



Soln:

Using $AB \cdot AC = AP^2$ *(Each length from* ***A****)*

$\Rightarrow \quad x(2x + 1) = 6^2$

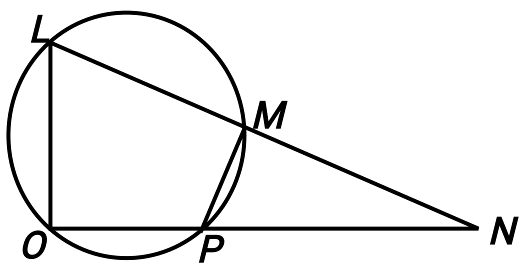
$2x^2 + x - 36 = 0$

$x = \frac{-1 \pm \sqrt{1 + 288}}{4}$

$x = 4$ *or* $-4{\cdot}5$

$\therefore \quad x = 4$

4. *In the circle below* ***OL= 4·5cm, PM = 3cm, NM = 4cm*** *and* ***LN = 7·5cm.***



Find the:

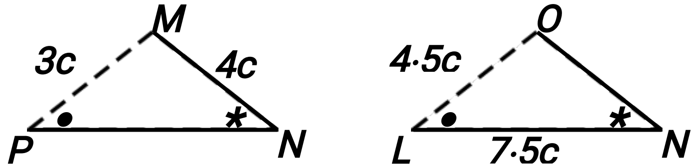
(i) *lengths of* ***ON*** *and* ***OP***

(ii) *radius of the circle*

(iii) *area of* ***OLMP***

Soln:

(i) *Using similar triangles* ***PMN*** *and* ***OLN,***



$\Rightarrow \quad \frac{ON}{4} = \frac{4 \cdot 5}{3}$

$\therefore$ ***ON = 6cm***

Also: $\frac{PN}{7 \cdot 5} = \frac{3}{4 \cdot 5}$

$\therefore$ ***PN = 5cm***

$\Rightarrow$ ***OP = ON – PN*** *= 6 – 5* ***= 1cm***

(ii) PMN *is a right triangle based on its dimensions* ***(∠PMN = 90°)***

$\Rightarrow$ ***∠POL = 90°,*** *thus* ***LP*** *is a diameter* ***(****angle in a semi–circle****)***

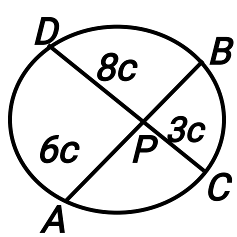
$\therefore \; LP = \sqrt{4 \cdot 5^2 + 1^2} = 4 \cdot 6098cm$

$\Rightarrow$ *Required radius =* $\frac{1}{2} \times 4 \cdot 6098 = 2 \cdot 3049cm^2$

(iii) *Required area =* $\frac{1}{2} \times 6 \times 4 \cdot 5 - \frac{1}{2} \times 4 \times 3 = 7 \cdot 5cm^2$

EER:

1. *In the circle below chords* ***AB*** *and* ***CD*** *intersect at* ***P.*** *If* ***CP = 3cm, DP = 8cm*** *and* ***AP = 6cm,***



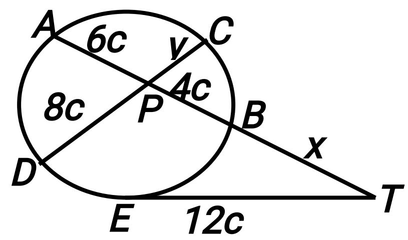
find the:

(i) length of ***PB***

(ii) *area of triangle* ***BPC*** *if the area of triangle* ***APD*** *is* $20cm^2$

(iii) *ratio of the areas of triangle* ***APC*** *to that of* ***BPD***

2. *In the circle below* ***TE = 12cm*** *is a tangent to the circle at* ***E.*** *Chords* ***AB*** *and* ***CD*** *intersect at* ***P*** *and* ***AB*** *is produced at* ***T.*** *If* ***AP = 6cm, PB = 4cm, BT = xcm, CP = ycm*** *and* ***DP = 8cm,***

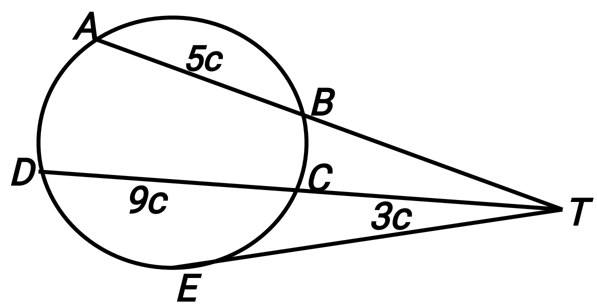


find the:

(i) *values of* ***x*** *and* ***y***

(ii) *ratio of the areas of triangle* ***APD*** *to that of* ***BPC***

3. *In the circle below* ***TE*** *is a tangent to the circle at* ***E.*** *Chords* ***AB*** *and* ***CD*** *are produced to intersect at* ***T.*** *If* ***AB = 5cm, DC = 9cm*** *and* ***CT = 3cm,*** *find the lengths of* ***BT*** *and* ***ET***



RELATION AND MAPPINGS

Summary:

A relation between members of the given set can be illustrated using a papygram

EXAMPLES:

1. *Draw a papygram illustrating the relation "is a prime factor of" in the set* ***{1, 2, 3, 4, 5, 6, 8, 12, 30}***

2. *Draw a papygram showing the relation "is a multiple of" in the set* ***{42, 28, 21, 14, 7}***

3. *Given that* ***T = {2, 5, 6, 8, 9, 10, 12, 13} ,*** *illustrate on papygrams the relations:* ***(i)*** *"Greater than by* ***3****"*
(ii) *"Factor of"*

FUNCTIONS

Summary:

1. *A function* ***f(x)*** *is a formula in terms of* ***x.***

2. *A function* ***f*** *that maps* ***x*** *on to* ***3x + 1*** *can either be written as follows:*

(i) $f(x) = 3x + 1$ ***(ii)*** $f : x \rightarrow 3x + 1$ ***(iii)*** $x \rightarrow 3x + 1$

3. (i) *A mapping diagram is an arrow diagram with a set of values of* ***x*** *and that*

of ***f(x).***

(ii) *A set of values of* ***x*** *is called The Domain*

(iii) *A set of values of* ***f(x)*** *is called The Range*

4. *The inverse of a function* ***f(x)*** *is denoted by* $f^{-1}(x)$. *This function maps the*

range back on to the domain.

__5.__ A function is undefined or meaningless if its denominator part is equal to zero.

6. (i) *A composite function* ***fg(x)*** *is a combination of two functions* ***f(x)*** *and* ***g(x).***

(ii)*A composite function* $f^{2}(x)$ *is the same as* ***ff(x).***

EXAMPLES:

1. (i) *Determine the range corresponding to the domain {**0, 1, 2, 3**} for the*

mapping ***f(x) = 3x + 1.***

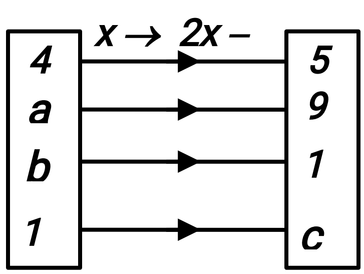
(ii) *Represent the mapping in* ***(i)*** *above on an arrow diagram.*

2. (i) *Determine the range corresponding to the domain {**–3, –2, 0, 1, 2, 3**} for the*

mapping $\boldsymbol{x \to x^2 + 1}$.

(ii) *Represent the mapping in* ***(i)*** *above on an arrow diagram.*

3. *Find the unknown values in the arrow diagram for the mapping* $\boldsymbol{x \to 2x - 3}$.



4. *Given the function* $f(x) = \dfrac{10x}{x^2 - 4}$*, find:*

(i) f(3)

(ii) f(6)

(iii) *the values of **x** for which **f(x)** is undefined*

(iv) *the values of **x** for which **f(x) = 6***

5. *Given the function* $f(x) = \frac{2x}{3x^2 - 10x - 8}$, *find:*

(i) f(2)

(ii) f(–1)

(iii) *the values of **x** for which **f(x)** is undefined*

6. *Given that **f(x) = 3x +5,*** $g(x) = \frac{2}{2x - 6}$ *and* $h(x) = \frac{4 - x^2}{x^2 + 3}$, *find:*

(i) $f^{-1}(x)$, *and hence* $f^{-1}(-1)$

(ii) $g^{-1}(x)$, *and hence* $g^{-1}(-2)$

(iii) $h^{-1}(x)$, *and hence* $h^{-1}(0)$

7. *Given that **f(x) = ax – 7** and **f(8) = 17,** find:*

(i) *the value of **a***

(ii) f(4)

(ii) $f^{-1}(x)$, *hence obtain* $f^{-1}(8)$.

8. *Given that **f(x) = a + bx, f(1) = 8** and **f(–1) = 2,** find:*

(i) *the values of **a** and **b***

(ii) $f(-2)$

(iii) $f(5)$

(iv) the value of **c** for which $f(c) = -7$

(v) $f^{-1}(x)$

(vi) $f^{-1}(-7)$

10. Given that $f(x) = ax + 9$ and $f^{-1}(13) = 1$, find:

(i) the value of **a**

(ii) $f^{-1}(1)$

11. Given that $f(x) = px + 7$ and $g^{-1}(x) = \frac{5 - 2x}{3}$, find:

(i) $g(x)$

(ii) the value of **p** for which $g(2x - 3) = f(x)$

12. Given that $g^{-1}(x) = \frac{1 + x}{x}$, find $g(3)$.

13. Given that $f(x) = \frac{2}{x + 2} + \frac{8x + 4}{x^2 - 4}$, express $f(x)$ in the form $\frac{ax}{x^2 + b}$.

Hence find:

(i) $f(3)$

***(ii)** the values of **x** for which **f(x)** is undefined*

***14.** Given that* $f(x) = \frac{2}{3x + 2} + \frac{5x + 3}{9x^2 - 4}$, *express **f(x)** in the form* $\frac{ax + b}{cx^2 + d}$.

Hence find:

(i) f(2)

***(ii)** the values of **x** for which **f(x)** is undefined*

***15.** Given that* $f(x) = x^2 - 12$ *and* $g(x) = 2x - 5$, *find:*

(i)** an expression for **gf(x)** and hence evaluate **gf(4)

(ii)** an expression for **fg(x)** and hence evaluate **fg(2)

(iii)** the values of **x** for which **gf(x) = fg(x)

(iv)** an expression for **gg(x)** and hence evaluate **gg(2)

(v)** an expression for **ff(x)** and hence evaluate **ff(–3)

***16.** Given that **f(x) = 5x – 7,** find **g(x)** for which:*

(i) fg(x) = 10x + 8,** hence evaluate **g(4)

(ii) $fg(x) = 20x^2 - 37$, *hence evaluate **g(2)***

***17.** Given that* $f(x) = x^2 - 7$ *and* $g(x) = x + 1$, *find the values of **x** for which*

fg(x) + gf(x) = 0.

18. *Given that* $f(x) = \frac{x+3}{2}$ *and* $g(x) = \frac{1-2x}{5}$, *find the values of* $\boldsymbol{x}$ *for which* $fg(x) = \frac{8x^2 + 24x + 9}{10}$.

19. *Given that* $f(x) = \frac{2}{2x - 6}$ *and* $g(x) = x^2 - 1$, *find the values of* $\boldsymbol{x}$ *for which* ***fg(x)*** *is undefined*

EER:

1. *Given that* $f(x) = \frac{14x}{x^2 - 9}$, *find:*

(i) f(4)

(ii) *the values of* $\boldsymbol{x}$ *for which* ***f(x)*** *is undefined.*

2. (i) *Determine the range corresponding to the domain {**4, 9, 16**} for the mapping*

$x \rightarrow 10 - 2\sqrt{x}$

(ii) *Represent the mapping in* ***(i)*** *above on an arrow diagram.*

3. *Express* $x^2 + 7x + 12$ *in the form* $(x + a)^2 + b$. *Hence solve the equation*

$x^2 + 7x + 12 = 0.$

*4. Given that **$f(x) = 2x + 4$** and **$g(x) = x + 5$**, find **$fg(x)$** and hence evaluate **$fg(4)$***

5. Given that $f(x) = (x - 3)^2$, find:

(i) $f(5)$

(ii) the values of x for which $f(x) = 16$.

*6. Given that **$f(x) = ax + 3$** and $f(5) = 33$, find:*

(i) the value of a

(ii) $f(-2)$

*7. Given that $f(x) = ax^2 + b$, **$f(-2) = 3$** and **$f(1) = -2$**, find:*

(i) the values of a and b

(ii) $f(4)$

*8. Given that **$f(x) = 2x - 5$**, find:*

(i) $f(-2)$

(ii) $f^{-1}(x)$

*9. Given that **$f(x) = 3x - 4$**, find $f^{-1}(5)$.*

10. Given that $f(x) = \frac{1}{2}(x + 7)$ and $g(x) = x(x - 6)$, find the values of x for *which*

which $f^{-1}(x) = g(x)$.

11. *Given that* $f(x) = \frac{2}{x + 4} + \frac{4}{x - 3} - \frac{4(x + 4)}{x^2 + x - 12}$, *express* ***f(x)*** *in the*

form $\frac{a}{x + b}$. *Hence evaluate* ***f(–3)***

12. *Given that* $f(x) = \frac{7}{x + 3} + \frac{7x + 21}{x^2 - 9}$, *express* ***f(x)*** *in the form*

$\frac{ax}{x^2 + b}$.

Hence find:

(i) f(4)

(ii) *the values of* ***x*** *for which* ***f(x)*** *is undefined*

13. *Given that* $f(x) = ax^2 + bx$, ***f(1) = 5*** *and* ***f(2) = 14,*** *find:*

(i) *the values of* ***a*** *and* ***b***

(ii) f(3)

14. *Express* $x^2 + x - 12$ *in the form* $(x + a)^2 + b$. *Hence solve the equation*

$x^2 + x - 12 = 0.$

15. *Express* $3x^2 + 4x - 4$ *in the form* $a(x + b)^2 + c$. *Hence solve the* *equation*

$$3x^2 + 4x - 4 = 0.$$

16. *Given that* $f(x) = \frac{3}{1 - x^2}$, *find the values of* ***x*** *for which* ***f(x) = 4.***

17. *Given the function* $f(x) = \frac{8x}{3x^2 + 9x - 30}$, *find:*

(i) f(–1)

(ii) *the values of* ***x*** *for which* ***f(x) = 1***

(iii) *the values of* ***x*** *for which* ***f(x)*** *is undefined*

18. *Given that* $f^{-1}(x) = \frac{\beta x - 2}{\beta + 4x}$ *and* ***f(2) = –4,*** *find the value of* β. *Hence find*

the value of ***x*** *for which* ***f(x)*** *is undefined.*

INDICES

Summary:

1. *The expression* 3^5 *is read as* ***3*** *to power* ***5*** *and is defined as follows:*

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$ ***("3"** is called the base and **"5"** is the index or power**)***

2. *The following rules apply to indices:*

(i) $a^m \times a^n = a^{m+n}$

To multiply expressions with the same base, copy the base and add the powers.

(ii) $a^m \div a^n = a^{m-n}$

To divide expressions with the same base, copy the base and subtract the

powers.

(iii) $\left(a^m\right)^n = a^{mn}$

To raise an expression with the ***nth*** *power, copy the base and multiply the*

powers.

(iv) $a^0 = 1$

Any number to power zero is ***1*** *except zero.*

(v) $a^{-n} = \frac{1}{a^n}$

Any number raised with a negative power, is the reciprocal of the number with *a positive power.*

$$\therefore \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$

(vi) $\sqrt{a} = a^{\frac{1}{2}}$

The square root of any number is that number raised to power a half.

(vii) $\sqrt[n]{a} = a^{\frac{1}{n}}$

The ***nth*** *root of any number is that number raised to power* $\frac{1}{n}$.

$$\therefore \left(\sqrt[n]{a}\right)^{m} = a^{\frac{m}{n}}$$

(viii) $a^{m} = a^{n}$ *implies that* $m = n$

(ix) $(a \times b)^{m} = a^{m} \times b^{m}$

EXAMPLES:

1. *Write the following expanded forms in index form:*

   (i) $5 \times 5 \times 5 \times 5$

   (ii) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

   (iii) $2 \times 2 \times 2 \times 2 \times 2$

   (iv) $7 \times 7 \times 7$

2. *Write each of the following index forms in expanded form:*

(i) 10^{3}

(ii) 7^{4}

(iii) 9^{5}

3. *Write the following numbers in index form using base* ***2****:*

(i) 8

(ii) 32

(iii) 16

(iv) 64

4. *Find the value of the following index forms:*

(i) 3^{2}

(ii) 2^{3}

(iii) 7^{2}

(iv) $(49)^{\frac{1}{2}}$

(v) $8^{\frac{2}{3}}$

(vi) $(125)^{\frac{1}{3}}$

(vii) $(0 \cdot 008)^{\frac{1}{3}}$

(viii) $\left(\sqrt[3]{125}\right)^{2}$

(ix) $\left(\frac{16}{81}\right)^{\frac{-3}{4}}$

5. Simplify the following:

(i) $y^7 \times y^3 \times y^5$

(ii) $y^{10} \div y^3$

(iii) $\frac{y^7 \times y^4}{y^5}$

(iv) $\frac{a^7 \times \left(ab^4\right)^2}{\left(a^2 \times b\right)^3}$

(v) $\frac{2^y \times 8^{y-1}}{16^{y-1}}$

(vi) $\frac{5^y \times 125^{y+1}}{625^{y+1}} + \frac{4^y \times 8^{y-2}}{32^{y-1}}$

6. Without using a calculator, simplify the following:

(i) $\frac{1}{2} - \left(\frac{25}{4}\right)^{\frac{-1}{2}}$

(ii) $(16)^{\frac{-1}{2}} + (32)^{\frac{-1}{5}} - \left(\frac{8}{125}\right)^{\frac{2}{3}}$

(iii) $4(0 \cdot 04)^{\frac{-1}{2}} - 8(4)^{-1}(16)^{\frac{3}{4}}$

(iv) $(27)^{\frac{-1}{3}}(25)^{\frac{1}{2}} - 5(0 \cdot 008)^{\frac{1}{3}}$

(v) $\left(\frac{1}{16}\right)^{\frac{-1}{2}} \times \left(\frac{1}{64}\right)^{\frac{-1}{3}}$

(vi) $\left(\frac{64}{81}\right)^{\frac{-1}{2}} \times \frac{1}{3}(32)^{\frac{3}{5}}$

(vii) $\left(\frac{8}{125}\right)^{\frac{-2}{3}} \times \left(\frac{5}{8^{1/2}}\right)^{-2}$

(viii) $\frac{2^{-2} \times 3^{-3}}{2^{-4} \times 3^{-6} \times 18}$

(ix) $\frac{(12)^{\frac{3}{2}} \times 9^{\frac{1}{4}}}{(36)^{\frac{1}{2}}}$

(x) $\frac{40 \times \sqrt{0 \cdot 36}}{(27)^{\frac{2}{3}} \div (81)^{\frac{3}{4}}}$

(xi) $\frac{(12)^{\frac{3}{2}} \times (16)^{\frac{1}{8}}}{(27)^{\frac{1}{6}} \times (18)^{\frac{1}{2}}}$

7. Solve for ***x*** and ***y*** in the following equations:

(i)
$$3^{x-y} = 27$$
$$3^{x+y} = 243$$

(ii)
$$2^{2x-3y} = 16$$
$$5^{x-2y} = 1$$

8. Solve for ***y*** in the following equations:

(i) $y^{\frac{2}{3}} = 4$

(ii) $4^{y} = 32$

(iii) $2^{2y} = \frac{1}{8}$

(iv) $\left(\frac{a}{b}\right)^{y-1} = \left(\frac{b}{a}\right)^{y-5}$

(v) $(32)^{\frac{3}{5}} \div y^{\frac{1}{2}} = 2$

(vi) $(32)^{y} \times \frac{1}{8} \times 4^{y+3} = 2^{24}$

(vii) $5^{3(y-1)} \times 125 = 625$

(viii) $\dfrac{243 \times 3^{2y}}{729 \times 3^{y} \div 3^{2y-1}} = 81$

(ix) $3^{y} \times 9^{\frac{-2}{y}} = 1$

(x) $27 \times 3^{2y} = \left(3^{y}\right)^{y}$

(vi) $(16)^{y^{2}} = 8^{4y-3}$

(vii) $4 \times 2^{3(y-1)} = 2^{-y}$

EER

1. Simplify the following:

(i) $\dfrac{3^{2n} \times 9^{n-1}}{81^{n-1}}$

(ii) $\dfrac{9^{n+1} \times 3^{n-1} \times 2^{n+1}}{27^{n-1} \times 2^{n}}$

(iii) $\dfrac{9^{n+1} \times 6^{n-1}}{3^{3n-1} \times 2^{n}}$

(iv) $\dfrac{243^{\frac{n}{5}} \times 3^{2n+1}}{9^{n} \times 3^{n-1}}$

2. Without using a calculator, evaluate: $\dfrac{3 \times 10^{4} \times \left(6 \times 10^{5}\right)^{2}}{90 \times 10^{3}}$

3. Without using a calculator, simplify $(27)^{\frac{-1}{3}}(25)^{\frac{1}{2}} - 5(0 \cdot 008)^{\frac{1}{3}}$

4. Solve for **x** and **y** in the equation: $\dfrac{3^x \times 2^{2y}}{3^y \times 2^x} = 72$

5. Without using a calculator, simplify $\left(\dfrac{81}{16}\right)^{\frac{-1}{4}} - \left(\dfrac{8}{27}\right)^{\frac{2}{3}}$

6. If $5^n = 3125,$ find the value of $5^{(n-3)}$

7. If $81^x = 729^y$, find the value of $\dfrac{y}{x}$

8. Without using a calculator, simplify $\left(\dfrac{16}{81}\right)^{\frac{-3}{4}} \div \left(\dfrac{36}{64}\right)^{\frac{3}{2}}$

9. Solve for **x** in the following equations:

(i) $5^{7x-2} = 125^x$ **(ii)** $3^x = 9^{x+3}$

(iii) $81^x = \left(\dfrac{1}{3}\right)^{x-10}$ **(vi)** $4^{6-9x} = \dfrac{1}{8^{x-2}}$

10. Solve for **x** in the following equations:

(i) $5^{x^2} = 5^{6-x}$ **(ii)** $16^{x^2} = 8^{4x-3}$

(iii) $3^{x^2} \times 9^{-2} = 1$ **(iv)** $3^x \times \dfrac{1}{729} = 9^{x^2}$

11. Solve for **x** and **y** in the equations:

$3^{x-y} = 9$

$2^{x+y} = 32$

12. *If* ***x = 32,*** *find the value of* $\left(x^{\frac{1}{5}}\right)^{-1} + 3\left(\frac{x}{2}\right)^{\frac{-1}{2}}$

13. *Given that* $\frac{4^{x} \times 2^{y}}{2^{x+2y}} = 2^{p}$, *express* ***p*** *in terms of* ***x*** *and* ***y***

14. *Given that* $3^{2x - y} = 27$ *and* $5^{x} \div 25^{y} = 1$, *solve for* ***x*** *and* ***y***

15. *Given that* ***a = –2*** *and* ***b = 2,*** *find the value of* $a^{b} - b^{a}$

16. *Without using a calculator or tables, evaluate:* $\sqrt{\frac{0{\cdot}24 \times 1{\cdot}56 \times 7{\cdot}2}{1{\cdot}3 \times 0{\cdot}16 \times 0{\cdot}09}}$

17. *Without using a calculator or tables, evaluate:* $\sqrt[3]{\frac{0{\cdot}064 \times 0{\cdot}125}{0{\cdot}008 \times 0{\cdot}001}}$

STANDARD FORM

Summary:

1. *Standard form is a way of expressing a number in the form* $A \times 10^{n}$ *where* $1 \leq A < 10$ *and* ***n*** *is an integer*

2. (i) *To express a number in standard form, we shift the decimal point until the digit part* ***A*** *is between* ***1*** *and* ***10.*** *This digit* ***A*** *has a decimal point placed after the first digit.*

(ii) *The power part* (10^{n}) *shows how many places to move the decimal point*

3. *The rules of indices apply to calculations in standard form*

EXAMPLES:

1. *Express the following numbers in standard form:*

(i) 25000 (ii) 3860 (iii) 568·3 (iv) 74·8 (v) 3·584 (vi) 4000

(vii) 0·435 (viii) 0·000263 (ix) 0·007 (x) 0·00356 (xi) 248×10^{3}

(xii) 4500×10^{-7} ***(xiii)*** $58 \cdot 4 \times 10^{-4}$ ***(xiv)*** $0 \cdot 027 \times 10^{-3}$

(xv) $0 \cdot 0062 \times 10^{-4}$ ***(xvi)*** $0 \cdot 0364 \times 10^{5}$

2. *By expressing each of the numbers in standard form, evaluate the following:*

(i) $0{\cdot}0004 \times 0{\cdot}002$ ***(ii)*** $0{\cdot}005 \times 0{\cdot}00004$ ***(iii)*** $800000 \times 0{\cdot}0005$

(iv) $\frac{800}{0{\cdot}004}$ ***(v)*** $\frac{0{\cdot}00006}{2000}$ ***(vi)*** $\frac{0{\cdot}0009 \times 8000}{0{\cdot}002 \times 0{\cdot}3}$

3. *By expressing each of the numbers in the form* $a \times 10^{n}$ *where* n *is even, evaluate the following:*

(i) $\frac{0{\cdot}81}{0{\cdot}0027}$ ***(ii)*** $0{\cdot}02 \times 0{\cdot}0015$ ***(iii)*** 1500×40000

4. *By expressing each of the numbers in the form* $a \times 10^{n}$ *where* n *is even, find the square root of:*

(i) $0{\cdot}25 \times 0{\cdot}64$ ***(ii)*** $0{\cdot}16 \times 0{\cdot}09$ ***(iii)*** $0{\cdot}0036 \times 0{\cdot}25$ ***(iv)*** $0{\cdot}0081 \times 0{\cdot}0004$

5. *Without using a calculator, evaluate the following give your answer in standard form:*

(i) $(2 \times 10^{2}) \times (6 \times 10^{5})$ ***(ii)*** $(4 \times 10^{-3}) \times (5 \times 10^{4})$

(iii) $(9 \times 10^{-2}) \times (4 \times 10^{-3})$ ***(iv)*** $(15 \times 10^{-3}) \times (20 \times 10^{7})$

6. Without using a calculator or tables, evaluate: $\frac{1\cdot21 \times 10^{-2} \times 40}{2\cdot2 \times 11}$

7. Without using a calculator or tables, evaluate: $\frac{2\cdot25 \times 10^{-3} \times 3\cdot9}{1\cdot3 \times 1\cdot5}$

EER:

1. Without using a calculator, evaluate: $\frac{0\cdot0035}{0\cdot07 \times 0\cdot2}$

2. Without using a calculator, evaluate: $\frac{0\cdot625 \times 0\cdot009}{0\cdot0045}$

2. Without using a calculator, evaluate: $\frac{0\cdot42 \times 0\cdot35 \times 0\cdot0015}{0\cdot049 \times 0\cdot003}$

3. Without using a calculator, evaluate: $\frac{1\cdot69 \times 10^{-2} \times 1\cdot25}{1\cdot3 \times 2\cdot5}$

4. Without using a calculator, evaluate: $\frac{5600}{80000}$ give your answer in the form $a \times 10^{n}$ where $1 \leq a < 10$ and n is an integer

4.

mmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmmm mmmmmmmmm

SURDS

Summary:

1. *Surds are numbers left in square root form.*

2. *The following rules apply to surds:*

(i) $a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$

(ii) $a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$

(iii) $a\sqrt{b} - c\sqrt{d}$ *cannot be manipulated because the surds are different. Unlike*

*surds cannot be added **or** subtracted*

(iv) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

(v) $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$

(vi) $(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$

(vii) $\sqrt{a^2 b} = a\sqrt{b}$

(viii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(3) (i) *The process of eliminating surds from the denominator is called*

rationalizing the denominator.

(ii) *To rationalize* $\frac{a}{\sqrt{b}}$, *multiply top and bottom by* $\sqrt{b}$.

(iii) *The conjugate surd of* $a + \sqrt{b}$ *is* $a - \sqrt{b}$ *and vice versa*

(iv) *To rationalize* $\frac{c}{a + \sqrt{b}}$, *multiply top and bottom by the conjugate of*

the

denominator.

EXAMPLES:

1. *Without using a calculator simplify:*

(i) $\sqrt{18}$ **(ii)** $\sqrt{50}$ **(iii)** $\sqrt{48}$

(vi) $\sqrt{52}$ **(v)** $\sqrt{90}$ **(vi)** $\sqrt{108}$

(vii) $\sqrt{98}$ **(viii)** $\sqrt{300}$ **(ix)** $\sqrt{32}$

2. *Without using a calculator simplify:*

(i) $\sqrt{3} + \sqrt{3}$

(ii) $3\sqrt{7} + \sqrt{7}$

(iii) $7\sqrt{5} - \sqrt{5}$

(iv) $\sqrt{50} + \sqrt{32} - \sqrt{18}$

(v) $\sqrt{80} + \sqrt{20} - \sqrt{45}$

(vi) $3\sqrt{28} + \sqrt{63} - \sqrt{112}$

(vii) $\sqrt{125} + \sqrt{20} - \sqrt{45}$

(viii) $\sqrt{40} + \sqrt{243} + \sqrt{10} - \sqrt{75}$

(ix) $\sqrt{75} + \sqrt{44} + \sqrt{27} + \sqrt{99}$

3. *Expand the following and simplify:*

(i) $2(5 - \sqrt{7})$

(ii) $\sqrt{2}(\sqrt{6} - \sqrt{2})$

(iii) $\sqrt{5}(3\sqrt{5} + 4\sqrt{15})$

(iv) $(1 + \sqrt{3})(2 + \sqrt{3})$

(v) $(2 + \sqrt{5})(2 + \sqrt{5})$

(vi) $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})$

(vii) $(2 + \sqrt{3})^2$

(viii) $(4\sqrt{5} + 3\sqrt{2})^2$

(ix) $(3 + 2\sqrt{5})(4\sqrt{5} - 2)$

(x) $(4\sqrt{5} + 3\sqrt{2})(4\sqrt{5} - 3\sqrt{2})$

4. *Express* $(2\sqrt{5} + \sqrt{2})(\sqrt{5} + 3\sqrt{2})$ *in the form* $a + b\sqrt{c}$*, hence state the values of* ***a, b*** *and* ***c.***

5. *If* $(2\sqrt{7} + 3\sqrt{2})(4\sqrt{7} - 5\sqrt{2}) = a + b\sqrt{c}$*, find the values of* ***a, b*** *and* ***c.***

6. If $\sqrt{6} = 2 \cdot 44949$, without using tables or a calculator evaluate

$5\sqrt{2}(2\sqrt{3} - \sqrt{2})$ correct to **3** decimal places.

6. If $\sqrt{14} = 3 \cdot 74166$, without using tables or a calculator evaluate

$(2\sqrt{7} + 3\sqrt{2})(4\sqrt{7} - 5\sqrt{2})$ correct to **4** decimal places.

7. A rectangle of length $(x\sqrt{5} + y\sqrt{2})cm$ and width $(\sqrt{5} + 4\sqrt{2})cm$ has an

area of $(11 + 25\sqrt{10})cm^2$. Without using tables or a calculator, find the

the values of ***x*** and ***y***.

8. Express the following with a rational denominator:

(i) $\frac{14}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{3}}$

(iii) $\frac{12}{\sqrt{18}}$

(iv) $\frac{\sqrt{2}}{4\sqrt{5} - 3\sqrt{2}}$

(v) $\frac{\sqrt{3} - 1}{2\sqrt{3} + 1}$

(iv) $\frac{3 + 2\sqrt{3}}{3 - 2\sqrt{3}}$

(v) $\dfrac{1+2\sqrt{3}}{2-\sqrt{3}}$

(vi) $\dfrac{1}{1+\sqrt{8}+\sqrt{18}-2\sqrt{2}}$

9. Express $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ in the form $a+b\sqrt{c}$.

10. Express $\dfrac{(1+\sqrt{3})^2}{\sqrt{3}-1}$ in the form $p+q\sqrt{r}$.

11. Without using tables or a calculator, simplify $\dfrac{1}{1+\sqrt{8}+\sqrt{18}-2\sqrt{2}}$

12. Express $\dfrac{3}{\sqrt{5}-2}+\dfrac{1}{\sqrt{5}}$ in the form $a+b\sqrt{c}$.

13. Simplify $\dfrac{\sqrt{63}+\sqrt{28}}{\sqrt{175}-\sqrt{63}}$ as far as possible

14. Without using tables or a calculator, simplify $\dfrac{\sqrt{30}}{\sqrt{6}}+\dfrac{\sqrt{35}}{\sqrt{7}}$

15. If $\sqrt{3}=1\cdot 73205$, without using tables or a calculator evaluate $\dfrac{1+\sqrt{3}}{2+\sqrt{3}}$

correct to **4** decimal places.

16. Without using tables or a calculator, simplify $\dfrac{1}{\sqrt{3}+\sqrt{2}}+\dfrac{3}{\sqrt{3}-\sqrt{2}}$

17. If $\dfrac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}=a+\sqrt{b}$, find the values of **a** and **b.**

18. *If* $\frac{6}{3\sqrt{2} - 2\sqrt{3}} = a\sqrt{2} + b\sqrt{3}$, *find the values of* ***a*** *and* ***b.***

19. *If* $\frac{3 - 2\sqrt{3}}{3 + 2\sqrt{3}} - \frac{3 + 2\sqrt{3}}{3 - 2\sqrt{3}} = a\sqrt{3}$, *find the value of* ***a.***

EER:

2. *Without using a calculator simplify:* $\sqrt{20} - \sqrt{45} + \sqrt{125}$

20. *If* $\frac{5}{\sqrt{5}} + \sqrt{20} = a\sqrt{5}$, *find the value of* ***a.***

4. *Express* $(2\sqrt{3} - \sqrt{2})(4\sqrt{3} + 5\sqrt{2})$ *in the form* $a + b\sqrt{c}$.

4. *Without using tables or a calculator, find the area of a triangle whose base is*

$(5\sqrt{3} - 3\sqrt{5})cm$ *and height* $(5\sqrt{3} + 3\sqrt{5})cm$.

4. *A rectangle of length* $(x\sqrt{5} + y\sqrt{2})cm$ *and width* $(\sqrt{5} + 4\sqrt{2})cm$ *has an*

area of $(11 + 25\sqrt{10})cm^2$. *Without using tables or a calculator, find the*

the values of ***x*** *and* ***y.***

1. *Express* $\frac{2\sqrt{3} + 5\sqrt{2}}{\sqrt{2} - \sqrt{3}}$ *in the form* $p + q\sqrt{r}$.

6. *Express* $\frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}}$ *without a surd in the denominator.*

5. *Express* $\frac{9}{\sqrt{5}-\sqrt{2}}$ *in the form* $a(\sqrt{b}+\sqrt{c})$.

5. *Express* $\frac{8}{3-\sqrt{5}}$ *in the form* $p+q\sqrt{5}$.

5. *Express* $\frac{1}{\sqrt{5}-\sqrt{2}}$ *with a rational denominator.*

8. *If* $\sqrt{3}=1\cdot732$, *without using tables or a calculator evaluate* $\frac{1+\sqrt{3}}{2+\sqrt{3}}$

correct to ***4*** *decimal places.*

2. *Express* $\frac{1+\sqrt{3}}{1-\sqrt{3}}$ *in the form* $a+b\sqrt{3}$. *Hence evaluate* $\frac{1+\sqrt{3}}{1-\sqrt{3}}$ *correct to*

3 *significant figures if* $\sqrt{3}=1\cdot732$.

2. *Express* $\frac{(1+\sqrt{3})^2}{\sqrt{3}-1}$ *in the form* $a+b\sqrt{3}$.

6. *Without using tables or a calculator, simplify* $\frac{2}{1+\sqrt{3}}-\frac{3}{3-2\sqrt{3}}$

8. *Use the fact that* $\sqrt{7}=2\cdot6458$, *to find the exact value of*

$\frac{2}{3-\sqrt{7}}-\frac{2}{3+2\sqrt{7}}$

6. Use the fact that $\sqrt{7}=2\cdot6458$, to find the exact value of

$\frac{2}{3-\sqrt{7}}-\frac{2}{3+\sqrt{7}}$

STANDARD FORM

Summary:

1. *Standard form is a way of expressing a number in the form* $A \times 10^{n}$ *where* $1 \leq A < 10$ *and **n** is an integer*

2. (i) *To express a number in standard form, we shift the decimal point until the digit part **A** is between **1** and **10.** This digit **A** has a decimal point placed after the first digit.*

(ii) *The power part* (10^{n}) *shows how many places to move the decimal point*

3. *The rules of indices apply to calculations in standard form*

EXAMPLES:

1. *Express the following numbers in standard form:*

(i) 25000 ***(ii) 3860*** ***(iii) 568·3*** ***(iv) 74·8*** ***(v) 3·584*** ***(vi) 4000***

(vii) 0·435 ***(viii) 0·000263*** ***(ix) 0·007*** ***(x) 0·00356*** ***(xi)*** 248×10^{3}

(xii) 4500×10^{-7} ***(xiii)*** $58 \cdot 4 \times 10^{-4}$ ***(xiv)*** $0 \cdot 027 \times 10^{-3}$

(xv) $0 \cdot 0062 \times 10^{-4}$ ***(xvi)*** $0 \cdot 0364 \times 10^{5}$

2. *By expressing each of the numbers in standard form, evaluate the following:*

(i) $0{\cdot}0004 \times 0{\cdot}002$ ***(ii)*** $0{\cdot}005 \times 0{\cdot}00004$ ***(iii)*** $800000 \times 0{\cdot}0005$

(iv) $\frac{800}{0{\cdot}004}$ ***(v)*** $\frac{0{\cdot}00006}{2000}$ ***(vi)*** $\frac{0{\cdot}0009 \times 8000}{0{\cdot}002 \times 0{\cdot}3}$

3. *By expressing each of the numbers in the form* $a \times 10^{n}$ *where **n** is even, evaluate the following:*

(i) $\frac{0{\cdot}81}{0{\cdot}0027}$ ***(ii)*** $0{\cdot}02 \times 0{\cdot}0015$ ***(iii)*** 1500×40000

4. *By expressing each of the numbers in the form* $a \times 10^{n}$ *where* ***n*** *is even, find the square root of:*

(i) $0{\cdot}25 \times 0{\cdot}64$ ***(ii)*** $0{\cdot}16 \times 0{\cdot}09$ ***(iii)*** $0{\cdot}0036 \times 0{\cdot}25$ ***(iv)*** $0{\cdot}0081 \times 0{\cdot}0004$

5. *Without using a calculator, evaluate the following give your answer in standard form:*

(i) $(2 \times 10^{2}) \times (6 \times 10^{5})$ ***(ii)*** $(4 \times 10^{-3}) \times (5 \times 10^{4})$

(iii) $(9 \times 10^{-2}) \times (4 \times 10^{-3})$ ***(iv)*** $(15 \times 10^{-3}) \times (20 \times 10^{7})$

6. *Without using a calculator, evaluate:* $\frac{1{\cdot}21 \times 10^{-2} \times 40}{2{\cdot}2 \times 11}$

7. *Without using a calculator, evaluate:* $\frac{2{\cdot}25 \times 10^{-3} \times 3{\cdot}9}{1{\cdot}3 \times 1{\cdot}5}$

EER:

1. *Without using a calculator, evaluate:* $\frac{0{\cdot}0035}{0{\cdot}07 \times 0{\cdot}2}$

2. *Without using a calculator, evaluate:* $\frac{0{\cdot}625 \times 0{\cdot}009}{0{\cdot}0045}$

3. Without using a calculator, evaluate: $\frac{1{\cdot}69 \times 10^{-2} \times 1{\cdot}25}{1{\cdot}3 \times 2{\cdot}5}$

4. Without using a calculator, evaluate: $\frac{5600}{80000}$ give your answer in the form $a \times 10^{n}$ where $1 \le a < 10$ and n is an integer

LOGARITHMS

Summary:

1. The logarithm of x to the base a is written as $\log_{a} x$

2. The following rules apply to logarithm:

(i) $\log_{c} a + \log_{c} b = \log_{c} ab$

(ii) $\log_{c} a - \log_{c} b = \log_{c} (a/b)$

(iii) $\log_{c} a^{m} = m\log_{c} a$

(iv) $\log_{c} c = 1$

(v) $\log_{c} 1 = 0$

(vi) $\log_{c} a = b$ implies that $a = c^{b}$

(vii) $\log_{c} a = \log_{c} b$ implies that $a = b$

(viii) $\log_{10} a$ is sometimes written as $\log a$

EXAMPLES:

1. Without using a calculator simplify:

(i) $\log_{10} 10$ (ii) $\log_{10} 100$ (iii) $\log_{10} 1000$

(vi) $\log_{10} 10000$ (v) $\log_{2} 16$ (vi) $\frac{1}{2}\log_{2} 64$

(vii) $\log_{5} 125$ (viii) $\frac{1}{3}\log_{3} 729$ (vii) $\log_{3} 81$

2. *Without using a calculator simplify:*

(i) $\log_{10} 40 + \log_{10} 50 - \log_{10} 20$

(ii) $2\log_{10} 50 + \log_{10} 80 - \log_{10} 2$

(iii) $\log_{2} 56 - \frac{1}{2}\log_{2} 49$

(iv) $\log 75 + 2\log 2 - \log 3$

(v) $4\log_{10} 5 + \frac{1}{2}\log_{10} 64 - \frac{1}{3}\log_{10} 125$

(vi) $\log_{10} 5 - \frac{1}{2}\log_{10} 16 - \frac{3}{2}\log_{10} 25$

(vii) $\log_{2} 18 + \log_{2} 32 - 2\log_{2} 6$

(viii) $\frac{1}{2}\log_{10} 16 + \log_{10} a^{2} - 2\log_{10} (a/5)$

(ix) $\frac{2}{3}\log 125 + \frac{1}{2}\log 49 - \log (7/4)$

(x) $\log_{a} a + 2\log_{a} a - \log_{a} a^{-3}$

(xi) $\dfrac{\log_{2} 8}{\log_{2} 4 + \log_{2} 16}$

(xii) $\dfrac{1}{4} + \dfrac{\log_{3} 27}{\log_{3} 9 + \log_{3} 81}$

(xiii) $\dfrac{\log 125}{\log 25} - \dfrac{\log 8}{\log 32}$

3. *Express the following as a single logarithm:*

(i) $2\log_{10} 3 + \frac{1}{3}\log_{10} 216 + \frac{1}{2}\log_{10} 16 - \log_{10} 12$

(ii) $2\log_{10} 8 + \frac{1}{3}\log_{10} 125 - 1$

(iii) $3\log_{3} 12 + \log_{3} 3^{-1} - \log_{3} 6^{2} + 1$

(iv) $\log_{10} 120 + \frac{1}{2}\log_{10} 25 - 2$

4. *Solve the simultaneous equations*

$$\log_{3} (2x + y) = 2$$

$$\log_{2} (5x + 3y) = 2$$

5. *Solve for* x *in the equations:*

(i) $2\log x + \log 6 = 1 + \log 15$

(ii) $\log_{10} (3x + 1) - \log_{10} (x - 1) = \log_{10} 4$

(iii) $\log_{10}(x+1) + \log_{10}(x-2) = \log_{10} 10$

(vi) $\log_{4}(x+3) - \log_{4}(x-3) = 2$

6. *Use logarithm tables to write down the logarithm of the following numbers:*

(i) 1826 ***(ii) 25*** ***(iii) 6·25*** ***(iv) 37·28***

(v) 0·0812 ***(vi) 0·2147*** ***(vii) 0·0025*** ***(viii) 0·0007***

7. *Use the fact that* $\log_{10} 3 = 0\cdot 4771$ *and* $\log_{10} 0\cdot 3 = \bar{1}\cdot 4771$ *to find:*

(i) $\log_{10} 3^{2}$ **(ii)** $\log_{10} 3^{3}$ **(iii)** $\log_{10} \sqrt{3}$

(iv) $\log_{10} \sqrt[3]{3}$ **(v)** $\log_{10} (0\cdot 3)^{2}$ **(vi)** $\log_{10} (0\cdot 3)^{4}$

(vii) $\log_{10} \sqrt{0\cdot 3}$ **(viii)** $\log_{10} \sqrt[3]{0\cdot 3}$ **(ix)** $\log_{10} (0\cdot 3)^{2/3}$

8. *Use logarithm tables to write down the logarithm of the following numbers:*

(i) $(24\cdot 68)^{2}$ **(ii)** $(195\cdot 6)^{3}$ **(iii)** $\sqrt{72\cdot 2}$ **(iv)** $\sqrt[3]{50\cdot 37}$

(v) $(7\cdot 38)^{2/3}$ **(vi)** $(0\cdot 567)^{2}$ **(vii)** $(0\cdot 0248)^{3}$ **(viii)** $\sqrt{0\cdot 0547}$

(ix) $\sqrt{0\cdot 5385}$ **(x)** $\sqrt[3]{0\cdot 472}$ **(xi)** $\sqrt[5]{0\cdot 0843}$ **(xii)** $(0\cdot 48)^{2/3}$

9. *Use the fact that* $\log x = 0\cdot 4771$, $\log y = 0\cdot 8451$ *and* $\log z = \bar{1}\cdot 3010$

to

find:

(i) $\log xy$ (ii) $\log xz$ (iii) $\log yz$ (iv) $\log x/y$

(v) $\log y/x$ (vi) $\log x/z$ (vii) $\log z/x$ (viii) $\log y/z$

(ix) $\log z/y$ (x) $\log xy/z$ (xi) $\log x/yz$ (xii) $\log x^2 y^3$

(xiii) $\log yz^4$ (xiv) $\log xz^{\frac{2}{3}}$ (xv) $\log \sqrt[3]{yz}$ (xvi) $\log x^{\frac{1}{3}} z^{-\frac{1}{2}}$

10. Use the fact that $\log_{10} 2 = 0 \cdot 3010$ and $\log_{10} 3 = 0 \cdot 4771$ to find:

(i) $\log_{10} 6$ (ii) $\log_{10} 60$ (iii) $\log_{10} 72$ (iv) $\log_{10} 5$

(v) $\log_{10} 150$ (vi) $\log_{10} 6 \cdot 4$ (vii) $\log_{10} 0 \cdot 72$ (viii) $\log_{10} 1 \cdot 5$

11. Use logarithm tables or a calculator to find the value of x for which:

(i) $\log_{10} x = 0 \cdot 7781$

(ii) $\log_{10} x = 1 \cdot 3010$

(iii) $\log_{10} x = 1 \cdot 6020$

(iv) $\log_{10} x = \bar{1} \cdot 4771$

(v) $\log_{10} x = \bar{2} \cdot 1761$

12. *Use the fact that* $\log 7 = 0 \cdot 8451$ *and* $\log 2 = 0 \cdot 3010$ *to find:*

(i) $\log(64/49)$, *hence obtain the value of* $\frac{64}{49}$ *correct to* ***3*** *decimal places.*

(ii) $\log(49/64)$, *hence obtain the value of* $\frac{49}{64}$ *correct to* ***3*** *decimal places.*

13. *Given that* $\log x = 2 \cdot 304$ *and* $\log y = \bar{2} \cdot 872$ *find the value of:*

(i) $\log x^{1/2}/y$, *hence find the value of* $x^{1/2}/y$ *correct to* ***3*** *decimal places.*

(ii) $x^{\frac{1}{4}} y^{-\frac{1}{3}}$ *correct to* ***one*** *significant figure.*

14. Given that $\log_{10} 2 = 0 \cdot 3010$ and $\log_{10} 3 = 0 \cdot 4771,$ without using tables or a

calculator, find the value of $\boldsymbol{x}$ for which:

(i) $\log_{10} x = 0 \cdot 7781$

(ii) $\log_{10} x = 0 \cdot 1761$

(iii) $\log_{10} x = 1 \cdot 4771$

(vi) $\log_{10} x = 1 \cdot 3010$

(v) $\log_{10} x = 1 \cdot 6020$

15. Use logarithm tables to evaluate the following:

(i) $25 \cdot 6 \times 731$

(ii) $\dfrac{22 \cdot 6}{47 \cdot 8}$

(iii) $\dfrac{328 \times 0 \cdot 3465}{46 \cdot 82}$

(iv) $\dfrac{22 \cdot 6}{47 \cdot 8 \times 0 \cdot 329}$

(v) $\dfrac{0 \cdot 0721 \times 925}{0 \cdot 00322 \times 405}$

(vi) $5 \cdot 6 \times (14 \cdot 73)^2$

(vii) $\dfrac{3 \cdot 83 \times 5 \cdot 968}{(1 \cdot 597)^2}$

(viii) $\dfrac{\sqrt{3 \cdot 37} \times 0 \cdot 429}{76 \cdot 51}$

(ix) $\sqrt{\dfrac{32 \cdot 6}{17 \cdot 86}}$

(ix) $\sqrt[3]{(0 \cdot 3256)^2}$

(x) $\sqrt[3]{\dfrac{0 \cdot 3216 \times 62 \cdot 58}{41 \cdot 57}}$

(xi) $\left(\dfrac{24 \cdot 8 \times 205}{0 \cdot 04763}\right)^{2/3}$

16. Solve for $\boldsymbol{x}$ in the following equations:

(i) $5^x = 9$

(ii) $3^x = 5$

(iii) $7^{x} = 15$

(iv) $3^{x} = 12$

(v) $1 \cdot 2^{x} = 1 \cdot 728$

17. *Make* ***n*** *the subject in this formula* $A = P\left(1+\frac{r}{100}\right)^{n}$ *, hence find* ***n*** *when*

r = 20, P = 2000 *and* ***A = 3456.***

18 . *A sum of* ***Sh. 400,000*** *is invested in a bank that gives* ***15%*** *compound interest*

per annum. Find how long it will take to accumulate an amount of ***sh529,000.***

EER:

1. *Without using a calculator, simplify:*

(i) $\log_{10} 24 + 2\log_{10} 5 - \log_{10} 6$

(ii) $\log 12 + \log 5 - \log 6$

(iii) $\log_{10} 5 - \frac{1}{2}\log_{10} 16 - \frac{3}{2}\log_{10} 25$

(iv) $\log_{10} 20 + \frac{1}{3}\log_{10} 125$

(v) $\log_{2} 18 + \log_{2} 32 - 2\log_{2} 6$

(vi) $\frac{1}{2}\log_{10} 16a^{6} + 2\log_{10} 5 - \frac{3}{2}\log_{10} a^{2}$

(vii) $3\log_{10} 5 + 6\log_{10} 2 - \frac{1}{2}\log_{10} 64$

(viii) $\dfrac{\log 343}{\log 49} - \dfrac{\log 8}{\log 32}$

(ix) $\frac{2}{3}\log 125 + \frac{1}{2}\log 49 - \log (7/4)$

(x) $\frac{5}{2}\log_{a} a + \frac{1}{2}\log_{a} a^{9} - 3\log_{a} a$

2. Express the following as a single logarithm:

(i) $2\log_{10} 6 + \frac{3}{2}\log_{10} 9 - 4\log_{10} 3$

(ii) $\log_{10} 48 + \frac{1}{2}\log_{10} 25 - 1$

(ii) $\log_{10} 120 + \frac{1}{2}\log_{10} 25 - 2$

3. (i) Find the prime factors of **150.** Hence

(ii) find $\log_{10} 150$, given that $\log_{10} 5 = 0 \cdot 6690$, $\log_{10} 3 = 0 \cdot 4771$ and

$\log_{10} 2 = 0 \cdot 3010$

4. Solve for $\boldsymbol{x}$ in the equation:

(i) $2\log x + \log 2 = \log 18$

(ii) $\log_{10}(2x + 4) - \log_{10}(1 - x) = 1$

(iii) $4\log_4 x - \log_4 2x = 1$

(iv) $\log_8 x + \log_8 2x = 1$

(v) $2\log_{16}(x - 1) - 2\log_{16} 5 = \frac{1}{2}$

(vi) $\log_{12}(x + 9) - \log_{12}(x - 9) = 2$

5. Use the substitution $y = \log_{10} x$ to solve for $\boldsymbol{x}$ in the equation:

$$\left(\log_{10} x\right)^2 - 4\log_{10} x + 4 = 0$$

6. Use the fact that $\log_{10} 2 = 0 \cdot 3010$ and $\boldsymbol{x = 4}$ to find the value of $\log_{10} x^2$.

7. Given that $5^x = 7$, find the value of $\boldsymbol{x}$ correct to **two** decimal places

8. *Make* n *the subject in this formula* $T = ar^{n-1}$*, hence find* n *when*

r = 5, a = 0·48 *and* ***T = 60.***

9 . *A sum of* ***Sh. 400,000*** *is invested in a bank that gives* ***15%*** *compound interest*

per annum. Find how long it will take to accumulate an amount of ***sh529,000.***

10 . *A sum of* ***Sh. 200,000*** *is invested in a bank that gives* ***20%*** *compound interest*

per annum. Find how long it will take to accumulate an amount of ***sh345,600.***

11. *Givent that* $\log a = 0\cdot 234$ *and* $\log b = 1\cdot 185$*, find* $\log a^5 b^2$.

12. *Use the fact that* $\log x = 0\cdot 648$ *and* $\log y = \bar{1}\cdot 913$*, to find* $\log xy^{\frac{1}{3}}$.

13. *Given that* $\log_{10} x = 2\cdot 852$ *and* $\log_{10} y = \bar{2}\cdot 581$*, use tables to evaluate*

$x^{\frac{1}{2}}\big/y$ *correct to* ***three*** *significant figures*

14. *Given that* $\log_{10} p = 2\cdot 476$ *and* $\log_{10} q = \bar{1}\cdot 811$*, find* $\log_{10} (p/q^2)$.

15. Use the fact that $\log_{10} 2 = 0 \cdot 3010$, to find $\log_{10} 5$

16. (i) Show that $\log_{10} 1 = 0$

(ii) Without using tables or a calculator, find $\log_{10} 0 \cdot 001$

17. Use the fact that $\log_{10} 2 = 0 \cdot 3010$ and $\log_{10} 3 = 0 \cdot 4771$ to find:

(i) $\log_{10} 12$ **(ii)** $\log_{10} 80$ **(iii)** $\log_{10} 240$ **(iv)** $\log_{10} (27/16)$

(v) $\log_{10} 5$ **(vi)** $\log_{10} 15$ **(vii)** $\log_{10} 2 \cdot 5$ **(viii)** $\log_{10} 1 \cdot 8$

18. Use the fact that $\log_{10} 7 = 0 \cdot 8451$ and $\log_{10} 6 = 0 \cdot 7782$ to find $\log_{10} 25 \cdot 2$.

19. Use the fact that $\log_{10} 32 \cdot 5 = 1 \cdot 5119$ to find:

(i) $\log_{10} 32500$ **(ii)** $\log_{10} 0 \cdot 00325$ **(iii)** $\log_{10} 3 \cdot 25$

20. Use the fact that $\log_{10} 0 \cdot 225 = \bar{1} \cdot 3522$ to find:

(i) $\log_{10} \sqrt{0 \cdot 225}$ **(ii)** $\log_{10} \sqrt[3]{0 \cdot 225}$

21. Find **x** if:

(i) $7^{x} = 15$ (ii) $\log_{10} x = 1\cdot 3472$

(iii) $1\cdot 4^{x} = 2\cdot 744$ (iv) $\log_{10} x = \bar{2}\cdot 4865$

22. Given that $\log_{10} 2 = 0\cdot 3010$ and $\log_{10} 3 = 0\cdot 4771$, without using tables or a

calculator, find the value of:

(i) $\log_{10} 0\cdot 6$

(ii) $\log_{10} 1\cdot 5$

(iiii) **X** if $\log_{10} x = 1\cdot 9030$

23. Given that $x^{3} = 3\cdot 375$, use tables to find the value of **X** correct to **three**

significant figures

24. Use logarithm tables to evaluate the following:

(i) $75\cdot 6 \times 0\cdot 8563$ (ii) $\frac{12\cdot 75}{7\cdot 908}$ (iii) $\frac{12\cdot 75 \times 28\cdot 34}{7\cdot 908}$

(iv) $\frac{925}{0\cdot 00322 \times 405}$ (v) $\frac{0\cdot 0075 \times 986}{0\cdot 04562 \times 225}$ (vi) $(18\cdot 26)^{4}$

(vii) $\frac{5\cdot 6 \times (14\cdot 73)^{2}}{34\cdot 26}$ (viii) $\frac{22\cdot 6 \times 3\cdot 657}{(0\cdot 329)^{4}}$ (ix) $\sqrt{32\cdot 83}$

(ix) $\sqrt[3]{11\cdot86}$ (x) $(0\cdot48)^{3/5}$ (x) $\sqrt{(31\cdot35)^3}$ (xii) $\sqrt{\dfrac{0\cdot0722}{0\cdot00916}}$

(xiii) $\dfrac{\sqrt{3\cdot83}\times5\cdot968}{(1\cdot597)^2}$ (xiv) $\dfrac{9\cdot324\times0\cdot3639}{\sqrt[3]{81\cdot95}}$ (xv) $\sqrt[3]{\dfrac{0\cdot3215\times1\cdot439}{0\cdot00485}}$

(xvi) $\sqrt[3]{\left(\dfrac{12\cdot75\times28\cdot34}{7\cdot908}\right)^2}$ (xvii) $\dfrac{19\cdot76}{(8\cdot27)^3}$ (xviii) $\dfrac{0\cdot56}{\sqrt{0\cdot08356}}$

RATIOS

Summary:

1. (i) *A ratio is a comparison between two similar quantities*

(ii) *The ratio **x** to **y** is the same as **x: y** or* $\frac{x}{y}$

2. (i) *A ratio is in simplest form if it cannot be reduced any smaller. Thus the ratio*

5:20 = 1:4

(ii) *Before a ratio can be stated the units must be the same. Thus the ratio*

800g: 4kg = 800g: 4000g = 1:5

EXAMPLES:

1. *Find in simplest form the ratio **15** to **20***

Soln:

*Required ratio **15: 20** =* $(15 \div 5) : (20 \div 5) = \mathbf{3 : 4}$

2. *Find in simplest form the ratio **45** minutes to **2** hours*

Soln:

*Required ratio **45min: 2hrs** = 45min : 120 min*

$= (45 \div 15) : (120 \div 15)$

*= **3:8***

3. *Find in simplest form the ratio **5: 10:30***

Soln:

*Required ratio **5: 10:30** =* $(5 \div 5) : (10 \div 5) : (30 \div 5)$

= **1:2:6**

4. Find in simplest form the ratio **1: 0·25:0·75**

Soln:

Required ratio **1: 0·25:0·75** = 100: 25 : 75

$= (100 \div 25) : (25 \div 25) : (75 \div 15)$

= **4:1:3**

5. Find in simplest form the ratio $\frac{3}{4} : \frac{1}{3}$

Soln:

Hint: Multiply each term by the **LCM**

Required ratio $\frac{3}{4} : \frac{1}{3} = \left(\frac{3}{4} \times 12\right) : \left(\frac{1}{3} \times 12\right)$

= **9:4**

6. Find in simplest form the ratio $\frac{3}{8} : 1\frac{1}{2} : 2\frac{3}{4}$

Soln:

Hint: Multiply each term by the **LCM**

Required ratio $\frac{3}{8} : 1\frac{1}{2} : 2\frac{3}{4} = \frac{3}{8} : \frac{3}{2} : \frac{11}{4}$

$= \left(\frac{3}{8} \times 8\right) : \left(\frac{3}{2} \times 8\right) : \left(\frac{11}{4} \times 8\right)$

= **3:12:22**

7. *Arrange the ratios **2:3, 3:7, 3:4** and **2:5** in ascending order of magnitude*

Soln:

Ratio	Percentage form
2:3	$\frac{2}{3} \times 100 = 66\frac{2}{3}\%$
3:7	$\frac{3}{7} \times 100 = 42\frac{6}{7}\%$
3:4	$\frac{3}{4} \times 100 = 75\%$
2:5	$\frac{2}{5} \times 100 = 40\%$

*Required order is **2:5, 3:7, 2:3** and **3:4***

9. *Give that **a : b = 3:5** and **b: c = 2:7,** find the ratio **a:b:c***

Soln:

Hint: *In both ratios make the common term **b** correspond to the same value. Thus multiply **2** in the first ratio and **5** in the second ratio*

a: b *= 3:5 =* $(3 \times 2) : (5 \times 2) = 6 : 10$

b: c *= 2:7 =* $(2 \times 5) : (7 \times 5) = 10 : 35$

$\therefore \quad a : b : c = 6 : 10 : 35$

Method II

$$a : b = 3 : 5 = \frac{3}{5} : 1$$

Also $b : c = 2 : 7 = 1 : \frac{7}{2}$

$$\therefore\ a : b : c = \frac{3}{5} : 1 : \frac{7}{2} = 6 : 10 : 35$$

10. *A sum of* ***Shs 60,000*** *is divided among three boys* ***P, Q*** *and* ***R*** *in the ratio* ***2:3:7.*** *Find how much did each get*

Soln:

Total ratio = $2 + 3 + 7$ ***= 12***

P's share = $\frac{2}{12} \times 60{,}000 = \mathbf{10{,}000}$

Q's share = $\frac{3}{12} \times 60{,}000 = \mathbf{15{,}000}$

R's share = $\frac{7}{12} \times 60{,}000 = \mathbf{35{,}000}$

11. *A sum of* ***Shs 51,000*** *is divided among three boys* ***X, Y*** *and* ***Z*** *in the ratio* $\frac{1}{2} : \frac{1}{4} : \frac{2}{3}$. *Find the:*

(i) *smallest amount shared*

(ii) *largest amount shared*

Soln:

(i) Hint: *Multiply each term by the* ***LCM***

$$X : Y : Z = \frac{1}{2} : \frac{1}{4} : \frac{2}{3} = \mathbf{6:3:8}$$

Total ratio = $6 + 3 + 8$ ***= 17***

Smallest share = $\frac{3}{17} \times 51{,}000 \quad = \mathbf{9{,}000}$

(ii) Largest share = $\frac{8}{17} \times 51{,}000 \quad = \mathbf{24{,}000}$

12. *A sum of money is divided among three boys **A, B** and **C** in the ratio**2:3:5**. If **A** got **Shs 6,000,** find:*

(i) *the total amount shared*

(ii) *how much did each of the other two get*

Soln:

(i) Total ratio = *2 + 3 + 5* ***= 10***

$$\frac{2}{10} \times T \;=\; \mathbf{6{,}000}$$
$$\therefore\; T \;=\; \mathbf{30{,}000}$$

(ii) B's share = $\frac{3}{10} \times 30{,}000 \quad = \mathbf{9{,}000}$

C's share = $\frac{5}{10} \times 30{,}000 \quad = \mathbf{15{,}000}$

13. *A sum of money is divided among three boys **P, Q** and **R** in the ratio**18:16:11.** If **P** got **Shs 5,600** more than **R,** find how much did each get*

Soln:

Total ratio = *18 + 16 + 11* ***= 45***

Ratio difference = *18 – 11* ***= 7***

$$\frac{7}{45} \times T \;=\; \mathbf{5{,}600}$$
$$\therefore\; T \;=\; \mathbf{36{,}000}$$

P's share = $\frac{18}{45} \times 36{,}000 \quad = \mathbf{14{,}400}$

Q's share = $\frac{16}{45} \times 36{,}000 \quad = \mathbf{12{,}800}$

R's share = $\frac{11}{45} \times 36{,}000 \quad = \mathbf{8{,}800}$

14. *A sum of **Shs 24,000** is divided among three boys Tom, Bob and Ben such that Bob gets twice as much as Tom and Ben gets thrice as much as Tom. Find:*

(i) *the ratio in which the money is shared*

(ii) *how much does each get*

Soln:

(i) Tom = x, Bob = 2x, Ben = 3x

Required ratio = x: 2x: 3x = 1:2:3

(ii) Total ratio = *1 + 2 + 3* ***= 6***

Tom's share = $\frac{1}{6} \times 24{,}000 \quad = \mathbf{4{,}000}$

Bob's share = $\frac{2}{6} \times 24{,}000 \quad = \mathbf{8{,}000}$

Ben's share = $\frac{3}{6} \times 24{,}000 \quad = \mathbf{12{,}000}$

15. *A sum of **Shs 55,000** is divided among three girls **P, Q** and **R** such that **Q** gets one and a half times as much as **P** and **R** gets a quarter of what **P** gets. Find:*

(i) *the ratio in which the money is shared*

(ii) *how much does each get*

Soln:

(i) P = x, Q = $1\frac{1}{2}x$, ***R =*** $\frac{1}{4}x$

Required ratio = $x : 1\frac{1}{2}x : \frac{1}{4}x$ ***= 4:6:1***

(ii) Total ratio = *4 + 6 + 1* ***= 11***

P's share = $\frac{4}{11} \times 55{,}000 = \mathbf{20{,}000}$

Q's share = $\frac{6}{11} \times 55{,}000 = \mathbf{30{,}000}$

R's share = $\frac{1}{11} \times 55{,}000 = \mathbf{5{,}000}$

16. *Tom and Bob contributed* ***Shs 400,000*** *and* ***Shs 600,000*** *respectively to run a business. They agreed to share the earned profits of* ***Shs 250,000*** *in the ratio of their contributions. Find how much does each get*

Soln:

Ratio = 400,000: 600,000 = 2:3

Total ratio = *2 + 3* ***= 5***

Tom's share = $\frac{2}{5} \times 250{,}000 = \mathbf{100{,}000}$

Bob's share = $\frac{3}{5} \times 250{,}000 = \mathbf{150{,}000}$

17. *In a class of* ***90*** *students, the ratio of boys to girls is* ***7:2.*** *Find how many more girls are required to join the class so that the ratio of boys to girls is* ***5:4.***

Soln:

Total ratio = *7 + 2* ***= 9***

No of boys = $\frac{7}{9} \times 90 = \mathbf{70}$

No of girls = $\frac{2}{9} \times 90 = \mathbf{20}$

$$\frac{70}{20 + G} = \frac{5}{4}$$
$$\therefore \quad G = \mathbf{36}$$

18. *In a class of* ***126*** *students****,*** *the ratio of boys to girls is* ***5:2.*** *Find how many more girls are required to join the class so that the ratio of boys to girls is* ***3:2.***

Soln:

Total ratio = *5 + 2* ***= 7***

No of boys = $\frac{5}{7} \times 126 = \mathbf{90}$

No of girls = $\frac{2}{7} \times 126 = \mathbf{36}$

$$\frac{90}{36 + G} = \frac{3}{2}$$
$$\therefore \quad G = \mathbf{24}$$

19. *Two numbers are in the ratio* ***3:4.*** *If their sum is* ***63,*** *find the two numbers*

Soln:

x:y = 3:4

Total ratio = *3 + 4* ***= 7***

x = $\frac{3}{7} \times 63 = \mathbf{27}$

y = $\frac{4}{7} \times 63 = \mathbf{36}$

Method II

x : y = 3:4 = 3a:4a

$\Rightarrow$ *Originally, **x= 3a** and **y = 4a (**The ratio **3a:4a** simplifies to **3:4)***

If x + y = 63

$\Rightarrow$ *3a + 4a = 63*

a = 9

$\therefore$ ***x =** 3a = 3(9) **=27,** **y =** 4a = 4(9) **= 36***

***20.** The sum of three numbers is **196.** If the ratio of the first to the second is **2:3** and that of the second to the third is **5:8,** find the second number*

Soln:

*If **A : B = 2:3** and **B: C = 5:8***

$\Rightarrow$ ***A: B** = 2:3 =* $(2 \times 5) : (3 \times 5) = \mathbf{10 : 15}$

***B: C** = 5:8 =* $(5 \times 3) : (8 \times 3) = \mathbf{15 : 24}$

$\therefore \quad A : B : C = 10 : 15 : 24$

Total ratio =** 10 + 15 + 24 **= 49

Value of B = $\frac{15}{49} \times 196 = \mathbf{60}$

***21.** The ages of Tom and Bob are in the ratio in the ratio **12:23.** After **9** years**,** the ratio of their age will be **3:5.** Find their present ages*

Soln:

T : B = 12:23 = 12a:23a

$\Rightarrow$ *Originally, **Tom = 12a** and **Bob = 23a (**The ratio **12a:23a** simplifies to **12:23)***

$$\text{After } \mathbf{9} \text{ years}, \quad \frac{12a + 9}{23a + 9} = \frac{3}{5}$$

$$\therefore \quad a = 2$$

⇒ ***Tom =*** *12a = 12(2)* ***= 24 years***

Bob = *23a = 23(2)* ***= 46 years***

22. *The salaries of Tom and Bob are in the ratio in the ratio* ***5:9.*** *If the salary of each is increased by* ***Shs 80,000,*** *the ratio of their salary will be* ***3:5.*** *Find their salaries after the increment*

Soln:

T : B = 5:9 = 5a:9a

⇒ *Originally,* ***Tom = 5a*** *and* ***Bob = 9a (****The ratio* ***5a:9a*** *simplifies to* ***5:9)***

$$\text{After an increase}, \quad \frac{5a + 80{,}000}{9a + 80{,}000} = \frac{3}{5}$$

$$\therefore \quad a = 80{,}000$$

⇒ ***Tom =*** *5a + 80,000 = 5(80,000) + 80,000* ***= 480,000***

Bob = *9a + 80,000 = 9(80,000) + 80,000* ***= 800,000***

23. *Given that* ***x: y: z = 2:3:5,*** *find the value of* ***z*** *such that:*

(i) x + y + z = 250

(ii) y = 84

Soln:

(i) Total ratio = *2 + 3 + 5* ***= 10***

$$\therefore \quad z = \frac{5}{10} \times 250 = 125$$

(ii) $\frac{3}{10} \times T = 84$

$\therefore T = 280$

$\Rightarrow$ ***z =*** $\frac{5}{10} \times 280 = 140$

Method II

x : y: z = 2:3:5 = 2a: 3a:5a

$\Rightarrow$ *Originally,* ***x= 2a, y = 3a*** *and* ***z = 5a***

If x + y + z = 250

$\Rightarrow 2a + 3a + 5a = 250$

a = 25

$\therefore$ ***z =*** *5a = 5(25)* ***= 125***

(ii) If y = 84

$\Rightarrow$ ***3a = 84***

a = 28

$\therefore$ ***z =*** *5a = 5(28)* ***= 140***

23. *Given that* ***A: B: C = 1:3:6,*** *find the values of* ***A, B*** *and* ***C*** *such that:*

(i) ABC = 144

(ii) A + B + 2C = 80

(iii) $A^2 + B^2 + 2C^2 = 738$

Soln:

(i) A : B: C = 1:3:6= y: 3y:6y

$\Rightarrow$ *Originally,* ***A= y, B = 3y*** *and* ***C = 6y***

If ***ABC = 144***

$\Rightarrow$ *y(3y)(6y) = 144*

$18y^3 = 144$

y = 2

$\therefore$ ***A =*** *y* ***= 2,*** ***B =*** *3y = 3(2)* ***= 6*** ***C =*** *6y = 6(2)* ***= 12***

(ii) A : B: C = 1:3:6= y: 3y:6y

$\Rightarrow$ *Originally,* ***A= y, B = 3y*** *and* ***C = 6y***

If ***A + B + 2C = 80***

$\Rightarrow$ *y + 3y + 2(6y) = 80*

y = 5

$\therefore$ ***A =*** *y* ***= 5,*** ***B =*** *3y = 3(5)* ***= 15*** ***C =*** *6y = 6(5)* ***= 30***

(iii) A : B: C = 1:3:6= y: 3y:6y

$\Rightarrow$ *Originally,* ***A= y, B = 3y*** *and* ***C = 6y***

If $A^2 + B^2 + 2C^2 = 738$

$\Rightarrow$ $y^2 + (3y)^2 + 2(6y)^2 = 738$

$82y^2 = 738$

y = ±3

$\therefore$ ***A =*** *y* ***= ±3,*** ***B =*** *3y = 3(±3)* ***= ±9*** ***C =*** *6y = 6(±3)* ***= ±18***

25. *Give that the ratio* ***x:3 = 12:x,*** *find the positive value of* ***x***

Soln:

$$\frac{x}{3} = \frac{12}{x}$$

$$\therefore\ x = \sqrt{36} = 6$$

***16.** Give that **5x = 3y,** find the ratio:*

(i) x:y

(ii) (10x – 3y) : (3x + 2y)

Soln:

(i) $\quad \text{if } 5x = 3y,$

$$\Rightarrow \frac{x}{y} = \frac{3}{5}$$

$$\therefore\ \boldsymbol{x : y = 3 : 5}$$

(ii) $$\frac{10x - 3y}{3x + 2y} = \frac{10\frac{x}{y} - 3}{3\frac{x}{y} + 2} = \frac{10\left(\frac{3}{5}\right) - 3}{3\left(\frac{3}{5}\right) + 2} = \boldsymbol{\frac{15}{19} = 15 : 19}$$

Method II

x : y = 3:5 = 3a:5a

$\Rightarrow$ *Originally, **x= 3a** and **y = 5a** **(**The ratio **3a:5a** simplifies to **3:5)***

$$\therefore\ \frac{10x - 3y}{3x + 2y} = \frac{10(3a) - 3(5a)}{3(3a) + 2(5a)} = \frac{15a}{19a} = \boldsymbol{\frac{15}{19} = 15 : 19}$$

***26.** Give that **3x + 4y = 12y – 17x,** find the ratio:*

(i) x:y

(ii) (2x + y) : (x + 2y)

Soln:

(i) if $3x + 4y = 12y - 13x,$

$$\Rightarrow 3x + 13x = 12y - 4y$$

$$20x = 8y$$

$$\frac{x}{y} = \frac{8}{20} = \frac{2}{5}$$

$$\therefore \quad \boldsymbol{x : y = 2 : 5}$$

(ii) $$\frac{2x + y}{x + 2y} = \frac{2\frac{x}{y} + 1}{\frac{x}{y} + 2} = \frac{2\left(\frac{2}{5}\right) + 1}{\left(\frac{2}{5}\right) + 2} = \frac{\boldsymbol{9}}{\boldsymbol{12}} = \boldsymbol{3 : 4}$$

Method II

x : y = 2:5 = 2a:5a

⇒ *Originally, **x= 2a** and **y = 5a***

$$\therefore \quad \frac{2x + y}{x + 2y} = \frac{2(2a) + 5a}{2a + 2(5a)} = \frac{9a}{12a} = \frac{\boldsymbol{9}}{\boldsymbol{12}} = \boldsymbol{3 : 4}$$

27. *Give that **3(4p – 2q) : 5(p + q) = 2: 3,** find the ratio **q:p***

Soln:

If $$\frac{3(4p - 2q)}{5(p + q)} = \frac{2}{3},$$

$$\Rightarrow 36p - 18q = 10p + 10q$$

$$26p = 28q$$

$$\frac{q}{p} = \frac{26}{28} = \frac{13}{14}$$

$$\therefore \quad \boldsymbol{q : p = 13 : 14}$$

28. *Give that **2A = 3B = 4C,** find the ratio **A:B:C***

Soln:

If ***2A = 3B = 4C = x,***

$\Rightarrow A = \frac{x}{2}, \quad B = \frac{x}{3} \quad \text{and } C = \frac{x}{4}$

Required ratio ***A:B:C*** $= \frac{x}{2} : \frac{x}{3} : \frac{x}{4}$

$$= \left(\frac{x}{2} \times 12\right) : \left(\frac{x}{3} \times 12\right) : \left(\frac{x}{4} \times 12\right)$$

$$= 6x:4x:3x$$

$$= 6:4:3$$

INCREASING AND DECREASING WITH RATIOS

Summary:

(i) *To increase or decrease a quantity in the ratio* ***a:b,*** *we multiply the quantity by the fraction* $\frac{a}{b}$.

(ii) *For an increase,* $\frac{a}{b}$ *is an improper fraction and for a decrease it is a proper fraction*

EXAMPLES:

1. *Increase* ***Shs 3600*** *in the ratio* ***5:3***

Soln:

***Increased value** = $\frac{5}{3} \times 3{,}600 \ = \mathbf{6{,}000}$*

2.** Decrease **Shs 5,400** in the ratio **7:9

Soln:

***Increased value** = $\frac{7}{9} \times 5{,}400 \ = \mathbf{4{,}200}$*

3.** The fraction $\frac{x}{y}$ becomes $\frac{27}{28}$ when its numerator is increased in the ratio **3:1** and its denominator decreased in the ratio **2:3.** Find in simplest form the ratio **x:y

Soln:

***New fraction** = $\frac{\left(\frac{3}{1} \times x\right)}{\left(\frac{2}{3} \times y\right)} = \frac{9x}{2y}$*

$\Rightarrow \ \frac{9x}{2y} = \frac{27}{28}$

Required ratio x:y = 3:14

EER:

1.** Find in simplest form the ratio **500g** to **3kg

2.** Find in simplest form the ratio **50cm** to **2m

***3.** Find in simplest form the ratio **8** days to **2** weeks*

***4.** Find in simplest form the ratio **6** seconds to **4** minutes*

5.** Find in simplest form the ratio **1: 0·6:0·8

***6.** Find in simplest form the ratio $\frac{1}{4} : \frac{1}{2} : 1\frac{1}{3}$*

7. Give that ***a : b = 2:3*** *and* ***b: c = 4:5,*** *find the ratio* ***a:b:c***

8. The dimension of a rectangular plot of land is ***3·6m*** *and* ***9m.*** *Find the ratio of length to breadth in simplest form*

9. Give that ***p : q = 5:8*** *and* ***z: q = 4:3,*** *find the ratio* ***p:z,*** *hence find* ***p*** *when* ***z = 96***

9. Arrange the ratios ***5:6, 4:9, 7:8*** *and* ***1:2*** *in descending order of magnitude*

10. A sum of ***Shs 3,600,000*** *was divided among six boys, three girls and one man in the ratio* ***4:5:3*** *respectively. Find how much each girl got*

11. There are ***12*** *boys and* ***15*** *girls in a class. Find in simplest form the ratio of:*

(i) *boys to girls*

(ii) *boys to the number of students in the class*

12. If $\frac{3}{8}$ *of the students in a class are boys, find in simplest form the ratio of boys to girls*

13. Given that ***x: y = 6:4*** *and* ***x + y = 30,*** *find the values of* ***x*** *and* ***y***

14. Give that ***3x = 2y,*** *find the ratio* ***x:y***

13. If ***(M + n) : (M − n) = 8: 3,*** *find the ratio* ***M: n.***

15. Give that ***x: y = 2:3,*** *find the ratio* ***(3x + 2y) : (9x – y)***

16. Give that ***(3x – 5y) : (x – y) = 5: 3,*** *find the ratio* ***y:x***

17. Give that ***x: y = 3:5,*** *find the ratio* ***(5x – 2y) : (x + 2y)***

18. A line of length ***40cm*** *is divided in the ratio* ***3:5.*** *Find the length of each division*

19. Tom and Bob are ***12*** *years and* ***9*** *years old respectively. They agreed to share a sum of* ***Shs 28,000*** *in the ratio of their ages. Find how much does* *each get*

20. *The angles of a triangle are in the ratio* ***3:5:7.*** *Find the size of the:*

(i) *smallest angle*

(ii) *largest angle*

21. *A sum of* ***Shs 782,000*** *is divided among three boys* ***X, Y*** *and* ***Z*** *in the ratio* $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$. *Find how much did each get*

22. *The sides of a right angled triangle are in the ratio* ***3:4:5.*** *If the length of its hypotenuse is* ***70mm,*** *find the length of the other two sides*

23. *A sum of* ***Shs 90,000*** *is divided among three boys* ***X, Y*** *and* ***Z*** *in the ratio* ***2:3:7.*** *Find how much more did* ***Z*** *get than* ***X***

24. *A sum of money is divided among three boys* ***P, Q*** *and* ***R*** *in the ratio* ***9:7:5.*** *If* ***P*** *got* ***Shs 32,000*** *more than* ***R,*** *find how much did each get*

25. *A sum of money is divided in the ratio* ***2:3:7*** *such that the largest amount shared is* ***Shs 21,000.*** *Find the:*

(i) *total amount shared*

(ii) *smallest amount shared*

26. *A sum of* ***Shs 40,000*** *is divided among three girls* ***P, Q*** *and* ***R*** *such that* ***R*** *gets six times as much as* ***Q*** *and* ***P*** *gets half of what* ***R*** *gets. Find:*

(i) *the ratio in which the money is shared*

(ii) *how much does each get*

27. *In a school of* ***300*** *students, the ratio of boys to girls is* ***31:44.*** *Find how many more girls are there in the school than boys*

28. *In a mixture of* ***80*** *litres of milk and water, the ratio of milk to water is* ***1:3.*** *Find how much water should be added to the mixture so that the ratio becomes* ***2:7.***

29. *A sum of* ***Shs 60,000*** *is divided among three girls* ***X, Y*** *and* ***Z*** *such that* ***Y*** *gets one and a half times as much as* ***X*** *and* ***Z*** *gets three and a half times as much as* ***X.*** *Find:*

(i) *the ratio in which the money is shared*

(ii) *how much does each get*

30. *A sum of* ***Shs 240,000*** *is divided among* ***Tom, Bob*** *and* ***Ben*** *such that Bob gets twice as much as Tom and Ben gets* ***Shs 15,000*** *more than Bob. Express the ratio of their share* ***Tom: Bob: Ben*** *in its simplest form*

31. *Increase* ***60kg*** *in the ratio* ***7:5***

32. *Decrease* ***90kg*** *in the ratio* ***2:3***

33. *If* ***40%*** *of a number is equal to two–third of another number, find the ratio of the first number to the second number*

34. *A sum of* ***Shs 12,000*** *is divided between* ***Tom*** *and* ***Bob*** *such that* $\frac{4}{15}$ *of Tom's share is equal to* $\frac{2}{5}$ *of Bob's share. Find:*

(i) *the ratio in which the money is shared*

(ii) *how much does each get*

35. ***Shs 12,000*** *is shared among* ***P, Q*** *and* ***R. P*** *takes one–fifth of it,* ***Q*** *takes one–sixth of the remainder and* ***R*** *takes what is left. Find:*

(i) *the ratio in which the money is shared*

(ii) *how much does each get*

36. *The ages of Tom and Bob are in the ratio in the ratio* ***3:5.*** *If* ***9*** *years ago, the ratio of their age was* ***12:23,*** *find their present ages*

37. *The ages of Tom and Bob are in the ratio in the ratio* ***4:1.*** *After* ***6*** *years, the ratio of their age will be* ***5:2.*** *Find their present ages*

38. *Give that* ***x: y = 1:2,*** *find the ratio* ***(2x + 3y) : (x + 4y)***

39. *Give that* ***3A – B = 2A + B,*** *find the ratio:*

(i) A:B

(ii) (2A + B) : (A + 2B)

40. *Find the value of* ***x*** *in the ratio* ***x: 15 = 2:5***

41. *Find the value of* ***x*** *in the ratio* ***(3x + 1) : 5 = (9x + 10) : 20***

42. *Give that* $\frac{10x - 4y}{x + 4y} = 2$, *find the ratio* ***y:x***

3. *Tom and Bob started a business and they realized a profit of* ***Shs 81,000.*** *The profit was to be allocated to development, dividends and reserves in the ratio* ***4:5:6*** *respectively. The dividends were shared in the ratio of their ages. If their ages were* ***25*** *years and* ***20*** *years respectively, find how much each of them got.*

4. *Tom, Bob, Ben, Abel and Adam were given a certain amount of money to share amongst them. Tom got* $\frac{3}{8}$ *of the total amount, while Bob got* $\frac{2}{5}$ *of the remainder. The remaining amount was shared equally among Ben, Abel and Adam each of which received* ***Shs 6,000.***

(a) *Find how much was shared among the five business men*

(b) *Find how much did Tom get*

(c) *Tom, Bob and Ben invested their money and earned a profit of* ***Shs***

12,000. $\frac{1}{3}$ *of the profit was left to maintain the business and the rest was shared in the ratio of their investments. Find how much each got.*

MAP SCALES

Summary:

1. *A map whose scale is* ***1: 250,000,*** *means that* ***1cm*** *on the map is equal to* ***250,000cm*** *on the ground.*

2. *A map scale written in fraction form is called* ***a representative fraction (RF).*** *Thus* $\frac{1}{250,000}$ *is the representative fraction of a map with scale* ***1: 250,000.***

3. *To determine distances and areas on the map, the following results apply:*

(i) Actual distance *= map distance × converted scale*

(ii) Actual area *= map area × (converted scale)* ^{2}

4. *To convert the scale to different units, the following conversions apply:*

(i) 1m = 100cm = 1,000mm

(ii) 1km = 1000m = 100,000cm

EXAMPLES:

1. *Two towns are **8cm** apart on a map whose scale is **1: 250,000.** Find the distance in **km** between the two towns*

Soln:

Actual distance** = map distance × scale in **km

$$\begin{aligned} d &= 8 \times \frac{250,000}{100,000} \\ &= 20km \end{aligned}$$

2. *Two schools are **3·6cm** apart on a map whose representative fraction is $\frac{1}{150,000}$. Find the distance in **km** between the two schools*

Soln:

Actual distance** = map distance × scale in **km

$$\begin{aligned} d &= 3 \cdot 6 \times \frac{150,000}{100,000} \\ &= 5 \cdot 4km \end{aligned}$$

3. *Two districts are **18km** apart. Find the distance in **cm** between them on a map whose scale is **1: 120,000.***

Soln:

Actual distance** = map distance × scale in **km

$$\begin{aligned} 18 &= d \times \frac{120,000}{100,000} \\ &= 15cm \end{aligned}$$

4. *Two points are **5cm** apart on a map whose scale is **1: 240.** Find the distance in **m** between the two points*

Soln:

Actual distance *= map distance × scale in* ***m***

$$\boldsymbol{d} = 5 \times \frac{240}{100}$$
$$= \boldsymbol{12m}$$

5. *On a certain map, a distance of* ***32cm*** *represents a distance of* ***40km*** *on the ground. Find the scale of the map*

Soln:

Actual distance *= map distance × scale in* ***km***

$$40 = 32 \times y$$
$$\boldsymbol{y} = 1 \cdot 25km$$
$$\boldsymbol{y} = 1 \cdot 25 \times 100{,}000 \quad = \boldsymbol{125{,}000cm}$$

$\therefore$ *Required scale =* ***1: 125,000***

6. *On a certain map, a distance of* ***2·5cm*** *represents a distance of* ***6km*** *on the ground. Find the representative fraction* ***(RF)*** *of the map*

Soln:

Actual distance *= map distance × scale in* ***km***

$$6 = 2 \cdot 5 \times y$$
$$\boldsymbol{y} = 2 \cdot 4km$$
$$\boldsymbol{y} = 2 \cdot 4 \times 100{,}000 \quad = \boldsymbol{240{,}000cm}$$

$\therefore$ *Required* ***RF*** *=* $\boldsymbol{\frac{1}{240{,}000}}$

7. *Given that* ***3·6cm*** *on a map represents a distance of* ***5·4km*** *on the ground, find the distance in* ***km*** *of a road represented* ***6cm*** *by on the map*

Soln:

Actual distance *= map distance × scale in* ***km***

$$5 \cdot 4 = 3 \cdot 6 \times y$$

$$\boldsymbol{y} = 1 \cdot 5km$$

$$\boldsymbol{y} = 1 \cdot 5 \times 100{,}000 \quad = \mathbf{150{,}000cm}$$

Actual distance *= map distance × scale in* ***km***

$$\boldsymbol{d} = 6 \times \frac{150{,}000}{100{,}000}$$

$$= \mathbf{9km}$$

8. *A forest covers an area of* $\mathbf{80cm^2}$ *on a map whose scale is* ***1: 50,000.*** *Find the area of the forest in* $\mathbf{km^2}$

Soln:

Actual area *= map area ×* $(scale\ \ in\ \ km)^2$

$$\boldsymbol{A} = 80 \times \left(\frac{50{,}000}{100{,}000}\right)^2$$

$$= \mathbf{20km^2}$$

9. *A farm covers an area of* $\mathbf{135km^2}$. *Find the area of the farm in* $\mathbf{cm^2}$ *on a map whose representative fraction is* $\frac{\mathbf{1}}{\mathbf{150{,}000}}$.

Soln:

Actual area *= map area ×* $(scale\ \ in\ \ km)^2$

$$135 = A \times \left(\frac{150{,}000}{100{,}000}\right)^2$$

$$A = 60cm^2$$

10.** A piece of land measures **33·6m** by **16·5m.** Find the area of this land, in cm^2 on a map whose scale is **1: 120.

Soln:

***Actual area** = map area $\times$ (scale in m)^2*

$$33 \cdot 6 \times 16 \cdot 5 = A \times \left(\frac{120}{100}\right)^2$$

$$A = 385cm^2$$

***11.** Given that an area of $50cm^2$ on a map represents an area of $4 \cdot 5km^2$ on the ground, find the scale of the map*

Soln:

***Actual area** = map area $\times$ (scale in km)^2*

$$4 \cdot 5 = 50 \times y^2$$
$$y = \sqrt{0 \cdot 09}$$
$$y = 0 \cdot 3km$$
$$y = 0 \cdot 3 \times 100{,}000 = 30{,}000cm$$

$\therefore$ *Required scale = **1: 30,000***

***12.** Given that an area of $60cm^2$ on a map represents an area of $135km^2$ on the ground, find the representative fraction **(RF)** of the map*

Soln:

***Actual area** = map area $\times$ (scale in km)^2*

$$135 = 60 \times \mathbf{y}^2$$
$$\mathbf{y} = \sqrt{2 \cdot 25}$$
$$\mathbf{y} = 1 \cdot 5km$$
$$\mathbf{y} = 1 \cdot 5 \times 100{,}000 = \mathbf{150{,}000cm}$$

$\therefore$ *Required **RF** =* $\frac{\mathbf{1}}{\mathbf{150{,}000}}$

***13.** Given that an area of $4 \cdot 8cm^2$ on a map represents an area of $120km^2$ on the ground, find the distance in **km** of a road represented **6cm** by on the map*

Soln:

***Actual area** = map area $\times$ (scale in km)^2*

$$120 = 4 \cdot 8 \times \mathbf{y}^2$$
$$\mathbf{y} = \sqrt{25}$$
$$\mathbf{y} = 5km$$
$$\mathbf{y} = 5 \times 100{,}000 = \mathbf{500{,}000cm}$$

Actual distance** = map distance $\times$ scale in **km

$$\mathbf{d} = 6 \times \frac{500{,}000}{100{,}000}$$
$$= \mathbf{30km}$$

EER:

***1.** Two towns are **8cm** apart on a map whose scale is **1: 250,000.** Find the* *distance in **km** between the two towns*

2.** Two towns are **2km** apart. Find the distance in **cm** between them on a map whose scale is **1: 40,000.

***3.** Two schools are **4·8cm** apart on a map whose representative fraction is* $\frac{1}{250{,}000}$. *Find the distance in **km** between the two schools*

***4.** On a certain map**,** a distance of **2·5cm** represents a distance of **6km** on the ground. Find the scale of the map*

***5.** On a certain map**,** a distance of **5cm** represents a distance of **6km** on the ground. Find the representative fraction **(RF)** of the map*

***6.** Two points are **5cm** apart on a map whose scale is **1: 160.** Find the distance in **m** between the two points*

***7.** A forest covers an area of* $300cm^2$ *on a map whose scale is **1: 15,000.** Find the area of the forest in* km^2

***8.** A farm covers an area of* $5 \cdot 6km^2$. *Find the area of the farm in* cm^2 *on a map whose representative fraction is* $\frac{1}{40{,}000}$.

***9.** Given that an area of* $4cm^2$ *on a map represents an area of* $5 \cdot 76km^2$ *on the ground. Find the representative fraction **(RF)** of the map*

***10.** A lake covers an area of* $4 \cdot 5km^2$. *Find the area of the lake in* cm^2 *on a map whose representative fraction is* $\frac{1}{30{,}000}$.

***11.** Given that **3·6cm** on a map represents a distance of **5·4km** on the ground, find the distance in **km** of a road represented **6cm** by on the map*

12.** A piece of land measures **33·6m** by **16·5m.** Find the area of this land, in* cm^2 *on a map whose scale is **1: 120.

***13.** Given that an area of* $80cm^2$ *on a map represents an area of* $20km^2$

on the ground, find the scale of the map

14. *Given that an area of* $60cm^2$ *on a map represents an area of* $135km^2$ *on the ground, find the representative fraction* ***(RF)*** *of the map*

15. *Given that an area of* $4 \cdot 8cm^2$ *on a map represents an area of* $120km^2$ *on the ground, find the distance in* ***km*** *of a road represented* ***6cm*** *by on the map*

EXPANSION

Summary:

(i) *To expand is to remove the brackets from an expression. Thus in expanded form* ***a(x + 5) = ax + 5a***

(ii) *To multiply two brackets, the first bracket is multiplied by each term in the second bracket*

EXAMPLES:

1. *Remove the brackets and simplify* **:**

(i) $a(3x+5) - x\left(a - \frac{2a}{x}\right)$ ***(ii)(x + 5)(x + 3)*** ***(iii) (3x + 2)(x– 4)***

(iv) 5(x – 3)(3x – 2) ***(v)*** $(x-5)^2$ ***(vi)*** $(2x+3)^2$ ***(vii)*** $a\left(1 - \frac{ax}{2}\right)^2$

(viii) $\left(x + \frac{1}{x}\right)^2$ ***(ix)*** $\left(x - \frac{1}{x}\right)^2$ ***(x)*** $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

(xi) $(a+b)^2$ ***(xii)*** $(a-b)^2$ ***(xiii)*** $(a+b)(a-b)$

QUADRATIC IDENTITIES

Summary:

The following expansions result into:

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $(a+b)(a-b) = a^2 - b^2$

EXAMPLES:

1. *Use the result* $(a + b)^2 = a^2 + 2ab + b^2$, *to evaluate the following:*

(i) $(102)^2$ **(ii)** $(10 \cdot 3)^2$ **(iii)** $(201)^2$

Soln:

(i) *If* $(a + b)^2 = a^2 + 2ab + b^2$,

$$\Rightarrow (102)^2 = (100 + 2)^2 = 100^2 + 2(100)(2) + 2^2$$
$$= 10000 + 400 + 4$$
$$= 10404$$

2. *Use the result* $(a - b)^2 = a^2 - 2ab + b^2$, *to evaluate the following:*

(i) $(99)^2$ **(ii)** $(9 \cdot 7)^2$ **(iii)** $(91)^2$

Soln:

(i) *If* $(a - b)^2 = a^2 - 2ab + b^2$,

$$\Rightarrow (99)^2 = (100 - 1)^2 = 100^2 - 2(100)(1) + 1^2$$
$$= 10000 - 200 + 1$$
$$= 9801$$

3. *If* $x^2 + 6x - 91 = (x + a)^2 + b$, *find the values of* ***a*** *and* ***b***

4. *Express* $x^2 + 6x - 12$ *in the form* $(x + a)^2 + b$

5. *Express* $3x^2 + 9x - 30$ *in the form* $a(x + b)^2 + c$.

6. *Given that* $x + y = 5$ *and* ***xy = 4,*** *find the values of:*

(i) $x^2 + y^2$

(ii) $x - y$

7. If $x + \frac{1}{x} = 3$, *find the values of:*

(i) $x^2 + \frac{1}{x^2}$ *(ii)* $x - \frac{1}{x}$

8. Find the coefficient of x^2 *in the expansion of* $(2x - 5)^3$

*9. Find the coordinates of point **P** on the **x**–axis which is equidistant from the points **A(5, 9)** and **B(–4, 6)***

*10. Find the equation of the locus of points equidistant from point **P(6, 0)** and **Q(–8, 4)***

EER:

1. Expand the following expressions:

(i) (2x + 5)(x + 3) *(ii) (3x + 2)(4x– 7)* *(iii) 3(x – 4)(5x – 3)*

(iv) $(x + y)^2$ *(v)* $(x - y)^2$ *(vi)* $(x + y)(x - y)$

*2. Make **P** the subject in the given formula* $p^2 = (p - q)(p - r)$

4. Expand the expression $\left(p + \frac{1}{p}\right)^2$

5. Expand the expression $h\left(1 - \frac{2}{3}h^3\right)^2$

6. Simplify the expression $\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2$

7. (i) Write down the expansion of $(a + b)^2$

(ii)*Use the result in* ***(i)*** *above to evaluate* $(10 \cdot 6)^2$

8. *Expand and simplify* $(3 - x)^3$

9. *Express* $2x^2 - 7x - 30$ *in the form* $a(x + b)^2 + c.$

10. *If* $3x^2 + 9x - 30 = a(x + b)^2 + c,$ *find the vales of* ***a, b*** *and* ***c***

11. *Given that* $x^2 + y^2 = 29$ *and* ***x + y = 3,*** *find the values of:*

(i) $x^2 + 2xy + y^2$

(ii) $2xy$

(iii) $x^2 - 2xy + y^2$

(iv) x – y, *hence solve for* ***x*** *and* ***y***

12. *If* $x^2 + \frac{1}{x^2} = 27,$ *find the values of* $x - \frac{1}{x}$

13. *Express* $2x^2 - 7x - 30$ *in the form* $a(x + b)^2 + c.$

14. *If* $(3x - 4)^2 = 9x^2 + ax + b,$ *find the vales of* ***a*** *and* ***b***

15. *If* $x + y = 8$ *and* ***xy = 10,*** *find the value of* $x^2 + y^2$

16. *If* $(x - b)^2 = x^2 - 20x + a,$ *find the vales of* ***a*** *and* ***b***

17. *If* $4x^2 - 32x + k - 20 = a(x + b)^2,$ *find the value of* ***k***

18. *Find the coordinates of point* ***P*** *on the* ***y****–axis which is equidistant from the points* ***A(–5, –2)*** *and* ***B(3, 2)***

19. *A point* ***P(x, y)*** *moves such that its distance from the origin,* ***O*** *is equal to its distance from the point* ***Q(1, 2).*** *Find the locus (equation) relating* ***x***

and ***y.***

FACTORING BY GROUPING

Summary:

1. (i) *To factorise is to write the expression as a product of its factors. Thus in factorised form* $3x^2 + 6x = 3x(x + 2)$ *and* $8ab^2 - 12a^2b = 4ab(2b - 3a)$

(ii) *To factorise by grouping, an expression must have four terms*

2. *This method of factoring is performed as follows:*

(i) *Factor out the* ***HCF*** *if any*

(ii) *Group the terms with common factors*

(iii) *Factor each group*

EXAMPLES:

1. *Factorise the following expressions completely:*

(i) $ah + ak + bh + bk$ ***(ii)*** $6mx - 3nx + 2my - ny$

(iii) $6x^2y + 2xy^2 - 27x - 9y$ ***(iv)*** $6ah - 3ak + bk - 2bh$

(vi) $2ab - 3 + 2a - 3b$ ***(vii)*** $2b^3 + 3a^3 + 3ab^2 + 2a^2b$

2. *Without using a calculator or tables, evaluate:*

(i) $0{\cdot}25 \times 2195 - 1795 \times 0{\cdot}25$ ***(ii)*** $65 \times 43{\cdot}6 + 65 \times 37{\cdot}2 - 65 \times 70{\cdot}8$

(iii) $3\cdot142^2 - 3\cdot042 \times 3\cdot142$ ***(iv)*** $\dfrac{30{\cdot}25^2 - 30{\cdot}15 \times 30{\cdot}25}{0{\cdot}0025}$

(v) $\dfrac{21{\cdot}35 \times 41{\cdot}35 - 21{\cdot}35^2}{0{\cdot}02}$

EER:

1. Factorise the following expressions completely:

(i) $ac - 2ab + 3bc - 6b^2$

(ii) $3a^2 + 2ab - 12ac - 8bc$

(iii) $2a^3 - 2ab + b^3 - ab^2$

(iv) $4q^4 - 8q^3 + 12q^2 - 24q$

(v) $2a^2 + 4ab + 3ac + 6bc$

(vi) $4xy^3 - x^3 - yx^2 + 4y^3$

(vii) $3x^3 + 7x^2 + 12x + 28$

(viii) $5x + 10y - ax - 2ay$

2. Without using a calculator or tables, evaluate: $\mathbf{0{\cdot}35 \times 2595 - 2495 \times 0{\cdot}35}$

3. Use factors to evaluate: $\mathbf{14 \times 398 - 198 \times 14}$

4. Use factors to evaluate: $\mathbf{617 \times 793 + 786 \times 793 + 597 \times 793}$

5. Without using a table or a calculator, evaluate: $3 \cdot 2 \times 758 - 658 \times 3 \cdot 2$

6. By first removing the brackets, factorise completely

$$3ph\left(\frac{1}{3p} - \frac{2q}{h}\right) - 6q\left(\frac{h}{3} - \frac{p}{2q}\right)$$

THE DIFFERENCE OF TWO SQUARES

Summary:

The result $a^2 - b^2 = (a + b)(a - b)$ *is called the difference of two squares*

EXAMPLES:

1. *Show that* $(x + y)(x - y) = x^2 - y^2$

2. *Factorise the following expressions completely:*

(i) $p^2 - q^2$ **(ii)** $x^2 - 9$ **(iii)** $x^2 - 1$ **(iv)** $25x^2 - 16$

(v) $9x^2 - 1$ **(vi)** $16x^2 - 25y^2$ **(vii)** $3x^2 - 27y^2$ **(viii)** $50x^3 - 18xy^2$

(ix) $8x^2 - 72$ **(x)** $x^2 - \frac{1}{x^2}$ **(xi)** $3x^2 - \frac{1}{3}$ **(xii)** $x^4 - y^4$

(xiii) $x^4 - 1$ **(xiv)** $ax^4 - a$ **(xv)** $x^4 - 16$ **(xvi)** $81x^5 - x$

3. *Factorise the following expressions completely:*

(i) $a^2 - b^2 + 2a + 2b$ **(ii)** $x^3 - 3x^2 - 16x + 48$ **(iii)** $18x^3 + 9x^2 - 2x - 1$

(iv) $2p^2q^3 - pq^3 + pq - 2p^2q$ **(v)** $(2x - 9)^2 - 25$ **(vi)** $(2x + 5)^2 - 81$

(vii) $(2x + 1)^2 - (x + 5)^2$

4. *Without using a calculator or tables, evaluate:*

(i) $251^2 - 250^2$ **(ii)** $7 \cdot 46^2 - 2 \cdot 54^2$ **(iii)** $5 \cdot 2 \times 3 \cdot 75^2 - 5 \cdot 2 \times 1 \cdot 25^2$

(iv) $67 \cdot 68^2 - 53 \cdot 6 \times 48 \cdot 69 - 32 \cdot 32^2 + 38 \cdot 69 \times 53 \cdot 6$ (v) $\frac{68 \cdot 75^2 - 31 \cdot 25^2}{3 \cdot 75}$

(vi) $\frac{6 \cdot 51^2 - 2 \cdot 81^2}{0 \cdot 932 \times 74}$ (vii) $\frac{32 \cdot 135^2 - 17 \cdot 865^2}{0 \cdot 7135}$

5. *Given that* $a^2 - b^2 = 63$ *and* $a + b = 21$, *find the values of:*

(i) $a - b$ (ii) a and b

6. *Given that* $x^2 - y^2 = 135$ *and* $x - y = 9$, *find the values of* x *and* y

7. *Given that* $x + y = 5$ *and* $xy = 4$, *find the values of:*

(i) $x^2 + y^2$

(ii) $x - y$

(iii) $x^2 - y^2$

8. *If* $x + \frac{1}{x} = 3$, *find the values of:*

(i) $x^2 + \frac{1}{x^2}$

(ii) $x - \frac{1}{x}$

(iii) $x^2 - \frac{1}{x^2}$

EER:

1. *Factorise completely* $27 - 3x^2$

2. *Factorise completely* $32x^2 - 18y^2$

3. *Factorise completely* $2a^4 - 162$

4. *Factorise completely* $x^3 - 3x^2 - x + 3$

5. *Factorise completely* $x^2 - y^2 + 3x + 3y$

6. *Factorise completely* $567 - 28y^2$

7. *Factorise completely* $3p^4 - 48$

8. *By first removing the brackets, factorise completely* $y(ay - x) + x(y - ax)$

9. *Without using a calculator or tables, evaluate the following:*

(i) $102^2 - 98^2$ **(ii)** $351^2 - 350^2$ **(iii)** $495^2 - 5^2$

(iv) $3 \cdot 1^2 - 1 \cdot 9^2$ **(v)** $7 \cdot 46^2 - 2 \cdot 54^2$ **(vi)**

10. *Without using a calculator or tables, evaluate:* $\frac{6{\cdot}25^2 - 3{\cdot}75^2}{0{\cdot}25}$

11. *Without using a calculator or tables, evaluate:* $\frac{30{\cdot}5^2 - 19{\cdot}5^2}{11}$

12. *Factorise the following expressions completely:*

(i) $4x^2 - 1$ **(ii)** $7 - 63x^2$ **(iii)** $9x^2 - 16$ **(iv)** $9x^2 - 4y^2$

(v) $4x^2 - 9$ **(vi)** $x^3 - xy^2$ **(vii)** $50x^3 - 18xy^2$ **(viii)** $3x^2 - 27y^2$

(ix) $x^2 - \frac{1}{9}$ **(x)** $a^2 - \frac{1}{4}$ **(xi)** $4x^2 - \frac{y^2}{100}$ **(xii)** $ax^4 - a$

(xiii) $a^4 - b^4$ **(xiv)** $x^2 - y^2 - x - y$ **(xv)** $x - x^2 + y + y^2$

13. *Factorize* $p^2 - q^2$*, hence find the value of* $7 \cdot 3^2 - 2 \cdot 7^2$

14. *Given that* $x^2 - y^2 = 16$ *and* $x + y = 9$*, find the values of* x *and* y

15. *Given that* $x^2 - y^2 = 33$ *and* $x - y = 3$, *find the values of* x *and* y

16. *Given that* $x^2 - y^2 = 24$ *and* $x + y = 12$, *find the values of* x *and* y

17 *Factorise completely* $2a^2 - 32$

18. *Factorise completely* $3x^3 - 12x$

19. *Factorise completely* $8a^3 - 18b^2$

20. *Factorise completely* $3x^3 + yx^2 - 12xy^2 - 4y^3$

21. *Factorise completely* $2x^3 - 50xy^2$

22. *Factorise completely* $9(x + y)^2 - x^2$

FACTORING QUADRATICS

Summary:

1. *A quadratic expression is written in the form* $ax^2 + bx + c$

2. *The following apply when factoring a quadratic expression* $ax^2 + bx + c$:

(i) *Find two numbers which multiply to give* ***ac*** *and add up to* ***b***

(ii) *Replace* ***b*** *with the sum of those numbers*

(iii) *Factor the terms by grouping*

EXAMPLES:

1. *Factorise the following expressions completely:*

(i) $3x^2 + 11x + 6$ **(ii)** $5x^2 - 13x + 6$ **(iii)** $3x^2 - 4x - 15$

(iv) $x^2 + 2x - 15$ **(v)** $x^2 + 6x + 9$ **(vi)** $x^2 - 4x + 3$

(vii) $x^2 - 5x + 6$ **(vii)** $a^2 + 2ab + b^2$ **(ix)** $3x^2 - 4xy + y^2$

(x) $2y^2 - 3xy - 2x^2$ **(xi)** $20x^2y^2 + xy - 1$ **(xii)** $6p^2 + pq^2 - 2q^4$

2. *Factorise the following expressions completely:*

(i) $25 - (x^2 + 2xy + y^2)$ **(ii)** $a^2 + b^2 - 4 + 2ab$

(iii) $a^2 - 5a - 36 + ay + 4y$ **(iv)** $a^2 - 2ab - 5a + 2b + 4$

(v) $(x^2 - 5x)^2 - 36$

3. *Simplify the following fractions as far as possible:*

(i) $\frac{x^2 - 9}{5x^2 - 13x - 6}$ **(ii)** $\frac{18x^2 - 2}{3x^2 + 2x - 1}$ **(iii)** $\frac{x^2 + 8x + 15}{x^2 - 25}$

(iv) $\frac{3x^2 + x - 4}{5x^2 - 7x + 2}$ **(v)** $\frac{3x^2 - 4x - 15}{5x^2 - 9x - 18}$

4. *Express* $\frac{2x^2 + x - 6}{x^2 - 4} + \frac{1}{x - 2}$ *in the form* $\frac{ax + b}{cx + d}$

5. *Express* $\frac{2}{x + 4} + \frac{4}{x - 3} - \frac{4(x + 4)}{x^2 + x - 12}$ *in the form* $\frac{a}{x + b}$

6. *Express* $\frac{2x^2 + x}{4x^2 - 1} - \frac{2}{2x - 1}$ *in the form* $\frac{ax + b}{cx + d}$

7. *Express* $\frac{x + 1}{x + 2} - \frac{x + 4}{3x + 6}$ *in the form* $\frac{ax + b}{cx + d}$

EER:

1. *Factorise the following expressions completely:*

(i) $5x^2 - 11x + 2$ **(ii)** $x^2 + 2x - 15$ **(iii)** $x^2 + 8x + 16$

(iv) $25x^2 - 10x + 1$ **(v)** $x^2 - 5x + 6$ **(vi)** $a^2 - 2ab + b^2$

(vii) $9x^2 - 12x + 4$ **(viii)** $2x^3 - 2x^2 - 4x$ **(ix)** $2x^2 - 21xy - 50y^2$

(x) $x^2 - 15xy + 56y^2$ **(xi)** $12x^2 - 17xy + 5y^2$ **(ix)** $2x^2y^2 + 13xy + 15$

2. By first simplifying the expression factorise completely
$6y^2 - 11y - 6 - (2y - 3)^2$

3. By first simplifying the expression factorise completely $(4 - x)^2 - 2x$

4. Factorise $35 - 2a - a^2$

5. Factorise $120 - 7x - x^2$

6. Factorise completely $2x^3 + 5x^2y - 12xy^2$

7. Factorise $x^2y^2 - 8xy - 48$

8. Factorise $6x^2 + xy^2 - 2y^4$

9. Factorise completely $2x^4 - 7x^2 - 4$

10. Factorise completely $y^2 - 2yb - 5y + 2b + 4$

11. Simplify the following fractions as far as possible:

(i) $\dfrac{9x^2 - 1}{3x^2 + 2x - 1}$ **(ii)** $\dfrac{x^2 + 6x - 16}{x^2 - 4}$ **(iii)** $\dfrac{x^2 + x - 2}{x^2 - x - 6}$

(iv) $\dfrac{x^2 - 2x - 3}{x^2 - x - 2}$ **(v)** $\dfrac{a^2 - a + \frac{1}{4}}{a^2 - \frac{1}{4}}$ **(vi)** $\dfrac{3x^2 - 4xy + y^2}{9x^2 - y^2}$

(vii) $\frac{2y^2 - 3xy - 2x^2}{4y^2 - x^2}$ **(viii)** $\frac{6a - 3ab - 2a - a^2}{a^2 + ab}$ **(ix)** $\frac{9a^2y - 16b^2y^3}{4by^2 - 3ay}$

12. Simplify $\frac{x^2 + 2x}{x^3 - x^2 - 6x}$, hence solve $\frac{x^2 + 2x}{x^3 - x^2 - 6x} = \frac{1}{4}$

13. Express $\frac{4}{x+5} + \frac{1}{x-2} - \frac{7}{x^2 + 3x - 10}$ in the form $\frac{a}{x + b}$

14. Express $\frac{2}{3x + 2} + \frac{5x + 3}{9x^2 - 4}$ in the form $\frac{ax + b}{cx^2 + d}$

15. Express $\frac{7}{x-2} - \frac{x + 5}{x^2 - 3x + 2}$ in the form $\frac{a}{x + b}$

16. Express $\frac{2}{x + 2} + \frac{8x + 4}{x^2 - 4}$ in the form $\frac{ax}{x^2 + b}$

SOLVING QUADRATIC BY FACTORING

Summary:

1. *A quadratic equation is written in the form* $ax^2 + bx + c = 0$

2. *A quadratic equation has two solutions often called the roots of the equation. It is possible for the two solutions to be the same*

3. *This method uses the fact that if* $m \times n = 0$, *then* $m = 0$ *or* $n = 0$

EXAMPLES:

1. *Solve the following equations:*

(i) $2x^2 + 7x + 6 = 0$ **(ii)** $x^2 - x = 12$ **(iii)** $x^2 + 12x + 36 = 0$

(iv) $x^2 - 5x = 0$ **(v)** $5x^2 = 3x$ **(vi)** $5x^3 - 5x^2 - 10x = 0$

(vii) $(x + 4)(x + 2) = 48$ **(vii)** $x + 2 = \frac{3}{x}$ **(ix)** $\frac{1}{x - 1} + \frac{4}{x} = 3$

(x) $\frac{1}{x + 1} = 1 - \frac{5}{2x - 4}$ (xi) $\frac{1}{x + 2} + \frac{1}{x^2 - 4} = \frac{2}{5}$

(xii) $\frac{x - 1}{x - 2} + \frac{5x - 13}{x^2 - x - 2} = 0$

2. Factorise $25x^2 - 64$, hence solve the equation $25x^2 - 64 = 0$

3. Factorise $(x^2 - 5x)^2 - 36$, hence solve the equation $(x^2 - 5x)^2 - 36 = 0$

4. Solve the following equations:

(i) $x^2 - 16 = 0$ (ii) $81x^2 - 64 = 0$ (iii) $(2x - 9)^2 = 25$

(iv) $(2x + 5)^2 - 81 = 0$ (v) $(2x + 1)^2 - (x + 5)^2 = 0$

SOLVING QUADRATIC BY FORMULA

Summary:

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXAMPLES:

1. Solve the following equations:

(i) $2x^2 + 7x + 6 = 0$ (ii) $3x^2 - 4x = 15$ (iii) $x^2 - x - 12 = 0$

(iv) $\frac{x}{x + 1} - \frac{x + 1}{3x - 1} = \frac{1}{4}$

EER:

1. Factorise $x^2 + 6x - 91$, hence solve the equation $x^2 + 6x - 91 = 0$

2. Use the factorisation method to solve the equation: $3x^2 - 19x - 14 = 0$

3. Factorise $25x^2 - 36$, hence solve the equation $25x^2 - 36 = 0$

4. Given that $p \uparrow q = p^2 + 2pq - q$, find the values of:

(i) $2 \uparrow 9$

(ii) p if $p \uparrow 4 = 29$

5. If $\frac{x^2}{x + c} = \frac{u}{w}$, make **x** the subject of the formula

6. Solve the equation: $6x^2 + 5x = 6$

7. Solve the equation $2x - 7 = \frac{15}{x}$

8. Solve the equation: $\frac{x - 1}{x + 1} = \frac{7x - 13}{6x + 1}$

9. Solve the following equations:

(i) $x^2 - 3x = 0$ (ii) $x^2 - 5x + 6 = 0$ (iii) $8x^2 - 10x + 3 = 0$
(iv) $x^2 + 5x = 0$ (v) $x^2 - 9 = 0$ (vi) $25x^2 - 36 = 0$
(vii) $x^2 + 6x + 9 = 0$ (viii) $x^2 + 7x + 12 = 0$ (ix) $10x^2 + 19x + 6 = 0$
(x) $4x^2 - 12x + 9 = 0$ (xi) $x(x - 1) = 6$ (xii) $(2x + 5)^2 - (x - 2)^2 = 0$ (xiii) $(x - 3)(x - 2) = 12$

10. Solve the following equations:

(i) $x = \frac{10}{3 + x}$ (ii) $\frac{x}{x + 3} = \frac{8}{x + 6}$ (iii) $\frac{x}{x + 3} - \frac{2}{x} = 1$
(iv) $\frac{x - 3}{2} = \frac{5}{x}$ (v) $x = \frac{15 - x^2}{2}$ (vi) $\frac{1}{x - 1} - \frac{1}{x^2 - 1} = \frac{1}{6}$

(vii) $\frac{x+1}{2x+5} = \frac{x-1}{3}$ **(viii)** $\frac{x}{2x-5} = \frac{3x}{x+11}$ **(ix)** $\frac{x}{x-1} - \frac{2}{x} = \frac{1}{x-1}$

(x) $\frac{1}{3x-4} + \frac{x}{x+1} = 1$ **(xi)** $\frac{x^2+4}{x^2-4} + \frac{2}{x+2} = \frac{5}{x-2}$

(xii) $x + 3 + \frac{3}{x-1} = \frac{4-x}{x-1}$

SOLVING QUADRATIC BY COMPLETING SQUARES

Summary:

(i) *A quadratic perfect square is written in the form* $a(x+b)^2$

(ii) *In completing squares, the original equation is first expressed in the form* $a(x+b)^2 + c = 0$ *and then solved*

EXAMPLES:

1. *Express* $x^2 - 8x - 20$ *in the form* $(x + a)^2 + b$. *Hence solve the equation* $x^2 - 8x - 20 = 0$.

2. *Express* $x^2 + 6x - 91$ *in the form* $(x + a)^2 + b$. *Hence solve the equation* $x^2 + 6x - 91 = 0$.

3. *Express* $2x^2 - 7x - 30$ *in the form* $a(x + b)^2 + c$. *Hence solve the equation* $2x^2 - 7x - 30 = 0$.

4. *Express* $3x^2 + 9x - 30$ *in the form* $a(x + b)^2 + c$. *Hence solve the equation* $3x^2 + 9x - 30 = 0$.

5. *Solve the equation* $3x^2 - 7x + 2 = 0$ *by the method of completing squares*

6. *Given that* $4x^2 - 12x + k$ *is a perfect square, find the value of* ***k***

EER:

1. If $x^2 + 8x - 20 = (x + a)^2 + b$, find the values of ***a*** and ***b***

QUADRATIC WORD POBLEMS

EXAMPLES:

1. *A rectangle of length **(3x + 1)cm** and width **(3x – 2)cm** has an area of* $130cm^2$. *Find the dimensions and perimeter of the rectangle*

2. *A man is **22** years older than his son. The product of their ages is **240** years. Find their present ages*

3. *Find the values of **x** for which the fraction* $\frac{5x + 7}{x^2 + 5x + 6}$ *is undefined*

4. *The distance between the point **(k, k+2)** and the origin is **10 units.** Find the possible values of **k***

5. *Find the value of **n** in the following equations:*

(i) $212_n = 25_{nine}$ **(ii)** $223_n = 63_{ten}$

6. *Solve the following simultaneous equations:*

(i) $2x - y = 5$, $xy = 12$ (ii) $6x + 7y = 4$, $x^2 + y^2 = 13$ (iii) $x + 3y = 10$, $3x^2 + 5y^2 = 16xy$

7. *Find the coordinates of the point of intersection of the line **y = 2 – 3x** and the*

curve $y = x^2 - 4x$

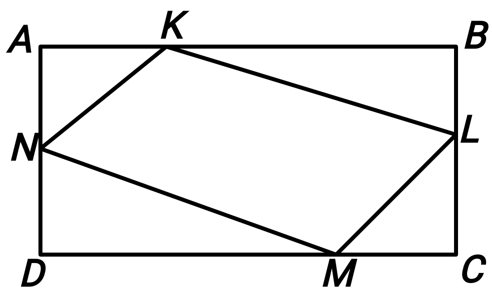
8. *Find the area of a right triangle whose hypotenuse is **13cm** long and its perimeter is **30cm.***

9. *The cost of **n** pens is **Shs 7,500.** This same amount of money can buy* *five extra pens if a discount of **Shs 50** per pen is given. Find the cost of each pen*

***10.** A basin can be filled by tap **P** in **10** minutes less than **Q.** If the two taps take **12** minutes to fill this basin when they are turned on at once, find the time taken by each tap separately to fill the basin.*

***11.** A train takes two hours less for a journey of **300km** if its speed is increased by $5kmh^{-1}$ from its normal speed. Find its normal speed*

*12. **ABCD** is a rectangle in which **AB = 12cm, BC = 7cm** and **AK = BL = CM = DN = y cm.***



*If the area of **KLNM** is $45cm^2$, find the value of **y***

FORMING A QUADRATIC EQUATION

Summary:

The relation $x^2 - (sum\ \ of\ \ roots)x + product\ \ of\ \ roots = 0$ is used to form a quadratic equation whose roots are known

EXAMPLES:

1.** Form a quadratic equation whose roots are **2** and **3

2.** Form a quadratic equation whose roots are **–5** and **2

3.** Form a quadratic equation whose solution set is **{–7, 3}

***4.** Form a quadratic equation whose roots are **–2** and $\frac{1}{3}$*

EER:

1. *The length of a rectangular floor is* **8** *metres longer than its width. If the area of the floor is* $65m^2$*, find the dimensions and perimeter of the floor.*

2. *If* $125_n = 85_{ten}$*, find the value of* **n**

3. *A number exceeds four times its reciprocal by* **3.** *Find the number.*

4. *Find the area of a right triangle whose hypotenuse is* **34cm** *long and its perimeter is* **80cm.**

5. *Find the number which when added to its square gives a total of* **42**

6. *Find the dimensions of the rectangle whose diagonal is* **10cm** *long and its length exceeds the width by* **2cm.**

7. *Find the dimension of a rectangle whose area is* $72cm^2$ *and its perimeter is* **34cm.**

8. *If* $124_n = 310_{four}$*, find the base that* **n** *represents*

9. *Express* $x^2 - x - \frac{3}{4}$ *in the form* $(x + p)^2 + q$*. Hence solve the equation* $x^2 - x - \frac{3}{4} = 0.$

10. *Solve for* **y** *in the following equations:*

(i) $27 \times 3^{2y} = \left(3^y\right)^y$ **(ii)** $(16)^{y^2} = 8^{4y-3}$

11. *Given that vector* $p = \begin{pmatrix} x \\ x+2 \end{pmatrix}$*, find the possible values of* **x** *for which* $|p| = 10$

12. *The distance between the points* **(4, 8)** *and* **(1, k)** *is* **5 units.** *Find the possible values of* **k**

13. *A rectangle of length* **(4x – 1)cm** *and width* **2x cm** *has an area of* $10cm^2$*. Find:*

(i) *the value of* ***x***
(ii) *its length and width*
(iii) *its perimeter*

14. *Find the coordinates of the points of intersection of the curve* $y = 5x^2 - 13$ *and the line* ***y = 7.***

15. *A right angled triangle of base* ***(x – 4)cm*** *and height* ***(x – 2)cm*** *has a hypotenuse of* ***xcm*** *long. Find:*

(i) *the value of* ***x***
(ii) *its dimensions*
(iii) *its perimeter and area*

16. *A right angled triangle of base* ***xcm*** *and height* ***(x – 7)cm*** *has a hypotenuse of* ***(x + 1)cm*** *long. Find:*

(i) *the value of* ***x***
(ii) *its base and height*
(iii) *its perimeter and area*

17. *Find the coordinates of the points of intersection of the curve* $y = x^2 - 5x$ *and the line* ***y + 6 = 0***

18. *Find the coordinates of the point of intersection of line* ***y – 5x + 9 = 0*** *and the curve* $y = x^2 - 3$

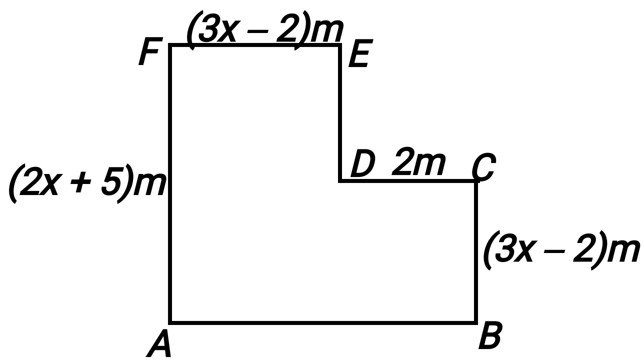
19. *Find the values of* ***x*** *for which the fraction* $\frac{5x + 7}{x^2 - x - 12}$ *is undefined*

20. *A group of people planned to contribute equally towards a water project which needed* ***Shs 2,000,000*** *to complete. However,* ***40*** *members of the group withdrew from the project. As a result, each of the remaining members was to contribute* ***Shs 2500*** *more.*
(a) *Find the original number of members in the group*
(b) *Forty five percent of the value of the project was donated by the Development Bank. Calculate the amount required to be contributed by each of the remaining members of the group*

(c) *Members contribution was in terms of labour provided and money contributed. The ratio of the value of labour to the money contributed was* ***6:19,*** *calculate the total amount of money contributed by the members*

21. *The figure below shows an* ***L****–shaped carpet* ***ABCDEF*** *in which* ***CD = 2m,***

BC = EF = (3x – 2)m *and* ***AF = (2x + 5)m.***



Given that the area of the carpet is $25m^2$,

(a) *Show that* $6x^2 + 17x - 39 = 0$

(b) *Calculate the:*

(i) *length of the longest side of the carpet*

(ii) *perimeter of the carpet*

GRAPHING QUADRATIC CURVES

Summary:

1. (i) *By drawing a suitable line, a quadratic graph can be used to solve related equations*

(ii) *The solution to the equation are the x–values at the point where the graphs meet*

2. *The appropriate line is obtained as follows:*

(i) *Compare the equation to that of the graph*

(ii) *Eliminate the term in* x^2 *to obtain the line*

3. (i) *The curve* $y = ax^2 + bx + c$ *has a vertical line of symmetry whose equation is given by* $x = -\frac{b}{2a}$

(ii) *The maximum or minimum value of the function occur at its turning points*

EXAMPLES:

1. *Find the equation of the line which should be drawn on the graph* $y = x^2 + 2x - 4$ *to solve each of these equations:*

(i) $x^2 + 2x - 4 = 0$ ***(ii)*** $x^2 + 2x - 7 = 0$ ***(iii)*** $x^2 + x - 5 = 0$

(iv) $x^2 - x - 6 = 0$ ***(v)*** $2x^2 - 5x - 3 = 0$ ***(vi)*** $1 + x - 2x^2 = 0$

2. (a) *Draw a graph of* $y = x^2 - 4x + 3$ *for* $\mathbf{0 \le x \le 4}$
(*use a scale of* ***2cm:1 unit*** *on both axes* ***)***

(b) *Use your graph to solve the equations:*

(i) $x^2 - 4x + 3 = 0$

(ii) $x^2 - 5x + 4 = 0$

(c) *State the:*

(i) *equation of the line of symmetry*

(ii) *minimum value of* ***y***

(iii) *value of* ***x*** *at which the minimum value of* ***y*** *occurs*

(iv) *range of values of* ***x*** *for which* $x^2 - 4x + 3 < 0$

3. (a) *Draw a graph of* $y = x^2 - x - 6$ *for* $-3 \le x \le 4$
(use a scale of **1cm:1 unit** *on both axes)*

(b) *Use your graph to solve the equations:*

(i) $x^2 - x - 6 = 0$

(ii) $x^2 - x - 2 = 0$

(iii) $x^2 + x - 2 = 0$

(iv) $2x^2 - 5x - 3 = 0$

(c) *State the:*

(i) *equation of the line of symmetry*

(ii) *range of values of* x *for which* $x^2 - x - 6 < 0$

4. (a) *Copy and complete the table below for the function* $y = 3 - 3x - x^2$

x	–5	–4	–3	–2	–1	0	1	2
$-x^2$				–4				–4
$-3x$				6				–6
3				3				3
y				5				–7

(b) *Draw a graph of* $y = 3 - 3x - x^2$ *for* $-5 \le x \le 2$
(use a scale of **1cm:1 unit** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $3 - 3x - x^2 = 0$ **(ii)** $x^2 + 3x - 4 = 0$ **(iii)** $x^2 + 4x + 3 = 0$

(iv) $x^2 + 2x - 8 = 0$ **(v)** $2x^2 + x - 6 = 0$

5. (a) *Copy and complete the table below for the function* $y = (x - 2)(x + 1)$

x	–3	–2	–1	0	1	2	3	4
(x – 2)		–4		–4				2
(x + 1)		–1		6				5
y		4		3				10

(b) *Draw a graph of* $y = (x - 2)(x + 1)$ *for* $-3 \le x \le 4$

(use a scale of ***1cm:1 unit*** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $x^2 - x - 2 = 0$ **(ii)** $x^2 - 2x - 5 = 0$

(iii) $2x^2 - x - 15 = 0$

(d) *State the:*

(i) *equation of the line of symmetry*

(ii) *minimum value of the function*

(iv) *range of values of* **x** *for which* $(x - 2)(x + 1) < 0$

EER:

1. *Use the graphical method to solve the simultaneous equations* $y = 3x^2 - 3x$ *and* $y = 10 - 5x$ *for* $-3 \le x \le 3$.

2. (a) *Draw a graph of* $y = x^2 - 2x + 1$ *for* $-3 \le x \le 3$

(use a scale of ***1cm:1 unit*** *on both axes)*

(b) *Use your graph to solve the equations:*

(i) $x^2 - 2x + 1 = 0$

(ii) $x^2 - x - 6 = 0$

3. (a) *Copy and complete the table below for the function* $y = 2 + x - x^2$

X	–3	–2	–1	0	1	2	3	4
$-x^2$		–4					–9	
x		–2					3	
2		2					2	
Y		–4					–4	

(b) *Draw a graph of* $y = 2 + x - x^2$ *for* ***–3 ≤ x ≤ 4***
(use a scale of ***1cm:1 unit*** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $x^2 - x - 6 = 0$

(ii) $x^2 - 3x - 4 = 0$

(iii) $3x^2 - 2x - 1 = 0$

4. *On the same axes, draw the graphs of* $y = 2x^2$ *and* $y = \frac{5x}{2} + 5$ *for* ***–2 ≤ x ≤ 3*** *(use a scale of* ***2cm : 1 unit*** *on the* ***x-axis*** *and* ***1cm : 2 units*** *on the* ***y-axis)***

*(**b**) Use your graphs to solve the equations:*

*(**i**)* $4x^2 - 5x - 10 = 0$

*(**ii**)* $6x^2 + 10x - 30 = 0$ *correct to **2** decimal places*

***5.** (**a**) Copy and complete the table below for the function*
$y = (x - 1)(x - 3)$

X	–1	0	1	2	3	4	5
(x – 1)		–1			2		
(x – 3)		–3			0		
Y		3			0		

*(**b**) Draw a graph of* $y = (x - 2)(x + 1)$ *for **–1 ≤ x ≤ 5***
*(use a scale of **1cm:1 unit** on both axes)*

*(**c**) Use your graph to solve the equations:*

*(**i**)* $x^2 - 4x + 3 = 0$

*(**ii**)* $x^2 - 4x + 1 = 0$

*(**d**) State the:*

*(**i**) minimum value of the function*

*(**iv**) range of values of **x** for which* $(x - 1)(x - 3) < 0$

***6.** (**a**) On the same axes, draw the graphs of* $y = 2x^2 + 5x - 3$ *and* $y = x + 1$ *for **–4 ≤ x ≤ 1** using intervals of **0·5** (use a scale of **2cm : 1 unit** on the **x-axis** and **1cm : 1 unit** on the **y-axis**)*

(b) *Use your graphs to solve the equation* $x^2 + 2x - 2 = 0$

7. **(a)** *Copy and complete the table below:*

X	–4	–3	–2	–1	0	1	2	3	4
$x^2 - 2$	14							7	
$-x^2 + 6$	–10							–3	

(b) *On the same axes, draw the graphs of* $y = x^2 - 2$ *and* $y = -x^2 + 6$ *for* **$-4 \le x \le 4$**. *(use a scale of* **1cm : 1 unit** *on the* **x-axis** *and* **1cm : 2 units** *on the* **y-axis)**

(c) *Use your graphs to solve the equation* $x^2 - 2 = 6 - x^2$

8. **(a)** *Draw a graph of* $y = x^2 - 2x - 3$ *for* **$-2 \le x \le 4$**
(use a scale of **1cm:1 unit** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $x^2 - 2x - 3 = 0$

(ii) $x^2 - 2x - 1 = 0$

(iii) $x^2 - 3x - 3 = 0$

9. **(a)** *Copy and complete the table below for the function* $y = 6 + 3x - 2x^2$

X	–2	–1·5	–1	–0·5	0	0·5	1	1·5	2	2·5	3
$-2x^2$	–8		–2	–0·5			–2	–4·5	–8		–18
$3x$	–6		–3	–1·			3	4·5	6		9

				5							
6	6	6	6	6	6	6	6	6	6	6	6
Y	–8		1	4			7	6	4		–3

(b) *On the same axes, draw the graphs of* $y = 6 + 3x - 2x^2$ *and* $y = 2x$ *for* ***–2 ≤ x ≤ 3*** *(use a scale of* ***2cm : 1 unit*** *on the* ***x-axis*** *and* ***1cm : 1 units*** *on the* ***y-axis)***

(c) *Use your graphs to solve the equation* $6 + x - 2x^2 = 0$

10. (a) *On the same axes, draw the graphs of* $y = 6x + x^2 - x^3$ *and* $y = 4x$ *for* ***–3 ≤ x ≤ 4*** *(use a scale of* ***1cm : 1 unit*** *on the* ***x-axis*** *and* ***1cm : 2 units*** *on the* ***y-axis)***

(b) *Use your graphs to solve the equations:*

(i) $6x + x^2 - x^3 = 0$

(ii) $2x + x^2 - x^3 = 0$

SUBJECT OF THE FORMULA

Summary:

1. *Subject of the formula is the formula for the required subject.*

2. *Making the subject of the formula means finding the formula for the required subject* **(***term***)***.*

EXAMPLES:

1. *Make* ***m*** *the subject in this formula* ***y = mx + c.***

2. *Make* ***s*** *the subject in this formula* $v^2 = u^2 + 2as$.

3. *If* $s = ut + \frac{1}{2}at^2$, *express* ***a*** *in terms of* ***s, u*** *and* ***t.*** *Hence find* ***a*** *when* ***s = 19, u = 8*** *and* ***t = 2.***

4. *Make* ***c*** *the subject in this formula* $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$, *hence find* ***c*** *when* ***a = 10*** *and* ***b = 15.***

5. *If* $T - mg = ma$, *express* ***m*** *in terms of* ***T, g*** *and* ***a.*** *Hence find* ***m*** *when* ***T = 52, a = 3·2*** *and* ***g = 9·8.***

6. *If* $k(n + 3) = 5n + 2$, *express* ***n*** *in terms of* ***k.*** *Hence find* ***n*** *when* ***k = 1.***

7. *Make* ***v*** *the subject in this formula* $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, *hence find* ***v*** *when* ***f = 6*** *and* ***u = 10.***

8. *If* $\frac{1}{a} = \frac{bc}{b + c}$, *make* ***c*** *the subject of the formula*

9. *Make* ***n*** *the subject in this formula* $I = \frac{nE}{nR + r}$

10. *Make* ***V*** *the subject in this formula* $R = \frac{Vr}{V - 2}$. *Hence find* ***V*** *when* ***R = 5r***

11. If $s = \frac{rk - a}{r - 1}$, make **c** the subject of the formula. Hence find **r** when **s = 93, a = 3** and **k = 48.**

12. If $V = \frac{1}{3}\pi r^2 h$, make **r** the subject of the formula.

13. If $a^2 + b^2 = c^2$, make **b** the subject of the formula. Hence find the values of **b** when **c = 10** and **a = 6.**

14. If $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, make **y** the subject of the formula. Hence find the values of **y** when **a = 4, b = 8** and **x = 5.**

15. If $b^2 = 4ac + (2ax + b)^2$, make **x** the subject of the formula. Hence find the values of **x** when **a = 2, b = –4** and **c = –6.**

16. Make **r** the subject in this formula $V = \frac{4}{3}\pi r^3$. Hence find **r** when **v = 38·808** and $\pi = \frac{22}{7}$.

17. Make **b** the subject in this formula $a = \frac{b^3}{b^3 + c}$. Hence find **b** when **c = 864** and **a = 0·2.**

18. Make **r** the subject in this formula $V = \frac{\pi pr^4}{8kl}$.

19. Make **r** the subject in this formula $A = P\left(1 + \frac{r}{100}\right)^n$, hence find **r** when

A = 8,820, p = 8,000 and **n = 2.**

20. If $k = \sqrt{\frac{w}{w + a}}$, make **w** the subject of the formula

21. Make **d** the subject in this formula $p = 2\pi\sqrt{\frac{d}{d-k}}$.

22. Make **k** the subject in this formula $p = \left(\frac{k-1}{k+1}\right)^{\frac{1}{2}}$.

23. Make **k** the subject in this formula $p = \sqrt[3]{\frac{b(x-k)}{k}}$.

23. Make **x** the subject in this formula $k = \sqrt[4]{px^2 - d}$.

24. If $y = \frac{1}{2}mv^2$ and $k = \frac{x}{v}$, express **y** in terms of **m, x** and **k.**

25. If $V = \frac{1}{3}\pi r^2 h$ and $A = 4\pi r^2$, express **V** in terms of **A** and **h.**

EER:

1. Make **a** the subject in this formula $v = u + at$. Hence find **a** when **v = 12t** and **u = 5t.**

2. If $c = \frac{5(f-32)}{9}$, express **f** in terms of **c**

3. Make **P** the subject in this formula $I = \frac{PRT}{100}$. Hence find **P** when **I = 740, T= 2** and **R = 5 .**

4. Make **s** the subject in this formula $v = \sqrt{u^2 + 2as}$. Hence find **s** when **v = 9, u= 5** and **a = 3·5 .**

5. Make **d** the subject in this formula $T = 2\pi\sqrt{\frac{d}{g}}$.

6. If $\frac{p}{q} = \frac{x}{x+c}$, make **x** the subject of the formula

7. If $p = \frac{k(x - a)}{a}$*, make **a** the subject of the formula*

*8. Make **V** the subject in this formula* $R = \frac{Vr}{V - 2}$*. Hence find **V** when **R = 5r***

9. If $p = \frac{4q + r}{r}$*,*

*(i) find the value of **p** when **q = r.***

*(ii) make **r** the subject*

10. If $p = \sqrt{\frac{k(x - a)}{a}}$*, make **a** the subject of the formula*

*11. Make **k** the subject in this formula* $p = \sqrt[3]{\frac{k - 1}{k + 1}}$*.*

*12. Make **y** the subject in this formula* $a = \frac{x}{y^2} - m$*. Hence find the values of **y** when **x = 80,** a **= 2** and **m = 3.***

13. If $m = (Ax - B)^2$*, make **x** the subject of the formula. Hence find the values of **x** when **m = 16, A = 2** and **B = 6.***

*14. Make **r** the subject in this formula* $A = P\left(1 + \frac{r}{100}\right)^4$*, hence find **r** when*

A = 20,736** and **P = 10,000.

*15. Make **b** the subject in this formula* $h = \sqrt{\frac{ab^3}{a + b^3}}$*. Hence find **b** when **h = 6** and **a = –108 .***

16. If $k = \sqrt{\frac{x + 1}{x}}$*, make **x** the subject of the formula*

17. *If* $y = \frac{4x + 3}{x + 1}$, *express* ***x*** *in terms of* ***y***

18. *If* $p = (1 + x)^{n}$, *express* ***x*** *in terms of* ***p*** *and* ***n***

PROBABILITY THEORY

Summary:

1. *Probability theory deals with events of chances.*

2. *An event is a single result of an experiment*

3. *A set of all possible outcomes of an experiment is called a sample space or possibility space*

4. *Probability of an event* **=** $\frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$.

5. *Probabilities can be expressed as fractions, decimals or percentages*

6. (i) *The probability of an event lies between* ***0*** *and* ***1***

(ii) *The probability of an impossible event is* ***0***

(iii) *The probability of a sure event is* ***1***

7. (i) *The probability of event* **A** *occurring is written as* ***P(A).***

(ii) *The probability of event* **A** *not occurring is written as* $P(\bar{A})$ *or* $P(A')$.

(iii) *For any event* ***A,*** $P(A) + P(\bar{A}) = 1$ *or* $P(\bar{A}) = 1 - P(A)$

EXAMPLES:

1. *An integer between* ***1*** *and* ***10*** *inclusive is chosen at random. Find the probability that the chosen integer is:*

(i) *more than* ***4***

(ii) *an even number*

(iii) *a multiple of* ***3***

(iv) a triangular number

2. A box contains 3 red, 5 green and 12 blue pens. Find the probability that the pen picked from the box is:

(i) red

(ii) green

(iii) blue

(iv) not green

(v) not blue

(vi) either red or blue

3. A box contains red, green and blue pens. The probability of picking a blue pen from the box is $\frac{4}{9}$ and that of a green pen is $\frac{1}{3}$. Find the probability of picking a red pen

4. A box contains 8 red pens and the rest are blue. The probability of picking a blue pen from the box is $\frac{3}{5}$. Find the number of blue pens in the box

5. A fair coin is thrown once.

(a) Write down its sample space

(b) Find the probability of obtaining:

(i) a head

(ii) a tail

6. Two fair coins are tossed together.

(a) Write down its sample space

(b) Find the probability of obtaining:

(i) two heads

***(ii)** a head and a tail*

***7.** Three fair coins are tossed simultaneously.*

***(a)** Write down its sample space*

***(b)** Find the probability of obtaining:*

***(i)** exactly two heads*

***(ii)** at least two heads*

***8.** A fair dice is thrown once.*

***(a)** Write down its sample space*

***(b)** Find the probability of obtaining:*

***(i)** an even number*

***(ii)** a square number*

(iii)** either a **3** or a **4

***9.** Two fair dice are thrown together.*

***(a)** Draw a table for the possible outcomes*

***(b)** Find the probability that:*

***(i)** both show even numbers*

***(ii)** both show similar faces*

***(iii)** one shows an even number and the other odd*

(iv)** the sum of the scores is **9

***10.** Two fair dice are thrown simultaneously. Find the probability that the sum of the scores is*

***(i)** five*

***(ii)** less than five*

(iii) *greater than six*

11. *A fair coin and a fair die are thrown together. Find the probability of obtaining a tail and an even number*

12. *Two fair coins and a fair die are thrown together. Find the probability of obtaining:*

(i) *two heads and a number greater than* ***3***

(ii) *at least one tail and a number less than* ***4***

13. *A two digit number is formed using the digits* ***2, 3*** *and* ***4*** *without repeating any digit in the number formed.*

(a) *Write down the possibility space for the numbers formed*

(b) *Find the probability that the number formed is:*

(i) *even*

(ii) *less than* ***30***

14. *A two digit number is formed using the digits* ***0, 3*** *and* ***5*** *without repeating any digit in the number formed.*

(i) *Write down the possibility space for the numbers formed*

(ii) *Find the probability that the number formed is less than* ***53***

Soln:

Hint: 03 *and* ***05*** *are non–two digit numbers since* ***03*** *is the same as* ***3*** *and* ***05*** *is* ***5***

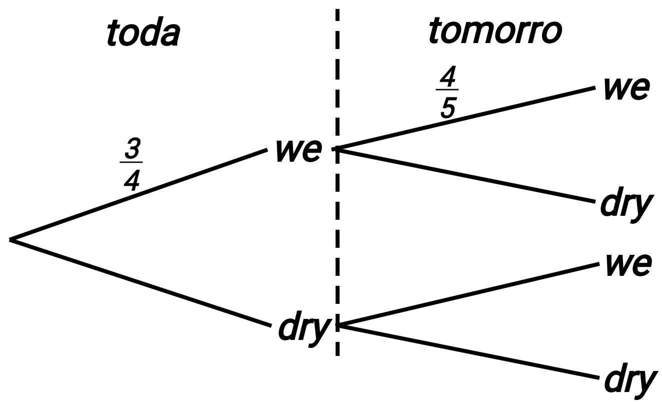
15. *A three digit number is formed using the digits* ***3, 4*** *and* ***5*** *without repeating any digit in the number formed.*

(i) *Write down the possibility space for the numbers formed*

(ii) *Find the probability that the number formed is even*

__16.__ The weather forecast says the probability that it will be wet today is $\frac{3}{4}$ *and that of tomorrow is* $\frac{4}{5}$.

(i) *Copy and complete the probability tree diagram below:*



(ii) *Find the probability that it will be wet on just one of the two days*

17. *A coin is biased so that the probability of tossing a head is* $\frac{2}{3}$. *If it is tossed three times,*

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of obtaining:*

(i) *exactly two heads*

(ii) *at least two tails*

(iii) *two tails on the first two tosses*

18. *A box contains **5** red and **3** blue pens. Two pens are drawn in succession at*

random from the box with replacement.

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of picking:*

(i) *pens of the same colour*

(ii) *exactly one red pen*

(iii) *at least one red pen*

(iv) *at most one blue pen*

19. *A box contains **8** red and **3** blue pens. Two pens are drawn in succession at*

random from the box without replacement.

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of picking:*

(i) *pens of different colours*

(ii) *exactly one red pen*

(iii) *at least one red pen*

(iv) *at most one blue pen*

20. *A bag contains **5** red, **7** blue and **8** green beads. Three beads are drawn in succession at random from the bag without replacement.*

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of picking:*

(i) *three green beads*

(ii) *one bead of each colour*

(iii) *beads of the same colour*

21. *A bag contains **30** white, **20** blue and **20** red balls. Three balls are drawn in*

succession at random without replacement. Find the probability that the first

ball and third ball is white.

EER:

1. *An integer between **10** and **30** inclusive is chosen at random. Find the probability that the chosen integer is:*

(i) *prime*

(ii) *divisible by **2, 3** or **5***

(iii) *a triangular number*

(iv) *a factor of **240***

2. *A letter is chosen from the word **"MISCELLANEOUS"**. Find the probability that it will be:*

(i) *an **S***

(ii) *a vowel*

(iii) *a consonant*

3. *In a game, a player tosses a fair coin once. When a **1** or **6** appears the player wins and when a **3** or **4** appears the player losses. Find the probability that the player neither wins nor losses.*

4. *A two digit number is formed using the digits **1, 2, 4, 5** and **7** without repeating any digit in the number formed.*

(a) *Write down the possibility space for the numbers formed*

(b) *Find the probability that the number formed is:*

(i) *divisible by **5***   ***(ii)*** *divisible by **2** or **5***   ***(iii)*** *greater than **50***

5. *Two fair dice are thrown together. Find the probability that the product of the scores is* *(i)* twelve

(ii) four

6. A fair coin is tossed three times.

(a) Draw a tree diagram showing the possible outcomes

(b) Find the probability of obtaining:

(i) three heads

(ii) exactly two heads

(iii) at least two tails

7. Two fair dice are thrown together. The score is the positive difference of the two numbers on which the dice land.

(a) Construct a table showing the possible outcomes

(b) Find the probability that the positive difference of the scores is

(i) two *(ii)* five

8. A box contains **15** red pens and the rest are blue. The probability of picking a red pen from the box is $\frac{3}{5}$. Find the:

(i) probability of picking a blue pen

(ii) number of pens in the box

9. A bag contains **3** white and **5** black balls. If two balls are picked in succession

at random from it, find the probability that the second ball picked is white if

picking is done:

(i) with replacement.

(ii) *without replacement.*

10. *Three balls are drawn at random one after the other without replacement from*

*a bag containing **4** white**, 8** blue **, 5** red and **3** pink balls. Find the probability*

*that the first ball is blue**,** the second red or blue and the third is white.*

11. *A box contains two types of balls **red** and **black**. When a ball is picked*

from the box, the probability that it is red is $\frac{7}{12}$. Two balls are selected at

*random from the box without replacement. Find the probability that**:***

the second ball is black

12. *A box contains **3** red, **4** green, and **5** blue beads. Two beads are selected at*

*random from the box without replacement. Find the probability of picking**:***

(i) *beads of the same colour*

(ii) *at least one red bead*

(iii) *beads of different colours*

13. *A fair die and a fair octahedral die are thrown together and the outcome on each die is recorded**.***

(a) *Draw a table showing the possible outcomes*

(b) *Find the probability that**:***

(i) *both show odd numbers*

(ii) *both show similar faces*

(iii) *one shows an even number and the other odd*

(iv) *the sum of the scores is greater than or equal to **5***

*(v) the product of the scores is **4** or **12***

***14.** A bag contains **x** red balls and **(x – 8)** blue ones. The probability of picking a red ball from the box is **0·75.** Find the:*

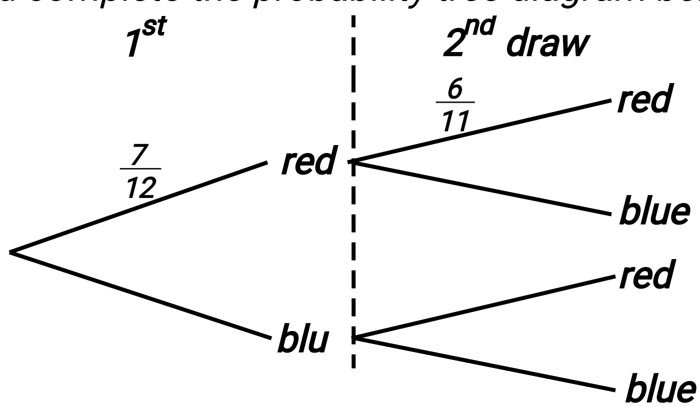
(i) probability of picking a blue ball

(ii) number of balls in the bag

***15.** A box contains **7** red and **5** blue pens. Two pens are drawn in succession at*

random from the box without replacement.

(i) Copy and complete the probability tree diagram below:



(ii) Find the probability of picking at least one red pen

***16.** A fair die and a fair tetrahedral die are thrown together and the outcome on each die is recorded.*

(a) Draw a table showing the possible outcomes

(b) Find the probability that:

(i) both show odd numbers

*(ii) the sum of the scores is greater than or equal to **5***

*(iii) the product of the scores is **4** or **12***

***17.** Two fair dice are thrown together.*

(a) *Draw a table for the possible outcomes*

(b) *Find the probability that:*

(i) *both show even numbers*

(ii) *both show similar faces*

(iii) *one shows an even number and the other odd*

(iv) *the sum of the scores is* ***9***

18. *Two fair tetrahedral dice each with faces numbered* ***1*** *to* ***4*** *are thrown together. The score is the sum of the two numbers on which the dice land.*

(a) *Construct a table showing the possible outcomes*

(b) *Find the probability that the sum of the scores is*

(i) *seven*

(ii) *less than six*

19. *Two fair dice are thrown together. Find the probability that the positive difference of the scores is*

(i) *two*

(ii) *five*

20. *A box contains* ***5*** *red and* ***3*** *blue balls. Two balls are drawn in succession at*

random from it without replacement. Find the probability that:

(i) *they are of the same colour*

(ii) *the second ball drawn is red.*

21. *A box contains* ***5*** *red and* ***3*** *blue balls. Three balls are selected in succession at*

random from it without replacement. Find the probability that:

(i) *they are of the same colour*

(ii) *the first and last are of the same colour*

(iii) *at most one blue ball is drawn.*

22*. Two fair tetrahedral dice are thrown up at once. Find the probability that the sum of the scores on the dice is less than* ***6.***

DEPARTMENT OF MATHEMATICS

S2 HOLIDAY WORK

SET 2

RELATIONS AND MAPPINGS

1. a.) Draw arrow diagrams for the following ordered pairs.
   (i.) {(5,5), (6,6), (7,7), (8,8)}
   (ii.) {(1,-2), (2,-2), (3,-3), (4,-5), (5,-5)}
   (iii.) {(2,1), (4,3), (6,5), (8,7)}
   b.) Write the domain and range in each of the following;
   c.) Which of the relations above represent functions and which ones do not?

2. A relation is defined as $f(x) = \sqrt{x}$
   a.) Write the range of the relation given the domain {1,4,9,16,25}
   b.) Draw a graph for the relation.
   c.) Is the relation a function? Justify your answer.

3. Use a papygram on the set M={1,2,3,4,5,6,7,8,9,10} to illustrate the relation "is twice as".

4. A relation "is a third of" is from set A={1,4,6} to the set B={2,4,5,3,12}, use an arrow diagram to illustrate this relation.

5. Illustrate the relation "is a factor of" on the sets M={2,4,5,6} and N={4,8,10,15}

6. a.) A relation "is less than" acts on the set A={1,5,6,4,2}, use a papygram to illustrate this relation.
   b.) Find the domain of this relation.
   c.) Find the range of the relation.

7. A relation is defined by $\{(x, y): x \in A$ and $y \in B$, where $y = 3x\}$
   A={1,2,3,4} and B={3,4,5,6,9}, find the;
   a.) Domain
   b.) Range

8. By using the mapping f such that $x \to 4x - 1$ on domain {4,2,3}, find the range of f

9. If $f(x) = \frac{1}{3}x$, find; $f(-1), f(2), f(3), f(-2)$

10. The relation f is defined by $x \to x^2 + 3$. By using the domain D={-2,-1,0,1,2,3}
    a.) Find the range of f on D
    b.) Show your answer on an arrow diagram.
    c.) What type of mapping is this?

11. The range corresponding to the mapping $x \to x + 2$ is {2, 3, 5, 6}. Find the domain

12. Given the functions; $f(x) = \frac{1-2x}{x+1}$, $g(x) = \frac{x+3}{2}$ and $h(x) = \frac{5x}{4} - 1$
    a.) Find the value of;
       (i.) $f(-2)$
       (ii.) $g(-1)$
    b.) Find the value of x for which;
       (i.) $f(0) = g(x)$
       (ii.) $g(x) = 0$
       (iii.) $g(x) = h(x)$

13. a.) Given that set P={2,3,4,6,12}, draw a papygram on set P showing the relation "is a multiple of"
    b.)Given the functions $f(x) = \frac{2x+1}{3}$ and $g(x) = \frac{x-1}{2}$, find;
    (i.) $f(1)$
    (ii.) $g(1)$
    (iii.) the value of x for which $f(x) + g(x) = 3\frac{1}{6}$

END

SET THEORY

Summary:

1. *A set is a collection of well defined objects.*

2. *An empty set is a set with no elements. It is denoted by curly brackets with nothing inside { }* ***or*** *ϕ.*

3. *A subset is a set that is part of a larger set*

4. *A universal set is a set of all other elements under consideration*

5. *A Venn diagram is a set diagram that shows relations between different sets.*

6. *For any two sets* ***A*** *and* ***B:***

(i) n(ε) *is read as number of members in the universal set. This means number of members in all the regions of the Venn diagram*

(ii) n(A) *is read as number of members in set* ***A****.*

(iii) $n(\overline{A})$ or $n(A')$ *is read as number of members in set* ***A*** *complement. This means number of members of the universal set that are not in set* ***A.***

(iv) $A \cup B$ *is read as* ***A*** *union* ***B.*** *This means members of either set* ***A*** *or* ***B*** ***(****the entire region covering the two sets****)****.*

(v) $A \cap B$ *is read as* ***A*** *intersection* ***B.*** *This means the region common to the two sets.*

(vi) $A \cap \overline{B}$ *is read as* ***A*** *intersection* ***B*** *complement. This means members of* ***A*** *only.*

(vii) $\overline{A} \cap B$ *is read as* ***A*** *complement intersection* ***B.*** *This means members of* ***B*** *only.*

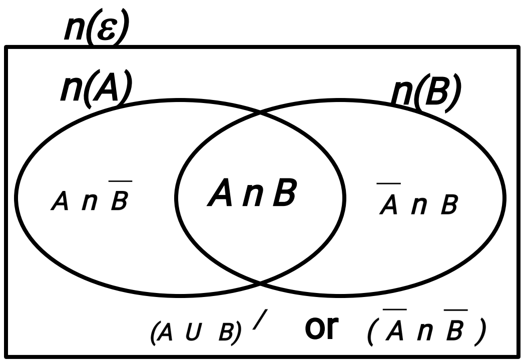
(ix) $\overline{A} \cap \overline{B}$ *is read as* ***A*** *complement intersection* ***B*** *complement. This means neither* ***A*** *nor* ***B.*** $\overline{A} \cap \overline{B}$ *is the same as* $(A \cup B)'$

(x) $A \cup \overline{B}$ *is read as* ***A*** *union* ***B*** *complement. This means of members of the universal set that are not in set* ***B*** *only.*

(xi) $\overline{A} \cup B$ *is read as* ***A*** *complement union* ***B****. This means of members of the universal set that are not in set* ***A*** *only.*

(xii) $\overline{A} \cup \overline{B}$ *is read as* ***A*** *complement union* ***B*** *complement. This means of members of the universal set that are not in the intersection.*

(xiii) The Venn diagrams illustrating the regions relating any two sets ***A*** *and* ***B*** *is as follows:*



EXAMPLES:

1. *Given the sets* $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 3, 4, 5, 8\}$ *and*

$B = \{1, 3, 5, 7, 8, 9\}$, *where* ε *is the universal set, find:*

(i) $A \cap B$ *(ii)* $A \cup B$ *(iii)* $A \cap \overline{B}$ *(iv)* $\overline{A} \cap B$ *(v)* $\overline{A} \cap \overline{B}$

(vi) $\overline{A}$ *(vii)* $\overline{B}$ *(viii)* $A \cup \overline{B}$ *(ix)* $\overline{A} \cup B$ *(x)* $\overline{A} \cup \overline{B}$

2. *Given the sets* $P = \{$ *All factors of* $90\}$ *and* $Q = \{$ *All factors of* $60\}$, *find:*

(i) $n(P \cap Q)$ *(ii)* $n(\overline{P} \cap Q)$ *(iii)* $n(P \cup Q)$

***3.** Given the sets **P = {** All triangular numbers less than **40}** and*

***Q = {** All factors of **60}**, find:*

(i) n(P n Q) ***(ii) n(P U Q)*** ***(iii)*** $n(P \cap \overline{Q})$ ***(iv)*** $n(\overline{P} \cap Q)$

***4.** If ε **= { x: 0 < x < 13},** **A = { x: 1 < x < 9}** and **B = { x: 4 < x < 11},** where **x** is an integer and ε is the universal set, find:*

(i) n(A n B) ***(ii) n(A U B)*** ***(iii)*** $n(A \cap \overline{B})$ ***(iii)*** $n(A \cup B)'$

***5.** If ε **= { x: 1 < x < 14},** **A = { x:** $2 \leq x \leq 9$ **}** and **B = { x:** $6 \leq x \leq 11$ **},** where **x** is an integer and ε is the universal set, find:*

(i) n(A n B) ***(ii) n(A U B)*** ***(iii)*** $n(A \cap \overline{B})$ ***(iii)*** $n(A \cup B)'$

6.** Given the sets ε **= {6, 7, 8, 9, 15, 16, 17, 18, 20, 21},

***M = { x: x** are multiples of **3}** and **N = { x: x** are odd numbers**},** where ε is the universal set, find:*

(i) n(M n N) ***(ii) n(M U N)*** ***(iii)*** $n(M \cap \overline{N})$

(iv) $n(\overline{M} \cap \overline{N})$ ***(v)*** $n(M \cup \overline{N})$ ***(vi)*** $n(\overline{M})$

***Hint:** The elements of **M** and N have to be chosen from the universal set*

***7.** Sets **A** and **B** are such that **n(ε) = 30, n(A) = 18, n(B) = 14** and* $n(A \cup B)' = 5$. *find:*

(i) n(A n B) ***(ii) n(A U B)*** ***(iii)*** $n(A \cap \overline{B})$ ***(iv)*** $n(A \cup \overline{B})$

8. *Sets* ***P*** *and* ***Q*** *are such that* ***n(P) = 12, n(Q) = 8, n(P U Q) = 15*** *and* $n(P \cup Q)' = 2$. *find:*

(i) n(P n Q)

(ii) n(ε), *where* ***ε*** *is the universal set*

9. *Sets* ***A*** *and* ***B*** *are such that* ***n(ε) = 40, n(A) = 25,*** $n(\overline{A} \cap B) = 10$ *and* $n(A \cap B) = n(A \cup B)'$, *where* ***ε*** *is the universal set. Use a Venn diagram to find:*

(i) n(A n B) ***(ii) n(A U B)***

10. *Sets* ***M*** *and* ***N*** *are such that* ***n(ε) = 19, n(M) = 8*** *and* $n(\overline{N}) = n(\overline{M} \cap N) = 7$, *where* ***ε*** *is the universal set. Use a Venn diagram to find:*

(i) n(M n N) ***(ii)*** $n(M \cup \overline{N})$

11. *In a class of* ***53*** *students,* ***36*** *drink tea,* ***18*** *drink coffee while* ***10*** *drink neither tea nor coffee. Find how many students drink both tea and coffee*

12. *In a class of* ***29*** *boys,* ***22*** *liked rice and* ***18*** *liked matooke. All the boys liked at least one of the foods. Find how many liked both.*

13. *In a class of* ***20*** *girls,* ***5*** *play golf but not netball,* ***9*** *play netball but not golf, while* ***3*** *play neither game. Find how many play:*

(i) *both games*

(ii) *either game*

(iii) *only one game*

14. *In a class of* ***80*** *boys,* ***70*** *play Tennis,* ***30*** *play golf, while all those that play golf also play Tennis.*

(i) *Represent the given information on a Venn diagram*

(ii) *Find how many play neither game*

15. *The number of people who play football* ***(F)*** *or basketball* ***(B)*** *is twice the number of those who play both* ***F*** *and* ***B****. If* ***n(F) = 9*** *and* ***n(B) = 6,*** *find how many play both games.*

EER:

1. *Sets* ***A*** *and* ***B*** *are such that* ***n(ε) = 28, n(A) = 10, n(B) = 17*** *and* ***n(A U B) = 22,*** *where ε is the universal set. Use a Venn diagram to find:*

(i) ***n(A n B)*** ***(ii)*** $n(A \cap \overline{B})$ ***(iii)*** $n(\overline{A} \cap B)$ ***(iv)*** $n(\overline{A} \cap \overline{B})$

(v) $n(A \cap B)'$ ***(vi)*** $n(A \cup \overline{B})$ ***(vii)*** $n(\overline{A})$

2. *Given the sets* ***M =*** *{ All multiples of* ***6*** *less than* ***72****} and*

N = *{ All multiples of* ***4*** *less than* ***50****}* ***,*** *find:*

(i) n(M n N) ***(ii) n(M U N)*** ***(iii)*** $n(M \cap \overline{N})$ ***(iv)*** $n(\overline{M} \cap N)$

3. *Sets* ***A*** *and* ***B*** *are such that* ***n(ε) = 23, n(A n B) = 8, n(B) = 14*** *and* $n(\overline{A})$ ***= 10,*** *where ε is the universal set. Use a Venn diagram to find:*

(i)* *n(A) ***(ii)*** $n(\overline{A} \cap \overline{B})$ ***(iii)*** $n(\overline{A} \cap B)$ ***(iv)*** $n(A \cup \overline{B})$

4. *In a class of **30** students, **18** play volley ball, **14** play hockey while **5** play neither. Find how many students play:*

(i) *both games*

(ii) *only one game*

5. *Given the sets **M =** { The first **10** rectangle numbers} and*

N = *{ The first **5** square numbers } , find:*

(i) n(M n N) ***(ii) n(M U N)*** ***(iii)*** $n(M \cap \overline{N})$ ***(iv)*** $n(\overline{M} \cap N)$

6. *In a class of **28** boys, **13** passed history and **25** passed physics. All the boys passed at least one subject. Find how many passed both subjects.*

7. *Sets **A** and **B** are such that **n(ε) = 35, n(A n B) = 8, n(A) = 17** and*

$n(A \cup B)' = 5$, *where **ε** is the universal set. Use a Venn diagram to find:*

(i) $n(\overline{A} \cap B)$ ***(ii)*** $n(\overline{A})$

8. *Given the sets **M =** { All multiples of **3** less than **20**} and*

N = *{ All odd numbers less than **20**} , find **n(M n N)***

9. *In a class of **50** boys, **23** passed history, **35** passed physics and **2** passed neither subject. Find how many passed:*

(i) *both subjects.*

(ii) *only one subject.*

10. *Given the sets* ***M = {*** *All integers greater than* ***4*** *but less than* ***10}*** *and*

N = { *All multiples of* ***3*** *between* ***1*** *and* ***20},*** *find:*

(i) n(M n N) ***(ii) n(M U N)*** ***(iii)*** $n(M \ n\ \overline{N}\ )$

11. *Given the sets* ***T = {*** *All triangle numbers less than* ***20}*** *and*

F = { *All factors of* ***12}*** ***,*** *find the members of* ***T n F.*** *Hence find* ***n(T n F)***

12. *Given the sets* ***P = {*** *All factors of* ***24}*** *and* ***Q = {*** *All factors of* ***30},*** *find*

$n(\ \overline{P}\ n\ Q\ )$

13. *Two sets* ***A*** *and* ***B*** *are such that* ***n (A) = 12, n (B) = 13, n (AUB) = 20*** *and*

n(ε) = 24, *find:*

(i) n (A∩B') ***(ii) n (A'∩B')*** ***(iii)n (AUB')***

14. *If* ***n (ε) = 60, n (A') = 32, n (A∩B) = 10*** *and* ***n (AUB)' = 17,*** *find:*

(i) n (B) ***(ii) n (A∩B')*** ***(iii)n (AUB)***

SENIOR THREE

*1. Sets **A, B** and **C** are such that **n(ε) = 100, n(A) = 46, n(B) = 40, n(C) = 49, n(A n B) = 14, n(A n C) = 17, n(B n C) = 15** and $n(\overline{A} \cap \overline{B} \cap \overline{C}) = 6$. Use a Venn diagram to find:*

(i) n(A n B n C)

(ii) n(A U B U C)

(iii) $n(\overline{A} \cap B \cap C)$

(iv) $n(A \cap \overline{B} \cap C)$

(v) $n(A \cap B \cap \overline{C})$

*2. In a class of **53** students, **30** study Art, **20** study French and **15** study Computer. **6** study both Art and French, **4** study Art and Computer, **5** study French and Computer. Each student studies at least one of the three subjects*

***(a)**Represent the information on Venn diagram*

***(b)**Find the number of the students who study:*

***(i)** all the three subjects*

***(ii)** at least two subjects.*

***(c)** Find the probability that a student chosen at random studies:*

***(i)** only one subject*

***(ii)** French only*

***(iii)** French but not Computer*

3. *In a class of* ***30*** *students****,*** ***18*** *play Tennis****,*** ***15*** *play Golf and* ***13*** *play Hockey. The number of students who play all the three games are equal to those who play neither game.* ***10*** *play both Tennis and Hockey****,*** ***8*** *play Tennis and Golf****,*** ***3*** *play only Golf and Hockey.*

(a)*Represent the information on Venn diagram*

(b) *Find the number of the students who play:*

(i) *all the three games*

(ii) *at most one game*

(c) *Find the probability that a student chosen at random plays at least two games*

3. *In a class of* ***56*** *students****,*** ***28*** *play Tennis****,*** ***24*** *play Chess and* ***32*** *play Hockey.* ***10*** *play both Tennis and Chess****,*** ***6*** *play both Chess and Hockey****,*** ***4*** *play all the three games.*

(a)*Represent the information on a Venn diagram*

(b) *Find the number of the students who play both Tennis and Hockey only*

(c) *Find the probability that a student chosen at random plays:*

(i) *at least two games*

(ii) *only one game*

4. *In a class of* ***100*** *students****,*** ***15*** *take Art only****,*** ***12*** *take French only and* ***8*** *take Computer only.* ***10*** *take both Art and French****,*** ***40*** *take Art and Computer****,*** ***20*** *take French and Computer****,*** ***65*** *take Computer.*

***(a)**Represent the information on Venn diagram*

***(b)** Find the number of the students who take:*

***(i)** all the three subjects*

***(ii)** Art*

***(iii)** French*

***(c)** Find the probability that a student chosen at random takes neither subject*

***5**. In a class of **52** students, an equal number of students visited Arua and Kasese. **24** visited Mbale**, 11** visited both Mbale and Arua**, 12** visited Arua and Kasese**, 13** visited Mbale and Kasese**. 8** visited all the three towns and **4** visited neither town.*

***(a)**Represent the information on Venn diagram*

***(b)**Find the number of the students who:*

***(i)** visited Kasese*

***(ii)** did not visit Arua*

***(c)** Find the probability that a student chosen at random visited at least two towns*

3.** In a class of **72** students, each student must at least take of the subjects Art **(A),** Computer **(C)** and French **(F).** None of the students takes **F** and **C, 26** take **F** only. Of the **35** students taking **A, 20** take the subject alone. The number of students taking **F** and **A** is three more than those taking **A** and **C.

***(a)**Represent the information on Venn diagram*

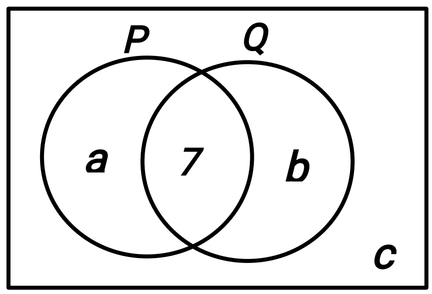
***(b)**Use the Venn diagram to find the number of the students who take;*

(i)Art and Computer

(ii) Computer.

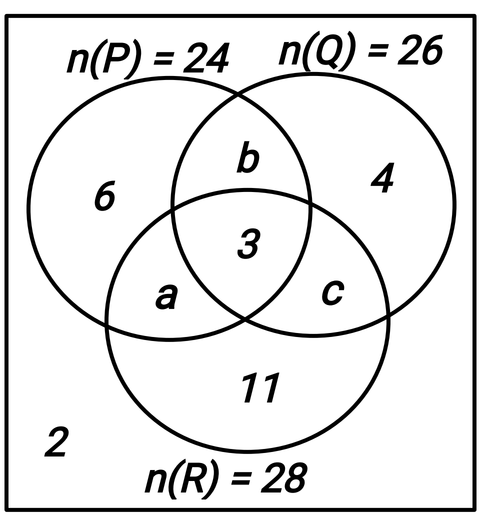
(c)What is the probability that a student chosen at random takes only one subject.

15.** In the Venn diagram below, sets **P** and **Q** are such that **n(PUQ) = 16, n(P') = 7,** and **n(Q') = 6.



*Find the values of **a, b** and **c.** Hence obtain **n(ε)***

***5.** Study the Venn diagram below:*



Find:

*(i) the values of **a, b** and **c***

*(ii) **n(ε),** where ε is the universal set*

*7. Sets **A, B** and **C** are such that **n(A) = 23, n(B) = 24, n(C) = 25, n(A n B n C) = 5,*** $n(A\ n\ \overline{B}\ n\ \overline{C}) = 8$, $n(\overline{A}\ n\ B\ n\ \overline{C}) = 12$, $n(\overline{A}\ n\ \overline{B}\ n\ C) = 9$ *and* $n(\overline{A}\ n\ \overline{B}\ n\ \overline{C}) = 2$. *Use a Venn diagram to find:*

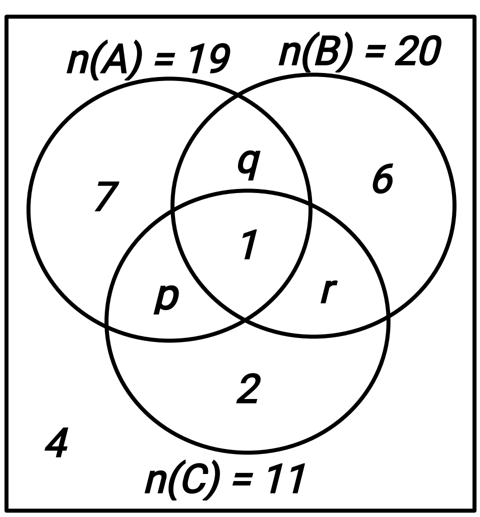
(i) $n(\overline{A}\ n\ B\ n\ C)$

(ii) $n(A\ n\ \overline{B}\ n\ C)$

(iii) $n(A\ n\ B\ n\ \overline{C})$

***(iv) n(ε) ,** where ε is the universal set.*

***6.** Study the Venn diagram below:*



Find:

(i) *the values of* ***p, q*** *and* ***r***

(ii) n(ε), *where* ***ε*** *is the universal set*

14. *In a class of* ***42*** *students,* ***15*** *like Chemistry* ***(C), 19*** *like Physics* ***(P),*** *and* ***28*** *like Mathematics* ***(M). 6*** *students like both Physics and Chemistry,* ***10*** *students like both Mathematics and Chemistry and* ***8*** *like Physics and Mathematics but not Chemistry. Given that the number of students who like all the three subjects is equal to those who do not like any of the subjects.*

(a) *Represent the above information on a Venn diagram.*

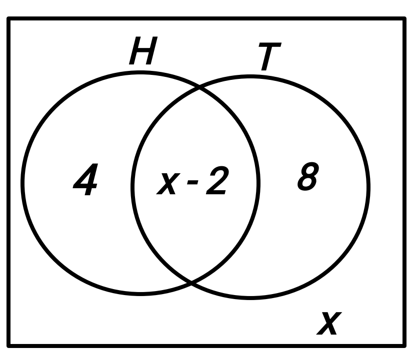
(b) *Find the number of students who like:*

(i) *all the three subjects.*

(ii) *at least two of the subjects.*

(c) *Find the probability that a student selected at random likes other subjects.*

6. *The Venn diagram below shows a group of students playing Hockey* ***(H)*** *or Tennis* ***(T)***

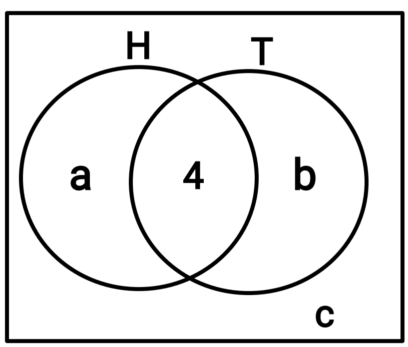


*If the probability that a student picked at random from the group plays both or none of the games is **0·4,** find the:*

(i) *value of **x.***

(ii) *number of students playing both of the games.*

6. *In the Venn diagram below**, 20** students play either Hockey **(H)** or Tennis **(T)***



*If **14** and **12** students do not play Hockey and Tennis respectively**,** find the**:***

(i) *values of **a, b** and **c.***

(ii) *probability that a student picked at random plays neither of the games*

5. *In a sports club **19** members play Hockey **(H), 18** play Rugby **(R), 17** play*
*Tennis **(T)** and **5** play neither of the games. **10** play both **H** and **R, 6** play*

both
***H** and **T, 7** play both **R** and **T, 20** play only one game.*
***(a)** Represent the above information on a Venn diagram.*
***(b)** Find the number of members:*
***(i)** who play all the three games.*
***(ii)** in the club.*
***(iii)** who play at most one game.*
***(c)** Find the probability that a member picked at random plays at least two games*

14. In a class of **42** students**, 15** like Chemistry **(C), 19** like Physics **(P),**

and **28** like Mathematics **(M). 6** students like both Physics and

Chemistry**, 10** students like both Mathematics and Chemistry and

8 like Physics and Mathematics but not Chemistry. Given that the

number of students who like all the three subjects is equal to those

who do not like any of the subjects.

(a) Represent the above information on a venn diagram.

(b) Find the number of students :

(i) who like all the three subjects.

(ii) atleast two of the subjects.

(c) Find the probability that a student selected at random from the class

likes other subjects.

16. *In a class of **40** students, **18** play Hockey **(H), 15** play Tennis **(T)** and **22** play football **(F). 7** play Hockey and Tennis**, 9** play Tennis and Football**, 8** play Hockey and Football and four students play all the three games.*

(a) *Represent the given information on a venn diagram*

(b) *Find the number of students who:*

(i) *did no play any of the games*

(ii) *played exactly two of the games*

(c) *Determine the probability that a student selected at random plays only one game.*

1. A group of tourists was asked which countries they visited in East Africa. It was found out that 90 tourists visited Tanzania, The number of tourists who visited Kenya was more than those who visited Tanzania by four, and the number of those who visited Uganda was less than those who visited Kenya by ten. Twenty four tourists visited only Uganda, forty six visited only Kenya and those who visited only Tanzania were ten more than those who visited only Uganda. It was found out that all tourists visited at least each of the countries while fifty eight visited only two countries and seventy four visited at least two countries.
   a) Represent the above information clearly on a venn diagram (03 marks)
   b) Using the venn diagram, find the number of tourists who
      (i). visited all the three countries
      (ii). were in the group (07 marks)

If a tourist was selected from the group at random, find the probability that the tourist selected visited Uganda and Tanzania.

MATRICES

Summary:

1. A matrix is a bracket with numbers in rows and columns. Thus $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 0 \\ -2 & 3 & 5 \end{pmatrix}$ are matrices.

2. The order of a matrix with **m** rows and **n** columns is written as $m \times n$ and is called an $m \times n$ matrix.

3. The numbers in a matrix are called its elements or entries.

4. (i) To add and subtract matrices of the same order, add and subtract corresponding elements

(ii) Two matrices are equal if their corresponding elements are equal

(iii) A scalar **k** multiplied by a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is treated as follows:

$$kA = k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

(iv) Matrix multiplication is treated as follows:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$$

Matrix product $AB \neq BA$.

Matrix product **AB** can be done if the number of columns in **A** is equal to the number of row in **B.**
If a **2 × 5** matrix is multiplied by a **5 × 3** matrix, then the resulting matrix has the outer dimensions **(**The new matrix is of order **2× 3)**
5. An identity matrix **I** is a matrix with ones along the major diagonal and zeros elsewhere. Thus a **2× 2** identity matrix is given by $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

6. *If matrix* $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,, *then:*

(i)*Determinant of* **A (Det A)** $= ad - cb$

(ii) *Adjoint of matrix* **A** = $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(iii) *The inverse of* **A,** $(A^{-1}) = \frac{1}{detA}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(iv) *The product* $AA^{-1} = I$

(v) *A matrix multiplied by an identity matrix remains unchanged*

7. *A singular matrix is the one whose determinant is zero and thus has no inverse.*

EXAMPLES:

1. *If matrix* $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$,

(a) *State the order of matrix* **A**

(b) *Determine the:*

(i) *determinant of* **A**

(ii) *inverse of* **A**

2. *Given that matrix* $P = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix}$ *and* $R = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, *find:*

(i) P + Q   (ii) Q – R   (iii) 3P – 2Q + R   (iv) PQ   (v) QP   (vi) QRP

(vii) P^2 ***(viii)*** Q^2 ***(ix)***$(P+Q)^2$ ***(x) 3P – 2I*** *where* ***I*** *is a* **2 × 2** *identity matrix*

3. *Given that matrix* $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ *and* $B = \begin{pmatrix} 4 & 8 \\ 2 & 6 \end{pmatrix}$, *find* ***det (AB)***

4. *Given that matrix* $P = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$, $Q = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ *and* $R = P^2Q$, *find* R^{-1}

5. *Find the order of the resulting matrix when a* $\mathbf{3 \times 4}$ *matrix is multiplied by a* $\mathbf{4 \times 5}$ *matrix*

6. *If matrix* $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}$ *and* $B = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$,

(i) *determine the order of matrix* ***AB***

(ii) *find matrix* ***AB***

7. *If matrix* $P = \begin{pmatrix} 3 & 1 & 7 \\ -1 & 3 & 2 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 2 \end{pmatrix}$ *and* $R = PQ$,

(i) *determine the order of matrix* ***R***

(ii) *find matrix* ***R***

8. *Given the matrix equation* ***AY = B,*** *use matrix inversion method to find:*

(i) *matrix* ***Y*** ***(ii)*** *matrix* ***A***

9. *If matrix* $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, *find matrix* ***B*** *such that* $AB = \begin{pmatrix} 17 & 13 \\ 21 & 14 \end{pmatrix}$

10. *If matrix* $B = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$, *find matrix* ***A*** *such that* $AB = \begin{pmatrix} 10 & 4 \\ -5 & 9 \end{pmatrix}$

11. *If matrix* $P = \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}$, $Q = \begin{pmatrix} 11 \\ 7 \end{pmatrix}$ *and* $PR = Q$, *determine:*

(i) the order of matrix ***R***

(ii) matrix ***R***

12. *If matrix* $P = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$, *find matrix* ***A*** *such that* $AP = I$, *where* ***I*** *is a* $\mathbf{2 \times 2}$ *identity matrix.*

13. *If matrix* $A = \begin{pmatrix} x & -7 \\ 4 & 6y \end{pmatrix}$ *and* $B = \begin{pmatrix} 17 - y & -21 \\ 12 & 36 \end{pmatrix}$, *find the values* ***x*** *and* ***y*** *such that* ***3A = B***

14. *Find the values of* ***a*** *and* ***b*** *such that* $\begin{pmatrix} 3 & b \\ 4 & a \end{pmatrix}\begin{pmatrix} 7a \\ 2 \end{pmatrix} = \begin{pmatrix} 43 \\ 30 \end{pmatrix}$

15. *Find the values of* ***k*** *and* ***n*** *such that* $\begin{pmatrix} 2 & 4 \\ -3 & 3 \end{pmatrix} + k\begin{pmatrix} 3 & 1 \\ 0 & n \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -3 & -1 \end{pmatrix}$

16. *Find the values of* ***x*** *and* ***y*** *such that* $(1 \quad 3 \quad 2)\begin{pmatrix} 4 & 3 \\ x & 2 \\ 10 & y \end{pmatrix} = (39 \quad 25)$

17. *Find the values of* ***x*** *and* ***y*** *such that* $\begin{pmatrix} 4 & 1 \\ x & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

18. *Given that matrix* $A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, *find the value of* λ *such that* $A^2 + \lambda I = 5A$, *where* ***I*** *is a* $\mathbf{2 \times 2}$ *identity matrix*

19. *Given that matrix* $A = \begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix}$ *and* $B = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix}$, *find the possible values of* ***x*** *such that* ***AB = BA***

20. *Find the values of* ***x*** *for which the matrix* $\begin{pmatrix} x & 6 \\ 8 & 3x \end{pmatrix}$ *has no inverse*

21. *Find the values of* ***x*** *for which the matrix* $\begin{pmatrix} x & 3 \\ 4 & x-4 \end{pmatrix}$ *is singular*

22. *Find the values of* ***x*** *for which the matrix* $\begin{pmatrix} 2x & 3x \\ 2 & x \end{pmatrix}$ *is singular*

23. *Given that matrix* $M = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, *find the values of* $\boldsymbol{\lambda}$ *such that the matrix* $(M - \lambda I)$ *is singular, where* ***I*** *is a* $\mathbf{2 \times 2}$ *identity matrix*

4. *Given that* $M = \begin{pmatrix} -1 & 2 \\ 2 & 4 \end{pmatrix}$, *show that* $\det(M^{-1}) = \dfrac{1}{\det M}$

7. *Given that m* $A = \begin{pmatrix} 2 & -1 \\ 0 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 6 & 8 \\ 10 & -12 \end{pmatrix}$, *find* $(AB)^{-1}$ *and* $B^{-1}A^{-1}$

1. *If* $A = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$ *and* I *is a* 2×2 *identity matrix, prove that* $A^2 = 7A + 2I$

2.

6. *Given the matrices* $A = \begin{pmatrix} 4 \cdot 5 & 1 \\ 0 & 7 \end{pmatrix}$ *and* $B = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, *find matrix* ***M*** *such*

that

$3M - 2I = 2A - B$, where ***I*** is a $\mathbf{2 \times 2}$ identity matrix

SOLUTION TO SIMULTANEOUS EQUATIONS BY MATRIX METHOD

Summary:

The following steps apply in solving simultaneous equation using matrix method:

(i) *Write the equations in matrix form*

(ii) *Find the inverse of the* $\mathbf{2 \times 2}$ *matrix*

(iii) *Pre multiply both sides of the matrix equation by the inverse matrix*

EXAMPLES:

1. *Use matrix method to solve the following simultaneous equations:*

(i) $x - y = 5$
$3x + 2y = 5$

(ii) $2x - 5y + 14 = 0$
$4x + 3y - 11 = 0$

(iii) $4x + 3y = 24$
$2y - 3y = -1$

(iv) $x + y = 15$
$$\frac{x}{3} + \frac{y}{9} = 3$$

2. *Find the inverse of* $\begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$, *hence solve the simultaneous equations*

$3x + 2y = 12$
$4x + 5y = 23$

3. *Tom bought* ***3*** *pens and* ***2*** *books at* ***Shs 4,800.*** *Bob bought* ***5*** *pens and* ***4*** *books from the same shop at* ***Shs 9,000.***

(i) *Form two equations to represent the above information*

***(ii)** Use matrix method to find the cost of each pen and that of each book*

***(iii)** How much would Ben pay for **10** pens and **6** books*

***4.** Shs **4000** can buy **10** bans and **5**cakes or **4**bans and **10**cakes.*

***(i)** Form two equations to represent the above information*

***(ii)** Find by matrix method the cost of each ban and that of each cake.*

MATRIX WORD PROBLEMS

***1.** Tom, Bob and Ben went to a supermarket for shopping .*
*Tom bought **3** pens and **5** books and **4** rulers*
*Bob bought **4** pens and **3** books and **2** rulers*
*Ben bought **6** pens and **3** rulers*
*The cost of a pen is **Shs 500,** a book is **Shs 800** and a ruler is **Shs 1500.***

***(a)** Write down:*

***(i)** a 3×3 matrix for the items bought by the three boys.*

***(ii)** a 3×1 cost matrix for each item*

***(b)**Use matrix multiplication to find the amount of money spent by each boy*

***2.** In a swimming competition, **7** points were awarded for each first-place finish, **4** points for second and **2** points for third.*
*Senior one had **4** first place finishes, **7** second place finishes and **3** third place finishes.*

*Senior two had **8** first place finishes, **9** second place finishes and **1** third place finish.*

*Senior three had **10** first place finishes, **5** second place finishes and **3** third place finishes.*

*Senior four had **3** first place finishes, **3** second place finishes and **6** third place finishes.*

(a) *Write down:*

(i) a $\mathbf{4 \times 3}$ *matrix for the number of finishes each class had.*

(ii) a $\mathbf{3 \times 1}$ *matrix for the points awarded for each finish*

(b)*Use matrix multiplication to determine the winner of the competition*

3. *Shops* ***A, B, C,*** *and* ***D*** *ordered for balls, bats and gloves as follows:*

	Balls	Bats	Gloves
Shop A	70	30	50
Shop B	60	20	25
Shop C	40	15	10
Shop D	50	40	30

The balls cost ***Shs 5,000*** *each, bats* ***Shs 3,000*** *each and gloves* ***Shs 2,000*** *each*

(a) *Write down:*

(i) a $\mathbf{4 \times 3}$ *matrix for the items ordered by each shop.*

(ii) a $\mathbf{3 \times 1}$ *cost matrix for each item*

(b) *By matrix multiplication, find the total cost of the items for each shop*

(c) *If the supplier had to pay a tax of* ***20%*** *of the cost of the items sold, find his expenditure on the order.*

EER:

*1. Given that **I** is an identity matrix of order **2 × 2** and matrix* $A = \begin{pmatrix} 2 & -1 \\ -2 & -5 \end{pmatrix}$,

*find matrix **B = A+2I***

2. Find the inverse of matrix $P = \begin{pmatrix} -2 & 3 \\ -4 & 5 \end{pmatrix}$

3. Use matrix method to solve the simultaneous equations:

$$\frac{x}{2} + \frac{y}{3} = 5$$
$$\frac{x}{3} + \frac{y}{4} = 1$$

*8. A hotel rents double rooms at **Shs 40,000** per day and single rooms at **Shs 25,000** per day. If **14** rooms were rented one day for a total of **Shs 470,000***

***(i)** Form two equations to represent the above information*

***(ii)** Find by matrix method how many rooms of each kind were rented.*

*4. In the morning, **5** breads and **8** cakes were bought.*

*In the afternoon, **7** breads and **6** cakes were bought.*

*The cost of a bread is **Shs 4000** and a cake is **Shs 1200***

***(a)** Write down:*

***(i)** a **2 × 2** matrix for the bought items*

***(ii)** a **2 × 1** cost matrix for each item*

***(b)** Use matrix multiplication to find the expenditure in each case.*

17. Given that matrix $A = \begin{pmatrix} 1 & 1 \\ x & y \end{pmatrix}$, *find the values of **x** and **y** such that*

$A^2 = I$, where **I** is a **2** × **2** identity matrix

4. Given that $P = \begin{pmatrix} 6 & -4 \\ 2 & -1 \end{pmatrix}$ and $PQ = \begin{pmatrix} 16 & -18 \\ 6 & -5 \end{pmatrix}$, find:

(i) the inverse of **P.**

(ii) matrix $Q = P^{-1}[\ PQ\ ]$.

5. Find the values of **x** for which the matrix $\begin{pmatrix} x & x+9 \\ 2 & x+5 \end{pmatrix}$ has no inverse

19. Find the values of **x** for which the matrix $\begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$ is singular

19. Find the values of **x** for which the matrix $\begin{pmatrix} x & 2x \\ x-1 & x+1 \end{pmatrix}$ is singular

19. Find the values of **x** for which the matrix $\begin{pmatrix} x-5 & 3 \\ -2 & x \end{pmatrix}$ is singular

19. Find the values of **x** for which the matrix $\begin{pmatrix} x & 4 \\ 1 & x-3 \end{pmatrix}$ is singular

21. Given that matrix $M = \begin{pmatrix} 2 & -1 \\ -6 & 1 \end{pmatrix}$, find the values of **k** such that the matrix $(kI - M)$ is singular, where **I** is a **2** × **2** identity matrix

21. Given that matrix $A = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$, find the values of λ such that the matrix $(A - \lambda I)$ is singular, where **I** is a **2** × **2** identity matrix

16. Find the values of **x** and **y** such that $(1 \quad 3)\begin{pmatrix} 4 & y \\ x & 2 \end{pmatrix} = (7 \quad 7)$

2. Given that matrix $P = \begin{pmatrix} x+7 & x \\ 3 & 0 \end{pmatrix}$, $Q = \begin{pmatrix} x-1 & 0 \\ 2 & 2 \end{pmatrix}$ and $R = P + Q$, find the value of **x** for which the determinant of **R** is **2**

21. Given that matrix $P = \begin{pmatrix} 4x+1 & 3x \\ 2x+1 & 2x \end{pmatrix}$, find the values of **x** for which the determinant of **P** is **6**

6. **Shs 244,000** can buy **5** bans and **6**cakes, while **Shs 356000** can buy **7** bans

and **9**cakes. Find by matrix method the cost of each ban and that of a cake.

7. Find the values of **y** for which the matrix $\begin{pmatrix} 2y & 5 \\ 4 & y + \frac{1}{y} \end{pmatrix}$ is singular

8. Given that matrix $A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, find the value of **λ** such that

$A^2 + \lambda I = 5A$, where **I** is a **2 × 2** identity matrix

9. Given the matrix $A = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$,, find the values of **x** and **y** such that

$xA + yI = A^2$, where **I** is a **2 × 2** identity matrix.

9. Given the matrix $A = \begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}$,, find the values of **x** and **y** such that

$$A^2 = \begin{pmatrix} x & -6 \\ -2 & y \end{pmatrix}$$

10. *Bob and Ben went to a supermarket for shopping.*

*Bob bought **2 kg** of sugar**, 4** bars of soap**, 5** counter books and one bottle of cooking oil.*

*Ben bought **5 kg** of sugar**, 3** bars of soap and a dozen of counter books.*

*The cost of sugar per kg was **Shs 1,500,** a bar of soap was **Shs 1,000,** a counter book was **Shs 3,000** and a bottle of cooking oil was **Shs 2,000.***

(a) *Write down:*

(i) *a 2×4 matrix for the items bought by the two people.*

(ii) *a 4×1 cost matrix for each item*

(b) *Calculate the:*

(i) *expenditure of each person by matrix multiplication*

(ii) *total expenditure of both Bob and Ben*

(c) *How much did Ben spend than Bob*

3. *A charity organization donated Ball pens, exercise books, graph books and table books to senior four, three and two Classes of a school as below;*

*Senior four students got **2** ball pens, **12** exercise books, **3** graph books and **1** table book each.*

*Senior three students got **2** ball pens, **8** exercise books, **1** graph books and **1** table book each.*

*Senior two students got **1** ball pens**, 6** exercise books and **1** table book each*

*There are **100** students in senior four, **120** in senior three and **130** students in senior two.*

The organization bought the items at the following rates:

*Ball pens at **Shs500** each, Exercise books at **Shs1500** each, graph book at*

Shs 2000 *each and table books at* ***Sh.6000*** *each.*

(a) *Write down*

(i) 1 × 3 *matrix for the number of students.*

(ii) 3 ×4 *matrix for the items*

(iii) 4 ×1 *cost matrix.*

(b)*By matrix multiplication, determine the*

(i) *number of items of each type distributed.*

(ii) *total amount spent by the organization in acquiring the items.*

(c) *If the organization had to pay* ***5%VAT*** *on the items bought, determine the total amount spent.*

10. *If matrix* $B = \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}$, *find matrix* ***A*** *such that* $AB = \begin{pmatrix} 17 & 13 \\ 21 & 14 \end{pmatrix}$

9. *If matrix* $P = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$, *find matrix* ***A*** *such that* $AP = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

15. *If* $\begin{pmatrix} x & -2 \\ -1 & y \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 18 & 0 \\ 0 & 18 \end{pmatrix}$, *find x and y*

16. *Given that* $\begin{pmatrix} 3 & x \\ 5 & 4 \end{pmatrix}\begin{pmatrix} 1 & y \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 13 & 7 \end{pmatrix}$, *find x and y.*

1. *Given that* $A = \begin{pmatrix} 2 & -7 \\ 1 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 8 & 3 \\ 0 & 2 \end{pmatrix}$ and $C = BA$, find;

2. *i)* $C + 3B$

3. *ii)* C^{-1}

Give that $P = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$, $Q = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$ *and* $R = \begin{pmatrix} 4 & 6 \\ 10 & 15 \end{pmatrix}$*, find matrix T such*

$T = P^2 + 3Q - R$

4. *If* $M = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$;

a) Determine ; *i)* M^2 *ii)* M^3

b) identify matrix M^2

12. Four Secondary schools football teams of Ntare H.S, Layibi College, Mvara S.S and Kitende S.S qualified for a football tournament , which was played in two rounds with other teams.

First round

Ntare H.S won three matches, drew one and lost one match.

Layibi college won two matches, drew one and lost two matches

Mvara S.S won one month , drew three and lost one match.

Kitende S.S won four matches , drew one and lost no match.

Second round:

Ntare H.S won three matches , drew two and lost no match.

Layibi college won two matches, drew two and lost one match.

Mvara S.S won no match, drew three and lost two matches.

Kitende S.S won three matches, drew two and lost no match.

a) Write down:

(i) a 4 x 3 matrix to show the performance of the four teams in each of the two rounds. (02 marks)

(ii) a 4 x 3 matrix which shows the overall performance of the teams in both rounds. (02 marks)

b) If three points are awarded for a win, one point for a draw and no point for a loss, use matrix multiplication to determine which school won the tournament.

(03 marks)

c) Given that MTN donated sh. 3, 450,000 to be shared by the four teams according to the ratio of their points scored in the tournament, find how much money each team got.
(05 marks)

SIMILARITY AND CONGRUENCE

Summary;

Similar shapes are identical in shape but not in size

The ratio of the corresponding sides of similar shapes is the same and is called linear scale factor.

For any two similar shapes;

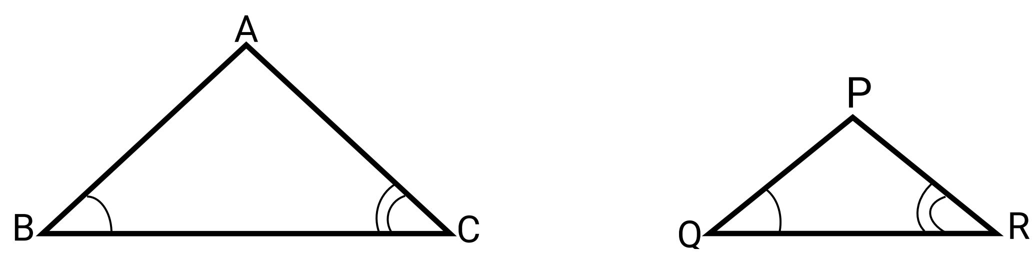
i) Ratio of length= linear scale factor (L.S.F)

ii) Ratio of areas = $(L.S.F)^2$

iii) Ratio of volumes = $(L.S.F)^3$

The following properties apply to similar shapes;

i) Corresponding angles are equal as shown



thus <A = <P, <B = <Q and <C = <R

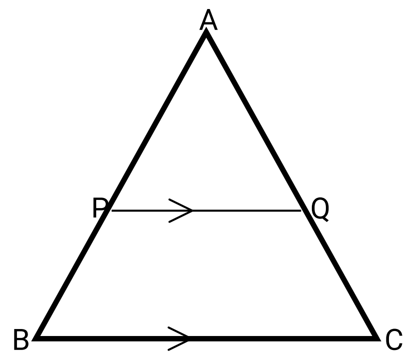
ii)The ratios between corresponding sides are equal

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \text{ or } \frac{PQ}{AB} = \frac{PR}{AC} = \frac{QR}{BC}$$

Congruent figures are the ones which are exactly the same size

Examples:

1. In the figure below, PQ is parallel to BC

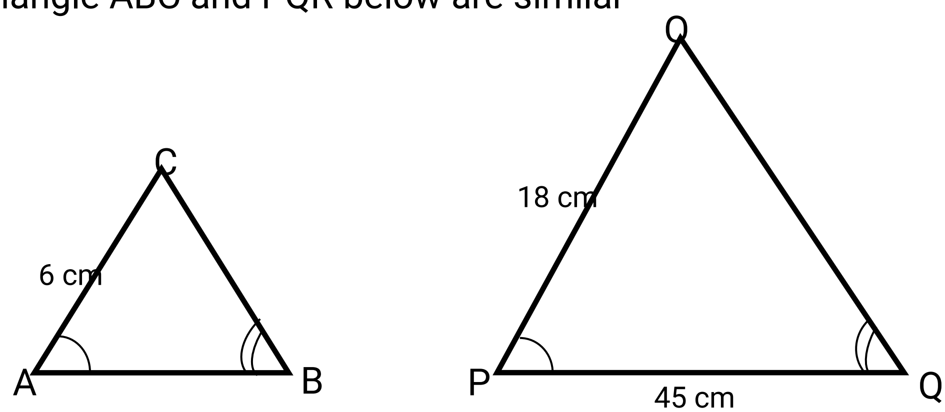


i)Show that triangles ABC and APQ are similar

(Hint: show that the triangles are equiangular)

ii) Write down the ratios between their corresponding sides

2. The triangle ABC and PQR below are similar



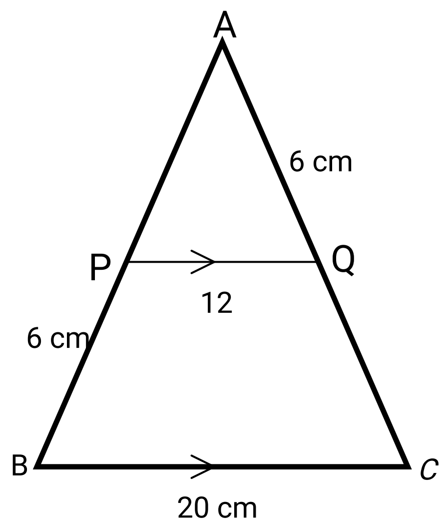
Determine the;

i) linear scale factor

ii) length AB

iii) ratio of area of triangle ABC that of PQR

3.



a) Find the lengths AP and QC

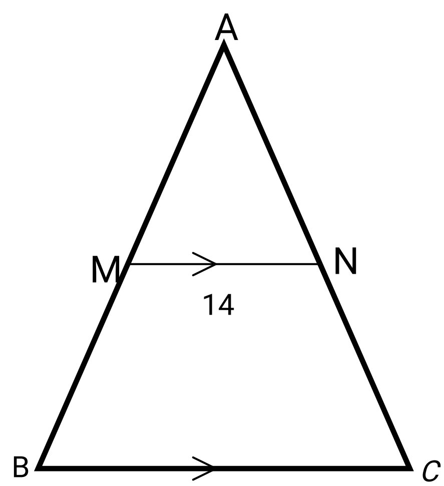
b) If the area of triangle APQ is 26.14 cm^2, find the;

i) area of triangle ABC

ii)Area of trapezium PBCQ

4. In the figure below, MN is parallel to BC. MN = 14 cm and

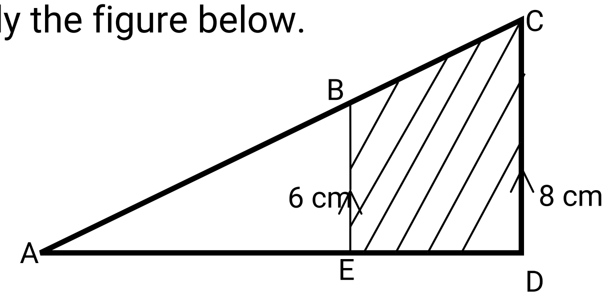
AM: MB = 2 :3



a) Find the length BC

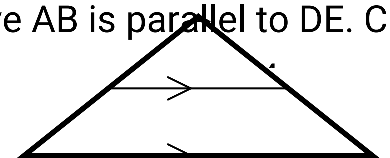
b) If the area of triangle ABC is 150 cm^2, find the area of trapezium MBCN.

5. Study the figure below.



If AD = 12 cm, find the area of the shaded region

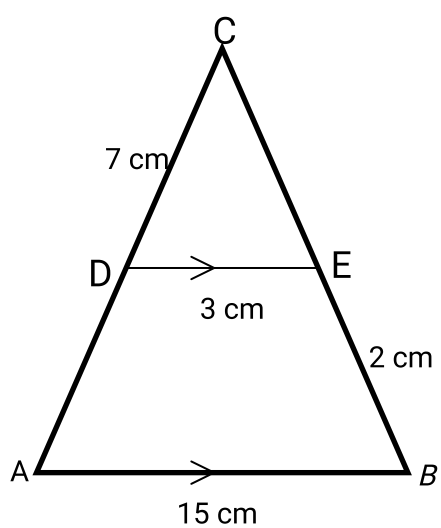
In the figure above AB is parallel to DE. Calculate the;



i) length AD

ii) ratio of area of triangle ABC to DEC

6.



1. s

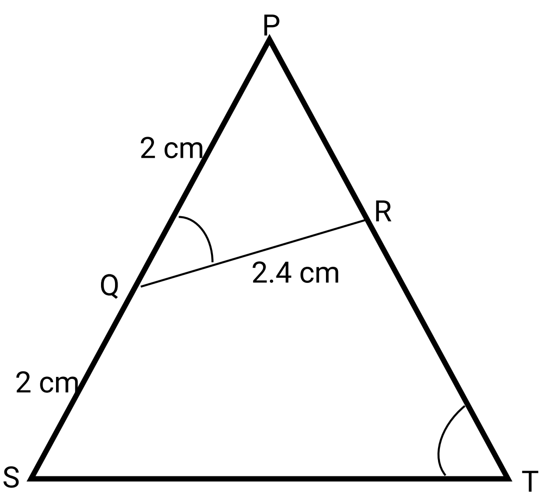
In the above AB is parallel to DE , DC = 7 cm, BE = 2 cm.

a) Calculate the length of;

i)AD ii)CE

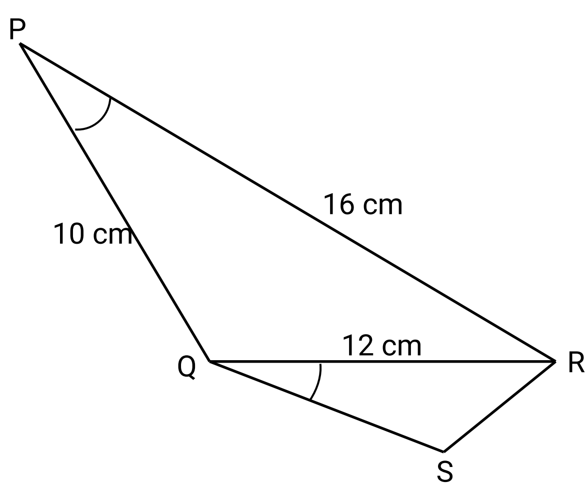
b) Given that the area of DEC is 30 cm^2, find the area of quadrilateral ABED.

2.



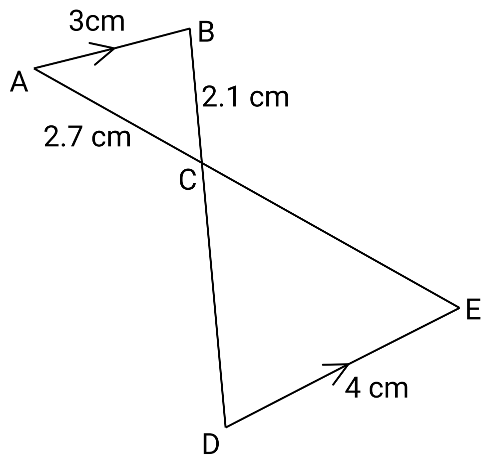
In the figure triangles PQR and PST are similar and PS= 5cm. Calculate the length of PR and ST.

3.



The triangles PQR and QSR are similar. Calculate the lengths QS and SR.

4.



In the figure AB is parallel to DE. Calculate the lengths CD and CE.

Word problems:

1. The areas of two similar triangles are 18 cm^2 and 32 cm^2 respectively. If the base of the smaller triangle is 6 cm, find the base of the larger triangle.

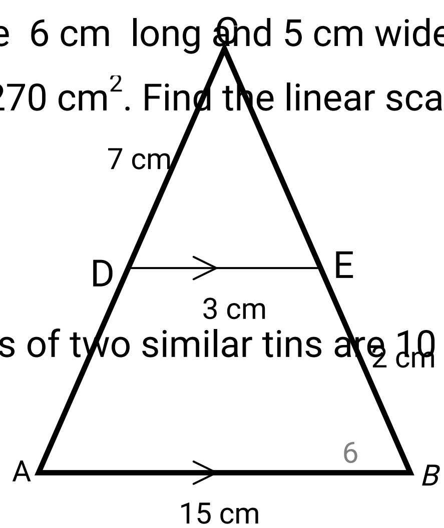
(8 cm)

2. Two similar figures have corresponding sides of length 3 cm and 5 cm. If the area of the larger figure is 100 cm^2 find the area of the smaller figure

(36 cm^2)

3. A rectangle 6 cm long and 5 cm wide is enlarged so that its area becomes 270 cm^2. Find the linear scale factor

(3)



4. The heights of two similar tins are 10 cm and 15 cm respectively. If the volume of the larger tin is 405 cm^3, find the volume of the smaller tin

($120\ cm^3$)

5. A beaker of height 21 cm has volume 135 cm^3. Find the height of a similar beaker whose volume is 40 cm^3

   (14 cm)

6. A tank has a volume of 6400 cm^3 and surface area 800 cm^2. Find the surface area of a similar tank whose volume is 2700 cm^3

   ($450\ cm^2$)

7. A cone has volume of 120 cm^3 and surface area of 48 cm^2. Find the volume of a similar cone whose surface area is 108 cm^2

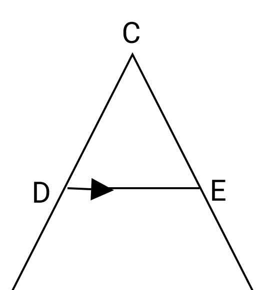
   ($405\ cm^3$)

8. A cylinder of radius 2 cm has height of 3.5 cm is enlarged so that its volume becomes 1188 cm^3. Find the linear scale factor

   (3)

EER

1. In the figure below DE is parallel to AB. AB= 15 cm , DE = 12 cm and AD = 4cm



find the;

i) length of CD

ii) area of trapezium ABED if area of triangle CDE is 288 cm^2

2. A tank of height 7.5 cm has capacity of 540 cm^3. Find the capacity of a similar tank whose height is 5 cm

3. A tank of height 2m and width 1.4 m has capacity of 3.08 m3. Find the height and width of a similar tank whose capacity is 83.16 m^3

4. Two solids have a linear scale factor of 3. Given that the larger one has surface area of 324 cm^2 and volume of 405 cm^3, find the volume and surface area of the other.

5. The volumes of two similar cups are 64 cm^3 and 343 cm^3 respectively. Find the ratio of their surface area.

6. A tank has volume of 2700 cm^3 and surface area of 450 cm^2. Find the surface area of a similar tank whose volume is 6400 cm^3

7. The heights of two similar cups are 10 cm and 15 cm resp. if the

larger cup has surface area of 108 cm^2 and a volume of 405 cm^3. Find the surface area and volume of a smaller cup

8. A cone has radius of 7 cm and vertical height of 30 cm. Find its;

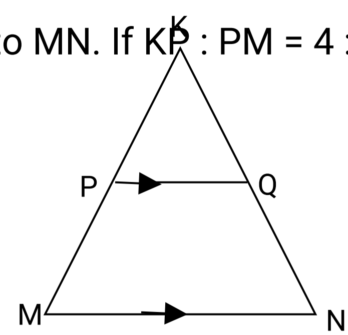
   i)its volume

   ii)the volume of another similar larger cone which has a linear scale factor of 2.

9. The heights of two similar jugs are 6 cm and 8 cm resp. if the capacity of the laeger jug is 252 cm^3, find the capacity of the smaller jug

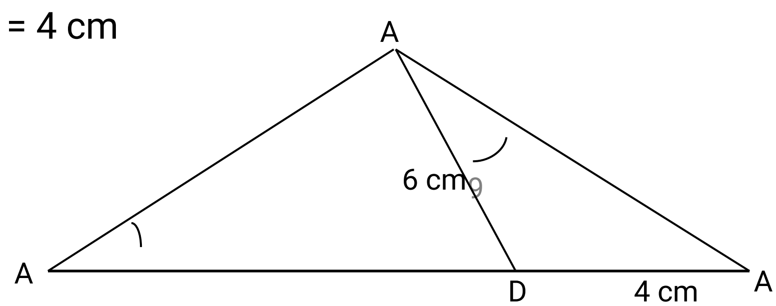
10. The areas of two similar triangles are 18 cm^2 and 32 cm^2 resp. if the height of the smaller triangle is 6 cm, find the height of the larger triangle.

11. In the figure below PQ is parallel to MN. If KP : PM = 4 : 13, KN = 20.4 cm and PQ = 6.4 cm



Find the lengths of KQ and MN.

12. In the figure below $\angle BAC = \angle DBC$, BC = 8 cm, DB = 6 cm and DC = 4 cm



Find the length of AD and AB

SIMULTANEOUS EQUATIONS

Summary:

1. *Equations with more than one unknown are called simultaneous equations*

2. *Simultaneous equations can be solved using the following methods:*
(i) *Elimination method* **(ii)** *Substitution method* **(iii)** *Graphical method*

(iv) *Matrix method*

3. *The solution to simultaneous equations must satisfy each equation.*

EXAMPLES:

1. *Use the elimination method to solve the following simultaneous equations:*

(i) $3x + y = 11$
$2x + y = 8$

(ii) $2x - y = 3$
$3x + 2y = 8$

(iii) $8x - 5y = -6$
$4y - 13x = -15$

(iv) $\frac{x}{2} + \frac{y}{3} = 5$
$\frac{x}{3} + \frac{y}{4} = 1$

(v) $\frac{x}{6} - \frac{2y}{3} = \frac{1}{15}$
$\frac{3x}{4} + \frac{y}{3} = \frac{5}{6}$

(vi) $\frac{x-3}{4} - \frac{y-2}{5} = \frac{7}{40}$
$\frac{4-x}{3} + \frac{3-y}{2} = \frac{3}{5}$

(vii) $\frac{5}{x} - \frac{2}{y} = 2$
$\frac{2}{x} + \frac{3}{y} = 16$

2. *Use the substitution method to solve the following simultaneous* equations:

(i) $4a + b = 14$ (ii) $3x + 4y = 11$ (iii) $4x + 3y = 24$

$3a - 4b = 7$ $2x + 3y = 8$ $2y - 3x = -1$

3. Solve graphically the following simultaneous equations:

(i) $x + y = 5$ (ii) $3x - y = 7$ (iii) $3x + 2y = 4$

$2x + y = 6$ $4x + y = 14$ $x + 2y = 0$

4. Solve the following simultaneous equations:

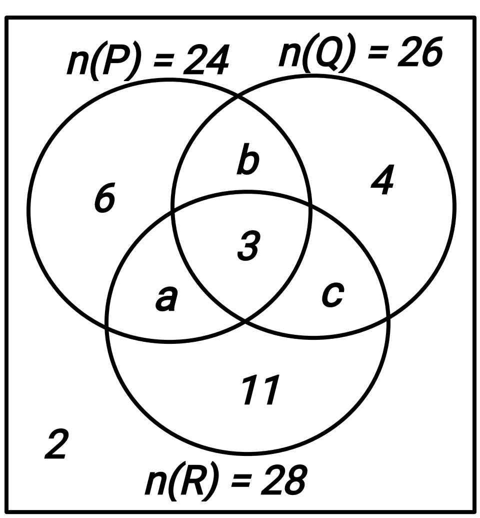
(i)
$$a + b = 12$$
$$b + c = 11$$
$$a + c = 9$$

(ii)
$$x + y = 16$$
$$y + z = 10$$
$$x + z = 18$$

Soln:

(i)
$$a + b = 12$$
$$+\ b + c = 11$$
$$\overline{a + c + 2b = 23}$$
$$\Rightarrow 9 + 2b = 23$$
$$\therefore \quad b = 7$$
$$\Rightarrow \quad a = 12 - 7 = 5, \quad c = 11 - 7 = 4$$

5. Study the Venn diagram below:



Find:

*(i) the values of **a, b** and **c***

*(ii) **n(ε),** where **ε** is the universal set*

WORD PROBLEMS ON SIMULTANEOUS EQUATION

Summary:

In solving word problems on simultaneous equation, read the problem carefully and form an equation using the conditions given in the problem

EXAMPLES:

1.** Find two numbers such that one exceeds the other by **9** and their sum is **25

Soln:

***2.** The sum of the present ages of a man and his son is **60** years. Six years ago, the man's age was **5** times the son's age. Find their present ages*

***3.** Shs **4000** can buy **10** bans and **5**cakes or **4**bans and **10**cakes. Find the cost of*

each ban and that of each cake.

***4.** The cost of **2** pens and **5** books is **Shs 5400.** If the cost of **4** pens and **3** books is **Shs 5200,** find the:*

***(i)** cost of each pen and that of each book*

***(ii)** cost of **3** pens and **2** books*

***(iii)** value of **n,** if **Shs 9000** can buy **n** pens and **n** books*

***5.** A total of **162** guests can be transported when a hired taxi makes **3** trips and the* *bus makes **2** trips. **250** guests can be transported when the taxi makes **5** trips*

*and the bus makes **3** trips. Find the carrying capacity of each vehicle*

***6.** A total of **120** tickets were sold for **Shs 570,000.** The cost of each adult ticket was **Shs 5,000** and that of each child was **Shs 4,000.** Find how many tickets of each kind were sold*

Soln:

*If **x =** number of adult tickets, **y =** number of children tickets*

$$\Rightarrow x + y = 120 \text{ - - - - - - - - - - - -(i)}$$

$$\text{Also : } 5000x + 4000y = 570{,}000$$

$$\Rightarrow 5x + 4y = 570 \text{ - - - - - - - - - - - - - - (ii)}$$

$$\text{On solving, } \mathbf{x = 90, \quad y = 30}$$

***7.** If Tom gives Bob **Shs 2,000,** they would have the same amount. While if Bob*

*gave Tom **Shs 2,200 ,** Tom would then have twice as much as Bob. Find how*

much does each has.

Soln:

*If **x =** Tom's original amount, **y =** Bob's original amount*

$$\Rightarrow x - 2000 = y + 2000$$

$$\Rightarrow x - y = 4000 \text{ - - - - - - - - - - - -(i)}$$

$$\text{Also : } x + 2200 = 2(y - 2200)$$

$$\Rightarrow x - 2y = -6600 \text{ - - - - - - - - - - - - - - (ii)}$$

$$\text{On solving, } \mathbf{x = 14{,}600, \quad y = 10{,}600}$$

***8.** A man covers a distance of **15km** in **3 hours,** partly by walking and partly by* running. If he walks at $3kmh^{-1}$ and runs at $9kmh^{-1}$, find the distance he covers by running

Soln:

If **x =** distance walked, **y =** distance ran

$\Rightarrow x + y = 15$ ------------(i)

Also : $\frac{x}{3} + \frac{y}{9} = 3$

$\Rightarrow 3x + y = 27$ -------------(ii)

On solving, $y = 9km$

9. The points **(–3, 9)** and **(–4, 20)** lie on the curve $y = px + qx^2$. Find the values of **p** and **q**

10. Find the fraction which becomes $\frac{2}{3}$ when its numerator and denominator are both decreased by one and is equal to $\frac{3}{4}$ when its numerator and denominator are both increased by one

11. A two digit number in base ten is equal to five times the sum of the digits. It is nine less than the number formed by interchanging the digits. Find the number

Soln:

If the required no **= xy**

$\Rightarrow \; xy_{ten} \; = \; 5\,(x + \; y)$

$(x \times 10^{1}) + (y \times 10^{0}) = 5\,(x + \; y))$

$10x \; + \; y = \; 5x \; + 5y$

$5x \; - \; 4y = \; 0$ ------------(i)

$Also \; : \; yx \; - \; xy \; = 9$

$10y \; + \; x \; - \; (10x \; + y) \; = 9$

$\Rightarrow \; y - x \; = 1$ -------------- (ii)

$On \; solving, \; x = \; 4, \; y = 5$

$\therefore$ $Required \; No \; = \boldsymbol{45}$

__12.__ A number consists of two digits whose sum is __7.__ If the number formed by reversing the digits is less than the original number by __27,__ find the original number

Soln:

*If the required no **= xy***

$\Rightarrow x \; + \; y \; = \; 7$ ------------(i)

$Also \; : \; xy \; - \; yx \; = 27$

$10x \; + \; y \; - \; (10y \; + x) \; = 9$

$\Rightarrow \; x - y \; = 3$ -------------- (ii)

$On \; solving, \; x = \; 5, \; y = 2$

$\therefore$ $Required \; No \; = \boldsymbol{52}$

__13.__ In a two digit number__,__ the ten's digit is three times the unit's digit. The sum of the number and its unit's digit is __64.__ Find the original number

Soln:

*If the required no **= xy***

$\Rightarrow x = 3y$ - - - - - - - - - - - -(i)

$Also: \quad xy + y = 64$

$10x + y + y = 64$

$\Rightarrow 5x + y = 32$ - - - - - - - - - - - - - -(ii)

On solving , $x = 6, \quad y = 2$

$\therefore$ Required No $=$ **62**

14. *In a two digit number, the ten's digit exceeds twice the unit's digit by **2** and the number obtained by interchanging the digits is **5** more than three times the sum of the digits. Find the original number*

Soln:

If the required no ***= xy***

$\Rightarrow x - 2y = 2$ - - - - - - - - - - - -(i)

$Also: \quad yx - 3(x + y) = 5$

$10y + x - 3(x + y) = 5$

$\Rightarrow 7y - 2x = 5$ - - - - - - - - - - - - - -(ii)

On solving, $x = 8, \quad y = 3$

$\therefore$ Required No $=$ **83**

EER:

1. *Solve the following simultaneous equations:*

(i)	(ii)	(iii)
$3a + 5b = 21$	$2x - 5y + 14 = 0$	$4x + 3y = 24$
$2a + 3b = 13$	$4x + 3y - 11 = 0$	$2y - 3x = -1$

2. *A total of **62** guests can be transported when car **A** makes **7** trips and car **B***

*makes **5** trips. With **6** trips each, the two cars can carry **60** guests. Find*

the

carrying capacity of each car

__3.__ Solve the following simultaneous equations:

(i) $\frac{x}{3} + \frac{y}{4} = 2$

$\frac{x}{2} - \frac{y}{3} = \frac{1}{6}$

(ii) $\frac{x-2}{3} + \frac{y-1}{4} = \frac{13}{12}$

$\frac{2-x}{2} + \frac{3+y}{3} = \frac{11}{6}$

4. *The cost of **3** plates and **5** cups is **Shs 7,600.** If **5** plates and **3** cups cost **Shs***

8,400,

(i) *find the cost of each plate and each cup*

(ii) *By buying **18** plates and **23** cups, a discount of **3%** and **8%** was allowed on*

each plate and each cup respectively. Calculate the percentage discount

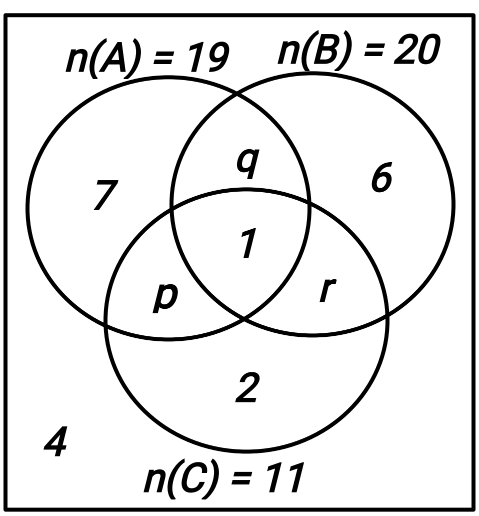
allowed on the whole purchase

5. *Solve graphically the following simultaneous equations:*

(i) $2x - y = 4$ ***(ii)*** $3x + y = 6$

$x + 4y = 11$ $x + 2y = 7$

6. *Study the Venn diagram below:*



Find:

*(i) the values of **p, q** and **r***

*(ii) **n(ε),** where **ε** is the universal set*

***7.** Two quantities **x** and **y** are related by the equation $y = px + qx^2$. When*

x = 2, y = 6** and when **x = 3, y = 24.** Find the values of **p** and **q

***8.** A man bought **30** cups for **Shs 12,600.** He bought some at **Shs 500** each and the other at **Shs 350** each. Find how many cups of each kind were bought*

***9.** The side of an equilateral triangle are **(x + 4y)cm, (3x + 2)cm** and **(6y – 1)cm.** Find the:*

*(i) values of **x** and **y***

(ii) perimeter of the triangle

10.** Find the fraction which becomes $\frac{1}{2}$ when its denominator is increased by **4** and is equal to $\frac{1}{8}$ when its numerator is reduced by **5

***11.** The sum of two angles of a triangle is **114°** and their difference is **48°,** find all*

the angles of the triangle

***12.** The sum of the present ages of a man and his son is **85** years. Five years ago, the man was twice as old as his son. Find their present ages*

***13.** The present ages of Tom and Bob are in the ratio of **5:4.** In three years time, the ratio of their ages will become **11:9**respectively. Find Bob's present age*

14. *Ten years ago, a man's age was thrice as old as his son. In ten years time, the man's age will be twice as old as his son. Find their present ages*

15. *A man spent* ***Shs 29,000*** *to buy* ***4 kg*** *of rice and* ***7 kg*** *of meat. Later he*

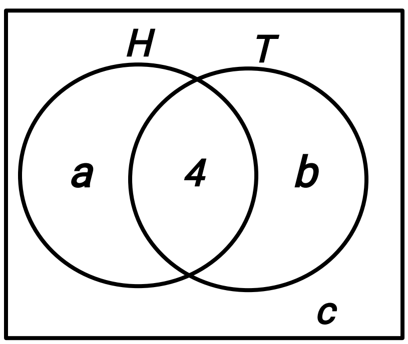
increased each of the quantities by ***1kg*** *thus increasing his expenditure by*

Shs 5,000. *Find*

(i) *the cost of each kg of rice and that of meat*

(ii) *how much would the man pay for buying* ***10 kg*** *of rice and* ***15 kg*** *of meat*

16. *In the Venn diagram below,* ***20*** *students play either Hockey* ***(H)*** *or Tennis* ***(T)***



Given that ***14*** *and* ***12*** *students do not play Hockey and Tennis respectively,*

Find the:

(i) *values of* ***a, b*** *and* ***c.***

(ii) *probability that a student picked at random plays neither of the games*

17. *In a two digit number, the unit's digit is thrice the ten's digit. If* ***36*** *is added to the number the digits are reversed. Find the original number*

18. *A number consists of two digits whose sum is* ***5.*** *If* ***9*** *is subtracted from* *the number the digits are reversed. Find the original number*

19. *A number consists of two digits whose sum is* ***12.*** *If the result of dividing the number by the number with the digits reversed is* $1\frac{3}{4}$*, find the original number*

20. *A number consists of two digits whose sum is* ***8.*** *The number obtained by interchanging the digits exceeds the original number by* ***18.*** *Find the original number*

21. *Solve for* ***x*** *and* ***y*** *in the equation:* $\frac{3^{x} \times 2^{2y}}{3^{y} \times 2^{x}} = 72$

22. *Find two numbers whose sum is* ***120*** *and their difference is* ***30***

23. *The points* ***(1, –1)*** *and* ***(2, 2)*** *lie on the curve* $y = px + qx^{2}$*. Find the values of*

p *and* ***q***

24. *Two quantities* ***f*** *and* ***m*** *are related by the equation* $f = am + bm^{2}$*. When*

f = 72, m = 8 *and when* ***f = 28, m = 4.*** *Find the:*

(i) *equation connecting* ***f*** *and* ***m***

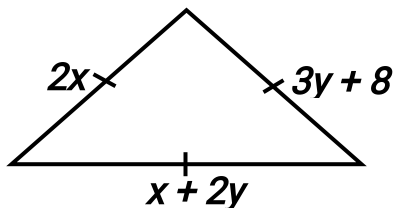
(ii) *value of* ***f*** *when* ***m = 6***

25. *Tom travelled a distance of* ***52km*** *in* ***6 hours.*** *He travelled partly on foot at* $4kmh^{-1}$ *and partly on a bicycle at* $12kmh^{-1}$*. Find the distance he travelled on foot*

26. *A cyclist travels a journey of* ***500m*** *in* ***22seconds,*** *part of the way at* $10ms^{-1}$ *and the remainder at* $50ms^{-1}$*. How far does he travel at each speed?*

27. *Tom is **6** years older than Bob. The ratio between the present ages of Tom and Bob is **7:5.** Find their present ages*

28. *Find the values of **x** and **y** in the given equilateral below*

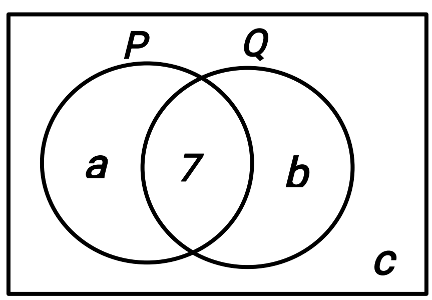


29. *The cost of two tables and three chairs is **Shs 705,000.** If the table costs **Shs 40,000** more than the chair, find the cost of each table and a chair*

30. *The cost of **4** pens and **5** books is **Shs 10,000.** If the cost of **3** pens and **2** books is **Shs 5,100,** find the cost of **7** pens and **6** books*

31. *A man spends **Shs 13,000** in two days. He spends **Shs 2,000** more on the first day than on the second day. Find his expenditure on the second day*

32. *In the Venn diagram below, sets **P** and **Q** are such that **n(PUQ) = 16, n(P') = 7,** and **n(Q') = 6.***



*Find the values of **a, b** and **c.** Hence obtain **n(ε)***

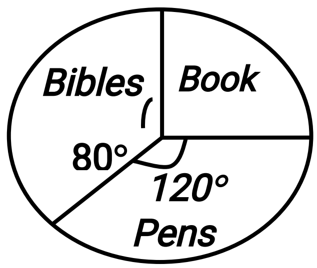
PIE CHART

Summary:

A pie chart shows information using sectors of a circle.

EXAMPLES:

1. *The pie chart below shows the various items sold by a certain shop*

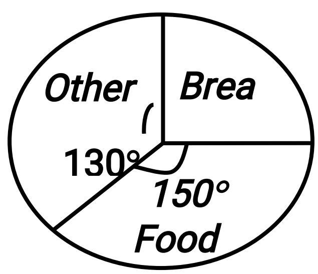


*If the total sale value of the items was **sh 720,000,** find the sales value of**:***

(i) *pens*

(ii) *books*

2. *The pie-chart below shows the daily expenditure of a certain family.*

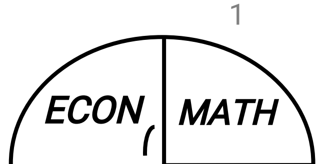


*If **sh 2000** is spent on bread, find that:*

(i) *total daily expenditure of the family*

(ii) *daily expenditure of the family on food*

3. *The pie chart below shows the number of students taking various*



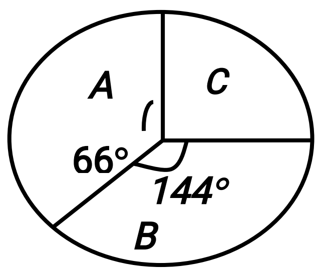
subjects in a certain class

*If the number of students taking **C·R·E** is **120,** find the:*

***(i)** population of the students in the class*

***(ii)** number of students taking mathematics*

***4.** The pie chart below shows the number of voters for party **A, B** and **C** in an election*



*Find the percentage of voters for party **C***

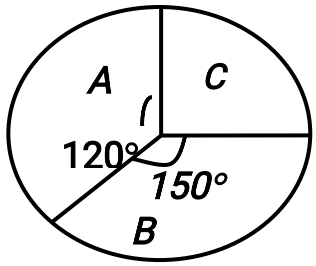
***6.** A family spends its income on the following items in a month*

Items	***food***	***wages***	***travel***	***others***
Amount (sh)	***42,000***	***9,000***	***6,000***	***4,000***

Show the family's expenditure in a pie chart

EER:

1. *The pie chart below shows the number of voters for party **A, B** and **C** in an election*



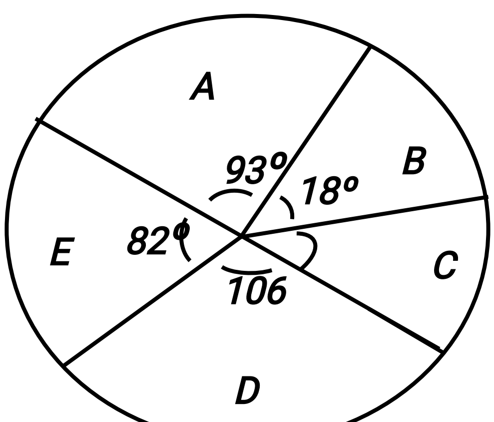
*If the total number of votes is **300,** find the number of votes for party **C***

2. *A family spends its income on the following items in a month*

Items	*food*	*wages*	*travel*	*Others*
Amount (sh)	*35,000*	*12,000*	*10,000*	*15,000*

Show the family's expenditure in a pie chart

3. *The pie chart below shows the number of voters from polling stations **A, B, C, D** and **E** in a constituency.*



*If the number of voters in station **A** is **6,231,** determine the:*

(i) *voter population in the constituency.*

(ii) *number of voters in polling station **C.***

4. *A company's cost is split as follows:*

Wages ***45%***

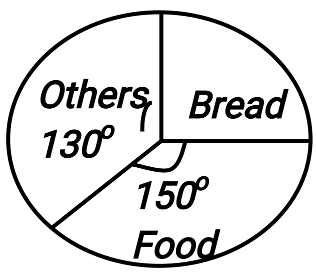
Food ***30%***

Travel ***15%***

Others ***10%***

Show this information in a pie chart

7. *The pie-chart below shows the daily expenditure of a certain family.*



*If **sh 2000** is spent on bread, find the:*

(i) *total daily expenditure of the family*

(ii) *daily expenditure of the family on food*

5. *The expenditure of a certain football club is as follows:*

Items	***food***	***wages***	***travel***	***others***
Amount (sh)	***35,000***	***12,000***	***10,000***	***15,000***

Draw a pie chart showing the club's expenditure

STATISTICS

Summary:

1. *For a set of* ***n*** *values:*

(i) *mean* **(Average)** $= \dfrac{\textit{sum of values}}{\textit{number of values}} = \dfrac{\textit{sum}}{\textit{count}}$

(ii) *Median is the middle value when the given data is listed in order of magnitude. If the number of items is even, the average of the middle two is used.*

(iii) *Mode is the value that occurs most frequently.*

There can be more than one mode in a given data.

(iv) *Range is the difference between the largest and smallest values*

EXAMPLES:

1. *Find the mean, mode and median of the following numbers:* ***7, 8, 10, 12*** *and* ***8.***

2. *Find the mean, mode and median of the following numbers:* ***31, 28, 30, 33, 25*** *and* ***30.***

3. *The mean of* ***3, 7, 10, 8*** *and* ***x*** *is* ***6.*** *Find* ***x***

4. *The marks scored by a boy in four tests were* ***45, 70, 35*** *and* ***40.*** *When he does a fifth test the mean mark of the five tests is* ***50.*** *Find his scored mark in the fifth test*

5. *If the mean of* ***6*** *numbers is* ***30,*** *find the sum of these numbers*

6. *The mean marks for a French test in a class of* ***30*** *boys and* ***20*** *girls are* ***60*** *and* ***70*** *respectively. Find the mean mark for the whole class*

Soln:

Required mean $= \dfrac{(30 \times 60) + (20 \times 70)}{30 + 20} = \mathbf{64}$

7. *In a class of boys and girls, the average age is* $15\frac{1}{2}$ *years. The class has* ***12*** *boys whose average age is* $16\frac{3}{4}$ *years. Find the size of the class, if the average age of the girls is* ***15*** *years.*

Soln:

If n = number of girls

$$\Rightarrow \quad \frac{(12 \times 16 \cdot 75) + (15 \times n)}{12 + n} = 15 \cdot 5$$

n = 30

$\therefore$ *Class size = 12 + 30 =* ***42***

8. *The mean age of a class of* ***30*** *students is* ***16*** *years* ***3*** *months. If* ***12*** *students whose mean age is* ***14*** *years* ***6*** *months left the class, find the mean age of those who remained.*

Soln:

Required mean $= \dfrac{(30 \times 16 \cdot 25) - (12 \times 14 \cdot 5)}{30 - 12} = \mathbf{17 \cdot 4167}$

9. *The mean weight of a class of* ***30*** *boys is* ***x kg.*** *When two boys with a total weight of* ***150kg*** *are absent, the mean weight of those present is* ***2kg*** *less than the mean weight of the whole class. Find the value of* ***x.***

Soln:

If mean for those present = x – 2

$$\Rightarrow \quad \frac{30x - 150}{30 - 2} = x - 2$$

$\therefore$ ***x = 47***

EER:

1. *The mean of **n** numbers is **5.** If the number **13** is included with the **n** numbers,*

*the new mean is **6.** Find the value of **n.***

2. *In a set of **10** numbers, the mean of **6** numbers is **64.5** and that of the **10***

*numbers is **68.** Find the mean of the other four numbers.*

*3. The mean of **3, 7, 3, x, 8, 10** and **x** is **7.** Find **x***

4. *The mean heights of **20** boys and **15** girls are **1·60m** and **1·52m** respectively.*

*Find the mean height of the **35** boys and girls.*

5. *The average age of **6** men is **45** and **5** of the men are **47, 40, 38, 46** and **43***

years old. Find the age of the sixth man.

6. *The average age of **6** men is **37** and one of them is **42** years old. Find the average age of the other five men.*

7. *The average age of a class of **30** boys is **14** years **4** months. If five boys whose*

*average age is **15** years **2** months leave the class, find the average age of the **25***

remaining boys.

8. *In a class of **30** students, there are **20** boys whose average age is **19** years **7** months and the rest are girls. Given that the mean age for the whole class is **18** years **4** months, find the mean age of the girls in the class.*

9. *A class of* ***15*** *boys took an examination in which* ***7*** *boys got an average mark of* ***40*** *and* ***7*** *others got an average mark of* ***50.*** *The average mark for the whole class was* ***46.*** *How many marks did the other boy get**?***

10. *The mean of four numbers is* ***20.*** *If two other numbers* ***(x + 3)*** *and* ***(x + 2)*** *are added, the new mean is* ***30.*** *Find the value of* ***x.***

FREQUENCY DISTRIBUTION TABLES

Summary:

1. *A frequency table shows a summary of values and their frequency*

2. (i) *Data that is listed is called ungrouped data*

(ii) *Data that is grouped together in classes is called grouped data*

3. *The following terms may be needed:*

(i) *Class boundaries are class groups in continuous form*

(ii) *Class width = upper class boundary – lower class boundary*

(iii) *Cumulative frequency is obtained by adding frequencies as you go along*

4. *In a frequency distribution table, mean can be computed as follows:*

(i) *mean* $= \dfrac{\sum fx}{\sum f}$, *where*

f = f*requency* ***(****number of times of occurrence****)***

x = *class mid values*

(ii) *mean* $= A + \dfrac{\sum fd}{\sum f}$, *where*

A = *assumed mean or working mean*

d = x – A *(deviation)*

5. (i) *The class which contains the mode is called the modal class*

(ii) *The modal class is the one with the highest frequency*

*6. For grouped data****,*** *mode is calculated as follows****:***

(i) *Determine the modal class*

(ii) $mode = L + \left(\dfrac{D_1}{D_1 + D_2}\right)c$, *where*

L = *lower boundary of the modal class*

D_1 ***=*** *modal frequency – premodal frequency*

D_2 ***=*** *modal frequency – post modal frequency*

c = *modal class width*

7. (i) *The class which contains the median is called the median class*

(ii) *The median class corresponds to a cumulative frequency of* $\frac{1}{2}\sum f$

8. *For grouped data, median is calculated as follows:*

(i) *Determine the median class*

(ii) $median = L + \left(\frac{\frac{1}{2}\sum f - Cf_b}{f_w}\right)c$, *where*

L *= lower boundary of the median class*

$\sum f$ *= total frequency*

Cf_b *= cumulative frequency before the median class*

f_w *= frequency within the median class*

c *= median class width*

9 (i) *The cumulative frequency curve* ***or*** *an ogive is a curve where cumulative frequencies are plotted against the upper class boundaries. It can estimate the median*

(ii) *A histogram consists of bars with frequency as the vertical and class boundaries as the horizontal. It can estimate the mode*

(iii) *A frequency polygon is a line graph drawn by plotting frequency against class mid values.*

NOTE:

(a) *The points are joined by straight lines.*

(b) *The polygon extends to the next lower and higher classes with zero frequencies*

EXAMPLES:

1. The marks of students in a test were as follows:

Marks	*4*	*5*	*6*	*7*	*8*
No of students	*2*	*6*	*4*	*5*	*3*

(a) *State the modal mark*

(b)*Find the:*

(i) *mean mark*

(ii) *median mark*

2. *The marks of students in a test were as follows:*

Marks	*3*	*4*	*5*
No of students	*3*	*x*	*4*

Given that the mean mark is ***4·1,*** *find* ***x***

3. *The ages in years of* ***40*** *students were as follows:*

12	13	14	12	15	14	13	16	14	15
13	14	16	15	14	12	13	14	15	13
15	16	15	14	15	12	15	13	12	15
13	15	12	15	16	14	15	14	16	14

(a) *Form an ungrouped frequency distribution table for the data*

(b) *State the modal mark*

(c) *Find the:*

(i) *mean mark*

(ii) *median mark*

4. *The marks of students in a test were as follows:*

Marks	***5***	***8***	***10***	***14***	***18***	***20***
No of students	***2***	***5***	***12***	***3***	***11***	***7***

Calculate the mean mark using an assumed mean of ***10,***

5. *The age distribution of* ***40*** *adults were as follows:*

Age	***Frequency***	***Cumulative frequency***
20 – 29	***4***	***4***
30 – 39	***12***	***16***
40 – 49	***8***	***…………***
50 – 59	***9***	***…………***
60 – 69	***7***	***…………***

(a) *Copy and complete the cumulative frequency column*

(b) *State the:*

(i) *class width*

(ii) *modal class*

(c) *Determine the median class*

(d) *Calculate the:*

(i) *mean*

(ii) *mode*

(iii) *median*

(c) *Display the data on a histogram and use it to estimate the mode.*

(d) *Draw an ogive for the data and use it to estimate the median*

(e) *Display the data on a frequency polygon*

6. *The marks of* ***40*** *students were as follows:*

26	11	10	12	14	16	20	25
21	22	13	17	18	27	30	32
27	35	40	44	39	28	37	26
44	37	36	39	28	46	32	15
16	19	34	43	26	38	48	40

(a) *Form a frequency distribution table with a lower class of* ***10 – 14.***

(b) *Calculate the:*

(i) *mean*

(ii) *mode*

(iii) *median*

(c) *Display the data on a histogram and use it to estimate the mode.*

(d) *Plot an ogive for the data and use it to estimate the median*

***7.** The age distribution of **40** adults were as follows:*

Ages	**2·0–2·4**	**2·5– 2·9**	**3·0–3·4**	**3·5–3·9**	**4·0–4·4**	**4·5– 4·9**
Frequency	**8**	**9**	**10**	**6**	**12**	**5**

***(a)** State the:*

***(i)** class width*

***(ii)** modal class*

***(b)** Determine the median class*

***(c)** Calculate the:*

***(i)** mean*

***(ii)** mode*

***(iii)** median*

***(d)** Display the data on a histogram and use it to estimate the mode.*

***(e)** Draw an ogive for the data and use it to estimate the median*

***8.** The cumulative distribution table shows the marks scored by **50** students.*

Marks	**Cumulative frequency**
30 – 39	**5**
40 – 49	**13**
50 – 59	**23**
60 – 69	**39**

70 – 79	***46***
80 – 89	***50***

(a) *Draw an ogive for the above data and use it to estimate the:*

(i) *median mark*

(ii) *pass mark of the test if **39** students passed*

(iii) *number of students who scored **75** marks and above*

(b) *Form a frequency distribution table for the above data to calculate the mean mark*

9. *The weights in kg of **40** students were as follows:*

Weights	***30 – 39***	***40 – 49***	***50 – 59***	***60 – 69***	***70 – 79***	***80 – 89***
Frequency	***1***	***7***	***9***	***8***	***10***	***5***

*Calculate the mean weight using an assumed mean of **54·5***

10. *The weights of **40** students were as follows:*

50	51	50	52	54	56	60	65
61	62	53	57	58	64	70	72
67	75	67	70	56	66	65	69
72	77	76	57	66	68	62	55
56	59	74	73	78	66	67	74

(a) *Form a frequency distribution table with class width of **5** starting with class of **50 – 54***

(b) (i) *Display the data on a histogram and use it to estimate the mode*

(c) *Calculate the:*

(i) *mean using a working mean of* ***62***

(ii) *mode*

(iii) *median*

EER:

1. *The weights in kg of* ***50*** *babies in a maternity ward were as follows:*

Age	Frequency	Cumulative frequency
2·0 – 2·4	8	…………
2·5 – 2·9	9	…………
3·0 – 3·4	10	…………
3·5 – 3·9	6	…………
4·0 – 4·4	12	…………
4·5 – 4·9	5	…………

(a) *Copy and complete the cumulative frequency column*

(b) *State the:*

(i) *class width*

(ii) *modal class*

(c) *Determine the median class*

(b) *Calculate the:*

(i) *mean*

(ii) *mode*

(iii) *median*

(c) *Display the data on a histogram and use it to estimate the mode.*

(d) *Plot an ogive for the data and use it to estimate the median*

(e) *Display the data on a frequency polygon*

2. *The weights in kg of **50** students were as follows:*

Weights	***20 – 24***	***25 – 29***	***30 – 34***	***35 – 39***	***40 – 44***	***45 – 49***
Frequency	***8***	***9***	***10***	***6***	***12***	***5***

(a) *Calculate the:*

(i) *mean*

(ii) *mode*

(iii) *median*

(b) *Display the data on a histogram and use it to estimate the mode*

(c) *Display the data on an ogive and use it to estimate the median*

3. *The age distribution of **40** adults were as follows:*

Age	F	x	fx	Cumulative frequency
20 – 29	4	24·5	98	4
30 – 39	12	……..	…………	16
40 – 49	………..	44·5	356	…………
50 – 59	9	……..	…………	…………
60 – 69	………..	64·5	451·5	…………
	∑f = ……		∑fx =……..	

(a) *Copy and complete the frequency distribution table above*

(b) *State the:*

(i) *class width*

(ii) *modal class*

(c) (i) *Determine the median class*

(ii) *Calculate the mean age*

4. *The cumulative distribution table shows the marks scored by **50** students.*

Marks	Cumulative frequency
30 – 39	5

40 – 49	***13***
50 – 59	***23***
60 – 69	***39***
70 – 79	***46***
80 – 89	***50***

(a) *Draw an ogive for the above data and use it to estimate the:*

(i) *median mark*

(ii) *pass mark of the test if* ***39*** *students passed*

(iii) *number of students who scored* ***75*** *marks and above*

(b) *Form a frequency distribution table for the above data to calculate the mean mark*

5. *The ages in years of* ***40*** *students were as follows:*

12	***13***	***14***	***12***	***15***	***14***	***13***	***16***	***14***	***15***
13	***14***	***16***	***15***	***14***	***12***	***13***	***14***	***15***	***13***
15	***16***	***15***	***14***	***15***	***12***	***15***	***13***	***12***	***15***
13	***15***	***12***	***15***	***16***	***14***	***15***	***14***	***16***	***14***

(a) *Form an ungrouped frequency distribution table for the data*

(b) *Find the:*

(i) mode

(ii) median

__6.__ The weights in kg of __50__ babies in a maternity ward were as follows:

4·2	***3·1***	***2·8***	***4·0***	***2·3***	***3·7***	***3·3***	***4·4***	***2·5***	***3·0***
3·6	***4·3***	***3·2***	***2·4***	***4·1***	***3·4***	***2·7***	***4·2***	***4·8***	***2·6***
2·2	***3·0***	***4·1***	***4·6***	***3·7***	***2·9***	***4·3***	***2·0***	***3·2***	***4·0***
4·7	***2·6***	***3·8***	***2·3***	***4·0***	***3·3***	***2·7***	***4·5***	***2·4***	***3·6***
2·0	***3·5***	***2·7***	***3·2***	***2·1***	***4·2***	***3·0***	***4·1***	***2·8***	***4·7***

__(a)__ Form a frequency distribution table with class width __0·5__ starting from __2·0– 2·4__

__(b)__ Calculate the:

(i) mean using an assumed mean of __3·2__

(ii) mode

(iv) median

__7.__ The marks of __40__ students were as follows:

11	***17***	***35***	***34***	***42***	***45***	***28***	***66***
16	***21***	***14***	***36***	***41***	***31***	***49***	***37***
20	***33***	***37***	***38***	***18***	***38***	***39***	***27***
26	***28***	***40***	***33***	***43***	***32***	***29***	***47***
29	***32***	***41***	***24***	***44***	***35***	***36***	***23***

__(a)__ Form a frequency distribution table for the data starting with a class of __10– 14__

__(b)__ State the:

__(i)__ class width

(ii) modal class

(c) Determine the:

(i) mean mark

(ii) median class

(d) Display the data on a histogram and use it to estimate the mode

(e) Draw an ogive for the data and use it to estimate the median

*8. The heights in **cm** of plants in a garden were as follows:*

10.3	9.7	10.2	9.8	10.1
9.9	10.1	9.9	10.1	10.2
10.3	10.0	10.2	10.1	9.8
9.9	10.1	10.0	10.1	9.9
10.1	10.1	10.1	10.1	9.9
9.8	9.8	10.0	9.9	10.2

(a) Copy and complete the frequency distribution table below:

Time (x)	Frequency(f)	Cumulative frequency	fx
9.7	…..	1	…..
9.8	4	5	…..
9.9	…..	…..	…..
10.0	3	…..	…..
10.1	…..	…..	…..
10.2	…..	…..	…..

10.3	**.....**	**.....**	**.....**
	$\sum f = ...$		$\sum fx = ...$

(b) *Use the table to;*

(i) *State the modal height*

(ii) *Calculate the mean and median height*

9. *The marks of students in a test were as follows:*

Marks	***3***	***4***	***5***	***6***	***7***	***8***
No of students	***2***	***3***	***6***	***4***	***3***	***2***

(a) *State the modal mark*

(b)*Find the:*

(i) *mean mark*

(ii) *median mark*

10. *The ages in years of* ***100*** *students were as follows:*

Age	***12***	***14***	***16***	***18***	***20***	***22***
Frequency	***15***	***25***	***18***	***22***	***12***	***8***

Find the:

(i) *mean age*

(ii) *mode*

(iii) *median*

INCOME AND TAXATION

TERM

1. GROSS INCOME

It is the employee's salary including the allowances. It is the money before taxation is done.

2. ALLOWANCES

It is income which is not taxed because it just aids the employee

3. TAXABLE INCOME

It is the income which is taxed.

Taxable income = Gross income – allowances.

4. INCOME TAX

It is the money calculated from taxable income as per given rates.

5. NET INCOME

This is the employee's income after income tax is deducted.

NOTE

In calculating family allowance, children with a higher allowance are considered first in case all are not to benefit.

Examples.

1. According to U.R.A tax department, income tax is calculated as follows

The first shs 120,000 is tax free and the remaining income is taxed at a rate of 25%.

Find the tax payable on earned income of;

(a)shs100,000

(b)shs440,000

2. *A manager of an industry earns a gross salary of shs2,000,000 per month which includes an allowance of shs500,000 tax free. The rest of her income is subjected to an income tax which is calculated as follows*

7.5% on the first shs800,000

12.5% on the next shs500,000

20% on the next shs100,000

30% on the next shs60,000

35% on the remainder

a) Find her taxable income

b) Calculate her monthly income tax

c) Express her monthly income tax as a percentage of her gross monthly salary.

***3.** The table below shows the income tax rates of a certain country for government employees*

Taxable income	Tax
1-100,000	5
100,001-200,000	13
200,001-300,000	20
300,001-400,000	30
400,001-500,000	40
500,001 and above	45

An employee has a gross monthly income of shs 753,500. He is entitled to the following monthly allowances

Marriage and children shs 115,500

Housing and transport 10% of the gross income

Medical care of shs81,600

Insurance premium of shs25,500

Calculate the

a) taxable income

b) income tax

c) net income

***4.**Opio is an employee with a construction firm that pays him a gross annual salary of shs6,600,000. He is married with 4 children two of whom are aged 17 and 19 years respectively while the other two are aged 11and 14 years. The company pays allowance for three children* *only for every employee*

A summary of allowances is as follows

Marriage	*one-tenth of gross monthly income*
Medical	*shs30,000 per annum*
Transport	*shs2,000 per day*
Children above 15 but below 19 years	*shs1,000*
Children above 10 years but below 15 years	*shs6,000*

a)Calculate Opio's taxable income for the month of June

b) The income tax structure is as below

Taxable monthly income	*Tax rate%*
1-100,000	*10*
100,001-150,000	*15*
150,001-220,000	*20*
220,001 and above	*25*

i) Calculate the income tax paid by Opio

ii) Express the income tax paid by Opio as a percentage of his taxable income.

5. *The table below shows the tax structure on taxable income of citizens of a country*

Taxable income per annum	*Tax rate%*
First shs 80,000	*7.5*
Next shs 80,000(80,001-160,000)	*12.5*
Next shs80,000(160,001-240,000)	*20.0*
240,001-320,000	*30.0*
320,001-400,000	*36.5*
400,001-480,000	*45.0*
480,001 and above	*52.6*

A man's gross annual income is shs 964,000. The allowances include

Housing	*shs 14,500 per month*
Marriage	*one tenth of his gross annual income*
Medical	*shs50,700 per annum*
Transport	*shs10,000 per month*
Insurance	*shs68,900 per annum*

Annual family allowance for four children at the following rates

Shs3,400 for each child above 18 years, shs4,200 for each child above 10 years but below 18years andshs5,400 for each child below 9 years.

Given that he has a family of five children with three of them below the age of 8, one 16 years and the elder child 20 years.

Determine the

a)taxable income

b) income tax he pays annually as a percentage of his gross annual income.

6. *The table below shows the tax structure on taxable income of a certain working class of people*

Income(sh)per month	*Tax rate(%)*
0-30,000	*10.0*
30,001-90,000	*16.5*
90,001-190,000	*23.5*
190,001-340,000	*32.0*
340,001-500,000	*40.0*
Above 500,000	*49.5*

An employee earnssh.750,000. His allowances include

Marriage allowance	*one fifteenth of his gross monthly income*
Water and electricity	*sh 15000 per month*
Relief and insurance	*sh 180,000 per annum*
Housing allowance	*sh 40,000 per month*
Medical	*sh 300,000 per annum*
Transport allowance	*sh 36,000 per month*

Family allowance for four children only as below

For children in the age bracket 0 to 10 years shs 12,500 per child, between 10 and 18 years shs 8,250 per child and over 18 years shs 5,000 per child.

a)Calculate the man's taxable income and the income tax he pays given that he has three children, two of whom are aged between 0 and 10 years, and the other child 13 years.

b) What percentage of his gross income goes to tax.

7. *In a certain country income tax is computed after deducting the following allowances*

Type of allowance	Amount (sh)
Marriage	10,000
Single	4,000
Each child above 10 but below 20 years	3,000
Each child under 10 years	2,000

Omoja is married with 3 children two below 10 years of age and the other 12 years old. Mbili is single but has two dependents aged 11 and 15 years. Each month Omoja and Mbili earn gross incomes of sh. 130,000 and shs 120,000 respectively. The income tax is calculated as follows

Taxable income	% rate
1-10,000	20
10,001-50,000	15
50,001- above	10

a) Calculate the

i) taxable income for Omoja and Mbili

ii) income tax for Omoja and mbili

b) Express the total income tax for each man as a percentage of their respective taxable incomes.

8. *In a certain school a teacher's salary includes the following tax free allowances*

Type of allowance	Amount
Legally married teacher	Sh 10,000
Each child under 10 years	Sh 2,500
Each child above 10 years	Sh 2,000
P.T.A	Sh 50,000
Head of department/ subject	Sh 10,000
Class teacher	Sh 5,000
House master/mistress	Sh 5000
Unmarried teacher	Sh 6,000

Mr. Mugisha and Mr Ofuti are senior teachers in this school. Mr Mugisha is married with two children under 10 years and one child above 10 years. He is also a class teacher and a head of department. Mr Ofuti is single but has two dependents under 10 years and is a house master and a class teacher. Their income is subjected to PAYE (pay as you earn) monthly at the following rates

For the first sh 10,000 taxable income tax is 20 % while the rest is taxed at 15 %.

At the end of each month, Mr Mugisha's gross income was sh 150,000 and Mr Ofuti's gross income is sh 130,000.

Calculate the

a)taxable income for each teacher

b) tax paid by each teacher

c)tax paid as a percentage of the gross income for each teacher.

9. The monthly income tax system of a certain country is given as below

Basic pay(ush)	Rate
0-150,000	Free
150,001-250,000	10.0
250,001-350,000	12.5
350,001-450,000	16.0
450,001-550,000	22.5
550,001-600,000	30.5

An allowance in excess of sh 80,000 is subjected to a tax of 25 % of the monthly allowance.

Two employees A and B are such that A earns a basic monthly pay of sh 355,000 and a top up allowance of sh 185,000 per month while B earns only a basic monthly pay of sh 540,000.

a) Who of the two employees pay more monthly income tax than the other and by how much

b) Express employee A's income tax as a percentage of his monthly earnings.

Calculation of taxable income for given tax

1. The income tax structure of a certain country is as below

Taxable income (sh)	Rate(%)
1-120,000	5
120,000-220,000	10
220,001 and above	15

Calculate the taxable income for a person who paid a tax of sh 20,500.

2. The table below shows the income tax rates on taxable income in a certain country.

Taxable income	Rate(%)

20,001-90,000	*8.5*
90,001-190,000	*12.0*
190,001-305,000	*13.5*
305,001-435,000	*14.0*
435,001 and above	*17.0*

Find the taxable income for a person who paid a tax of

i)sh 13,150

ii)sh 32,800

iii) sh 62, 725

3. *An accountant in a certain company is entitled to a tax free allowance of sh 500,000. The income tax rates on taxable income are as follows*

Taxable income	*Rate(%)*
1-800,000	*7.5*
800,001-1,300,000	*12.5*
1,300,001-1,400,000	*20.0*
1,400,001-1,460,000	*30.0*
1,460,001 and above	*35.0*

Given that the accountant pays a tax of sh 174,500 , calculate his

i) taxable income

ii) gross income

iii) net income

4. *In an organization employees are given the following allowances which are tax free*

Transport *sh 80,000 per month*

Medical *sh 600,000 per annum*

Housing *sh 100,000 per month*

Insurance *sh 360,000 per annum*

Water and electricity *sh 30,000 per month*

The following tax rates apply on taxable income

Taxable income	*Rate %*
1-230,000	*Free*
230,001-320,000	*10*
320,001-420,000	*15*
420,001 and above	*20*

Find the

i) total allowance given to each employee per month

ii) gross income for an employee who paid a tax of sh 32,000

iii) net income for the employee in ii) above.

INCOME AND TAXATION

Summary:

1. GROSS INCOME: *is the income before income tax is deducted.*

2. AN ALLOWANCE: *is a nontaxable income (Tax free income). In case of a discriminative policy on children allowance, children attracting a higher allowance take up the benefit*

3. TAXABLE INCOME: *is the income to be taxed.*

Taxable income = Gross income – allowances

4. INCOME TAX: *is a tax imposed on the taxable income*

5. NET INCOME: *is the income after income tax is deducted.*

Net income = Gross income – income tax

EXAMPLES:

3. *In a certain country, income tax is calculated as follows:*

Income(Shs)per month	*Tax rate(%)*
01-30,000	*10.0*
30,001-90,000	*16.5*
90,001-270,000	*23.5*
270,001-390,000	*32.0*
390,001-500,000	*40.0*
Above 500,000	*49.5*

A man earns a gross monthly income of Shs 900,000. He is entitled to the following allowances:

Married   *one– fifteenth of the gross income*

Unmarried   *10% of the gross income*

Transport   *sh 40,000 per month*

Medical   *sh 600,000 per annum*

Housing   *sh 100,000 per month*

Insurance   *sh 360,000 per annum*

Electricity   *sh 25,000 per month*
Family allowance for only four children at the following rates:
Shs 30,000 for each child above 18 years, Shs 45,000 for each child above 10 but below 18years and Shs 60,000 for each child below 9 years.

Given that the man is married with five children of whom three are aged below 8 years, one aged 16 years and the elder child 20 years

Calculate his

(i) *taxable income*

(ii) *income tax*

(iii) *net income*

(iv) *income tax as a percentage of his gross monthly salary*

2. *The income tax of an employee is calculated as follows:*

Income(Shs)per month	Tax rate(%)
50,001– 100,000	20
100,001– 200,000	30
200,001– 400,000	35
400,001– 550,000	40
Above 550,000	45

Calculate the taxable income of an employee whose tax pay is ***Shs 89,000***

3. *The table below shows the tax rates in* ***2018***

Income(Shs)per month	Tax rate(%)
01-450,000	18
450,001-800,000	25
800,001-950,000	30
Above 950,000	40

Bob earns a gross monthly salary of ***Shs 750,000*** *which includes an allowance of* ***Shs 120,000.*** *Calculate his:*

(i) *taxable income* ***(01 mark)***

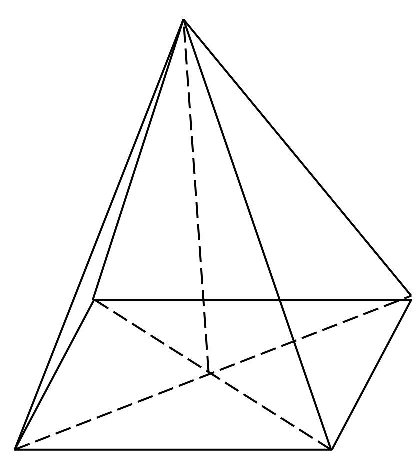
(ii) *monthly income tax* ***(03 marks)***

THREE DIMENSIONAL GEOMETRY

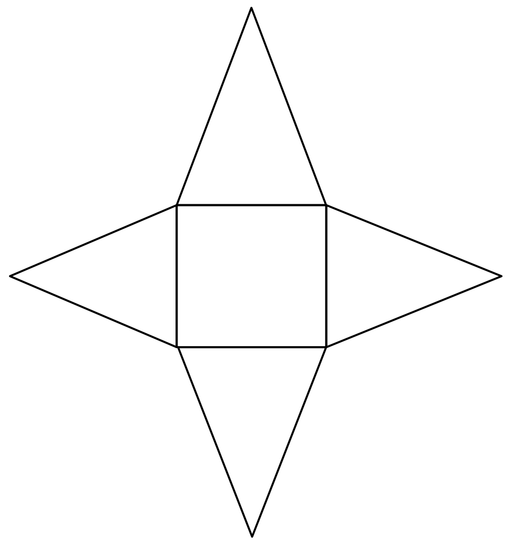
- This deals with closed figure whose interior cannot be seen e.g. Pyramids, cuboids, prisms, Tetrahedrons etc.
- Such figures consist of flat surfaces which are triangular, rectangular, pentagonal etc.
- The line where the flat surfaces meet is called an **edge**
- A flat surface is called a **plane**
- The lengths of lines can be got using Pythagoraś theorem or trigonometry of a triangle
- The angle between a line and a plane is the same as the angle between the line and its shadow on the plane.
- The angle between two planes lies at the mid-point of the common line of the planes or the common point.
- In finding the angle between two planes, the planes are bisected from the mid-point of the common line or the common point.

PYRAMIDS

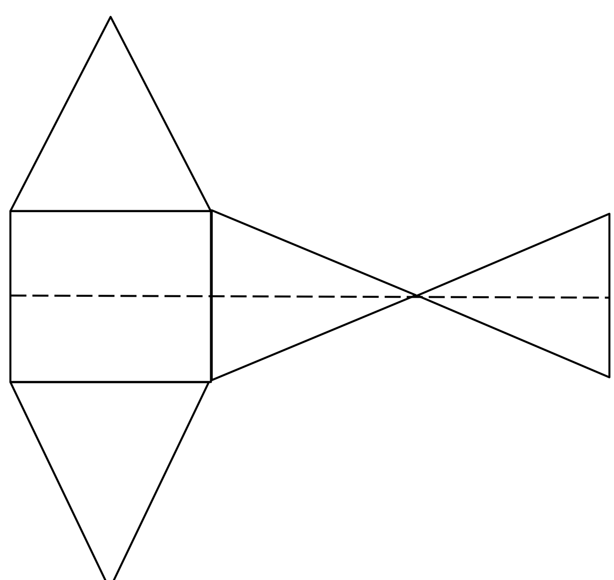
- It is a closed figure with flat surfaces resting on a flat base which is a square, rectangular, pentagon



- When opened, a pyramid gives the following possible nets

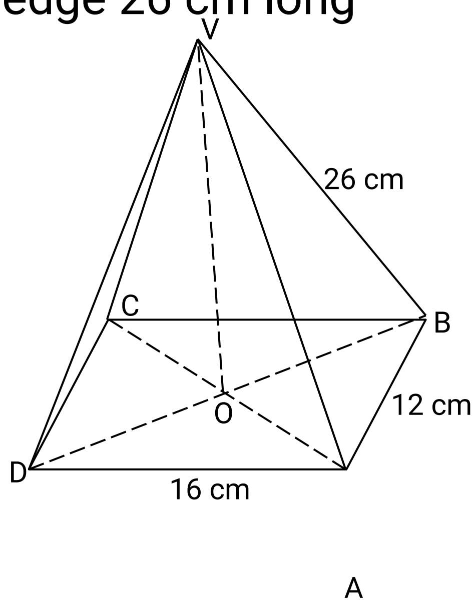


Or



Q. 1.

The figure below shows a rectangular pyramid with each slant edge 26 cm long



Calculate the;

i) Height OV

ii) Length AV

iii) Angle between line AV and the base ABCD

iv) Angle between the planes ABV and ABCD

v) Angle between planes ABV and DCV

vi) Angle between the planes ADV and BCV

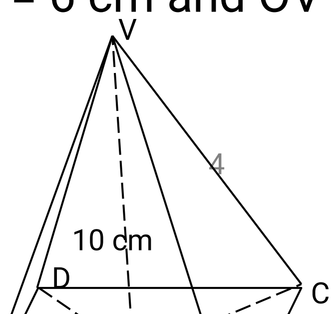
vii) Volume of the pyramid

viii) Surface area of the prism

Q. 2

The figure below shows a square based pyramid ABCDV in

which AB = BC = 6 cm and OV = 10 cm



Find the;

i)length VA

ii) angle between the AV and plane ABCD

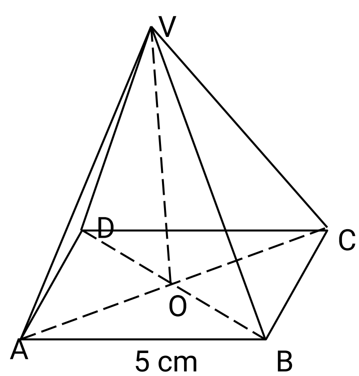
iii) angle between planes VBC and ABCD

iv) angle between planes VBC and VAD

v) volume of the pyramid

Q.3

The figure below shows a square based pyramid with equilateral triangular slant faces . Given that AB = 5 cm.



Calculate the;

i)height of the triangular faces

ii) length AC

iii) angle between VA and ABCD

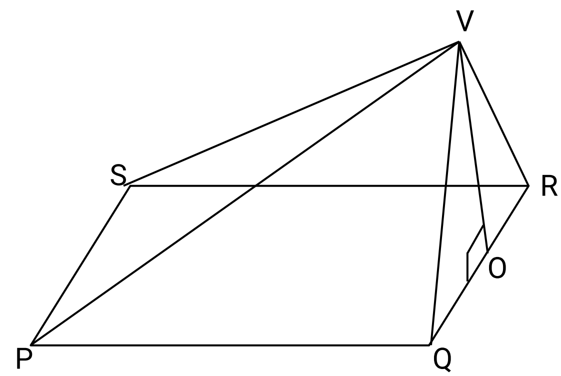
iV)angle between VAD and ABCD

V) angle between VAB and VBC

Q.4

PQRSV is a pyramid with a vertical plane VQR. PQRS is a

square with length 40 cm and VQR is an equilateral triangle



Find the;

i)lengths OV and PV

(**Hint; join P to O and use triangle POV**)

ii) angle between line PV and plane PQRS

iii) angle between the planes VSP and PQRS

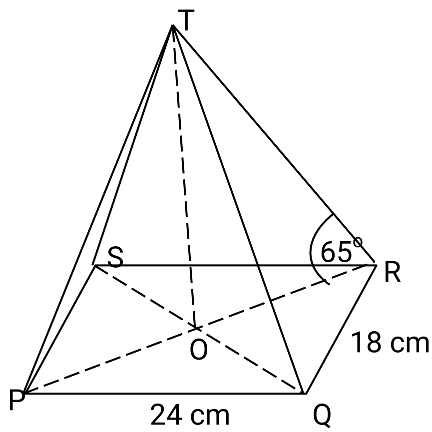
iv) total surface area

v) volume of the pyramid

Q.5

The figure shows a right pyramid with PQ = 24 cm , QR = 18

Cm and angle TRO = 65^{o}



Find the;

i) Length TO and TR

ii) Angle between the planes TQR and PQRS

iii) Angle between the planes TQR and TSP

iv) Volume of the prism

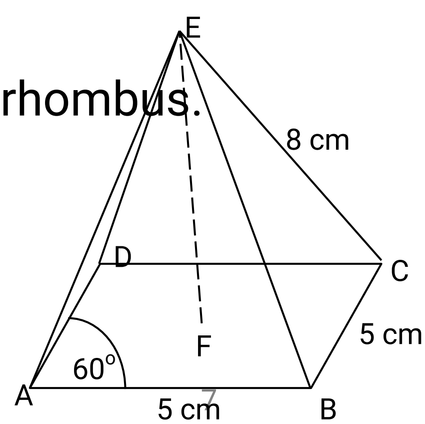
Q.6

The figure below shows a pyramid whose base ABCD is a

rhombus of side 5 cm and whose acute angle is 60^{o}.

AE = DE = CE = BE = 8 cm. F is a point of intersection of the

diagonals of the rhombus.



Find the;

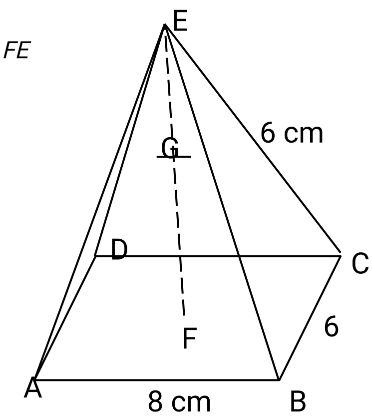
i) length EF

ii) angle AEB

iii)angle each of the slanting faces makes with base .

Q.7

The figure below shows a rectangular based pyramid of sides 8 cm by 6 cm in which AE = DE = BE = CE = 6 cm. F is the point of intersection of the diagonals and E is a point such that $FG = \frac{2}{3} FE$



Find the;

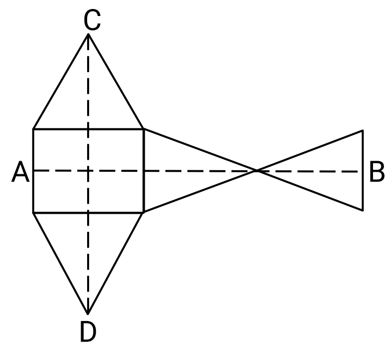
i)angle AEC

ii) length EF and AG

iii) angle which each of the slanting planes makes with the base

Q.8

The figure below shows a net of pyramid consisting of a square of side 12 cm and four congruent isosceles triangles



Given that AB = CD = 40 cm, calculate the;

(a) i)height of the vertex of the pyramid from the square

base

ii) angle between the triangular base and the base of

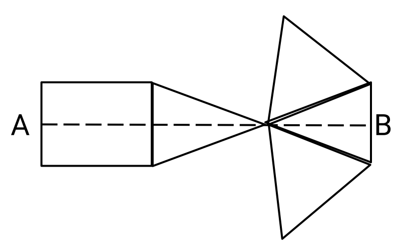
the pyramid

iii) volume of the pyramid

(b) If the pyramid is cut horizontally at a vertical height of 2.6 cm from the square base, and the upper part of the pyramid containing the vertex is thrown away, find the volume of the remaining solid.

Q.9

The figure below shows a net of a pyramid consisting a square of side 6 cm and four congruent isosceles triangles. The distance AB = 20 cm.



Calculate the;

a) Total surface area of the pyramid

b) Perpendicular height of the pyramid formed when the

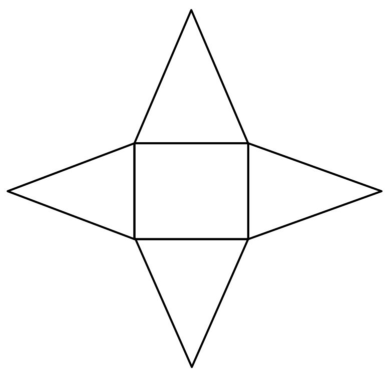
net is folded

c) angle of inclination of the triangular face to the base of

the pyramid.

Q.10

The figure below is a net of a pyramid consisting of a square of side 12 cm and four congruent isosceles triangles each of vertical height 10 cm.



Calculate the;

i) Vertical height of the pyramid

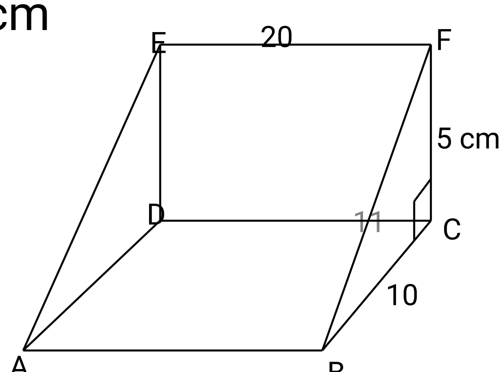
ii) Angle each sloping face makes with the base

iii) Angle between the opposite sloping faces

iv) Volume of the pyramid

Q.11

The figure shows the wedge with a rectangular base ABCD and a vertical rectangular face DCFE. EF = 20 cm, BC = cm and FC = 5 cm



Calculate the;

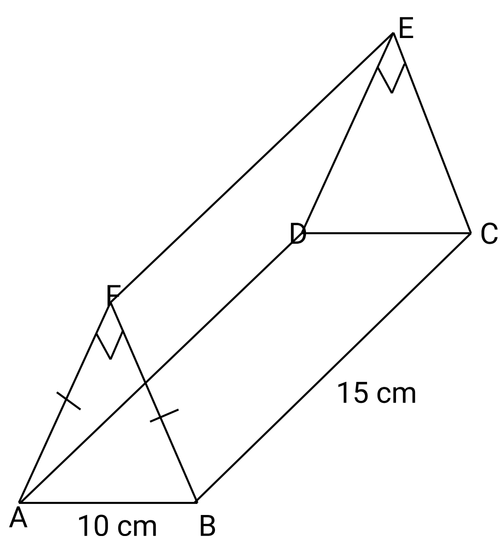
i)angle between planes ABCD and ABFE

ii) length of AF and its angle of inclination to the horizontal

iii)volume of the wedge.

Q.12

The figure below shows the a prism ABCDEF with an isosceles right angled triangle as cross section and horizontal rectangular base ABCD



Calculate the;

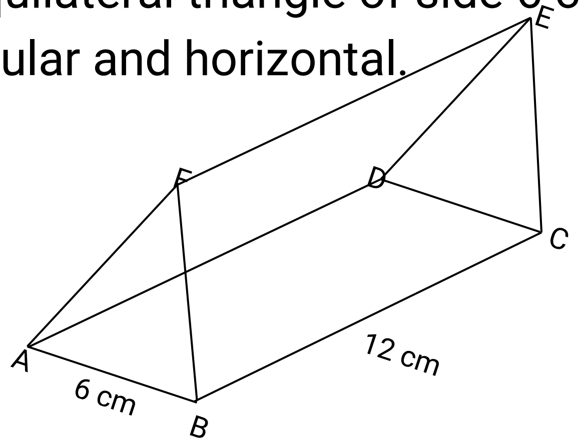
i) lengths AF and BE

ii) angle between BE and the base

iii) volume of the prism

Q.13

A right prism ABCDEF is 12 cm long and its cross section is an equilateral triangle of side 6 cm. The base ABCD is rectangular and horizontal.



Calculate the;

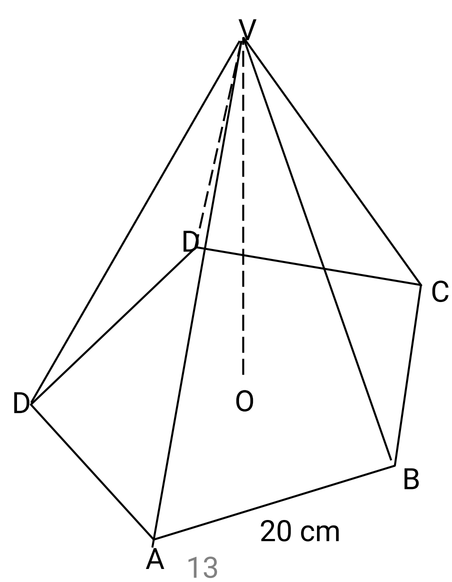
i) volume of the triangular face

ii) length EM where M is the mid-point of AB

iii)angle between EM and the base

Q.14

The diagram below shows a solid object with a regular pentagonal base of side 20 cm and centre O. The vertex V is vertically above O and VO = 30 cm.



Find the;

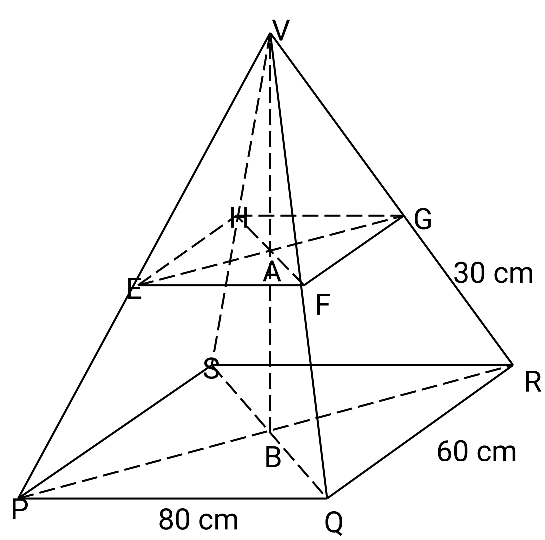
i)angle BCO

ii) length OC

iii)length of VC and the angle its inclination to the base

Q.15

The figure below shows a right pyramid with a rectangular base measuring 80 cm by 60 cm whose top part FGHEV is cut off. Given that FGHE is parallel to the base PQRS and slant length of the remaining part RG= 30 cm.



Given that 4VA = 3 BV, calculate the;

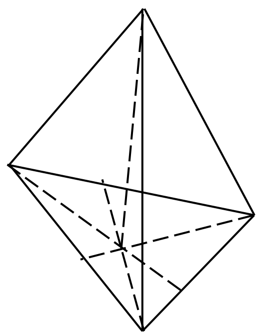
i) lengths EF, FG and VG.

ii) height BV of the pyramid

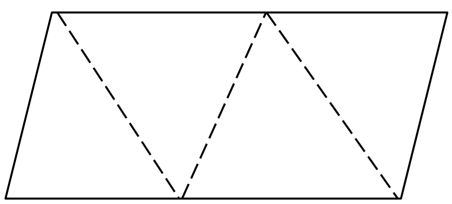
iii) volume of the frustrum PQRSHEFG.

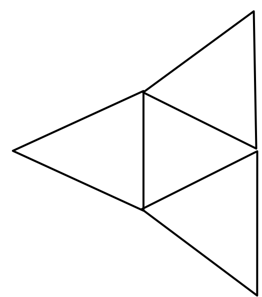
TETRAHEDRON

It is a closed figure with four triangular faces. A regular tetrahedron has all its sides equal.



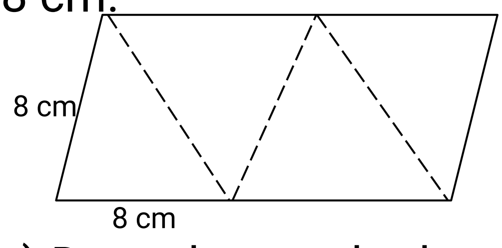
Nets of tetrahedron





Q.1

The figure below shows a net of a regular tetrahedron of side 8 cm.



a) Draw the tetrahedron

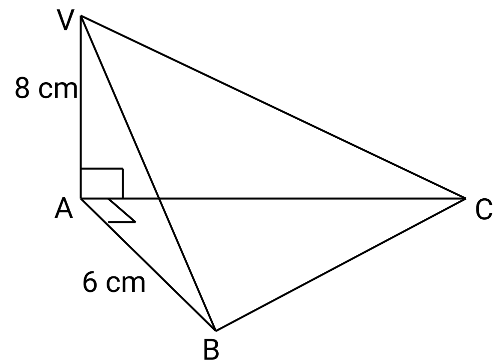
b) Calculate the;

i) height of each triangular face.

ii) total surface area of the tetrahedron

Q.2

The figure below shows a tetrahedron VABCD consisting of right angled triangular faces. AV = AC = 8 cm and AB = 6 cm.



Given that angle VAB = 90^o,

a) Calculate the;

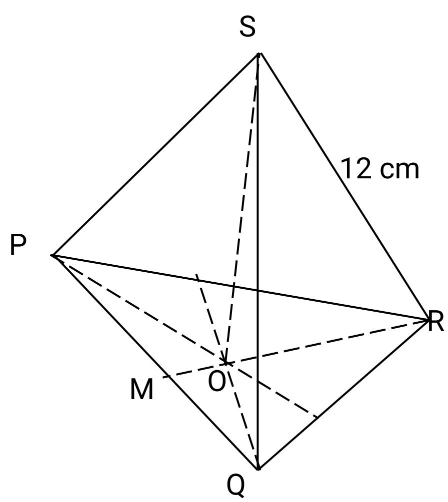
i) Lengths VB, BC and VC.

ii) Volume of the tetrahedron

b) Draw a net of the tetrahedron and hence find its total surface area.

Q.3

The figure below shows a regular tetrahedron PQRS of side 12 cm.



Calculate the;

i) lengths OM, OR and OS

ii) angle each slant edge makes with the base

iii) angle between the planes PQR and PQS

iv) volume of the tetrahedron

v) total surface area of the figure.

Q.4

A regular tetrahedron PQRV is of side 8 cm and the vertex V is directly above the center of the base.

a) Draw the tetrahedron

b)Calculate the;

i) height of V above the base

ii) angle each slant edge makes with the base

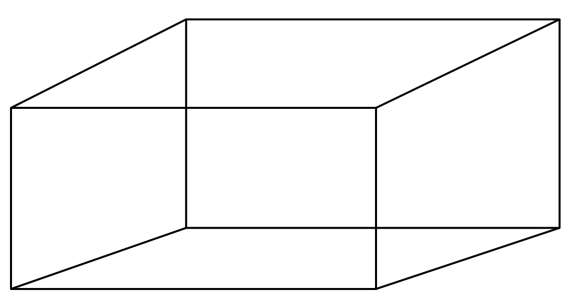
iii) angle each slant face makes with the base

iv) volume

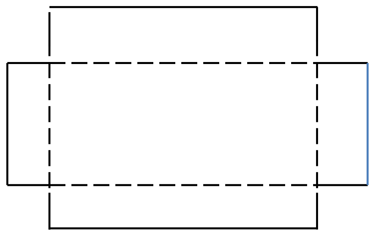
CUBOIDS AND CUBES

A cuboid is a closed figure bounded by six rectangles.

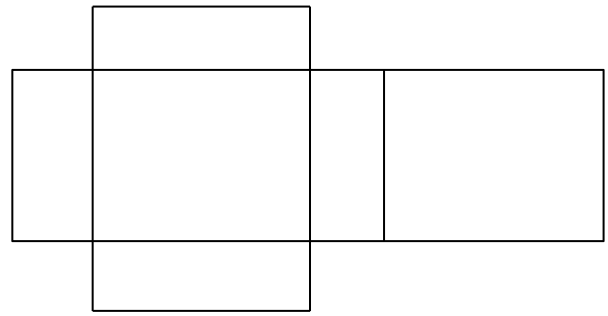
A cube is a cuboid having all its six faces as squares.



Possible nets



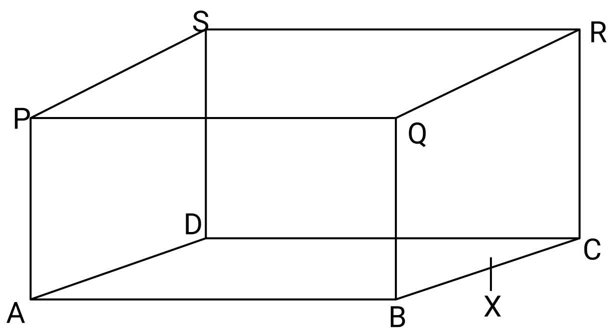
For Open cuboid



For closed cuboid

Q.1

The figure below shows a rectangular box measuring 30 cm by 40 cm by 20 cm. X is the mid-point of BC



Calculate the;

a) lengths AQ, BR , AC and AR

b) angle between AR and the base ABCD

c) angle between the planes;

i) ABRS and ABCD

ii) ADRQ and ABCD

iii) ACQ and ABCD

iv) PSX and PQRS

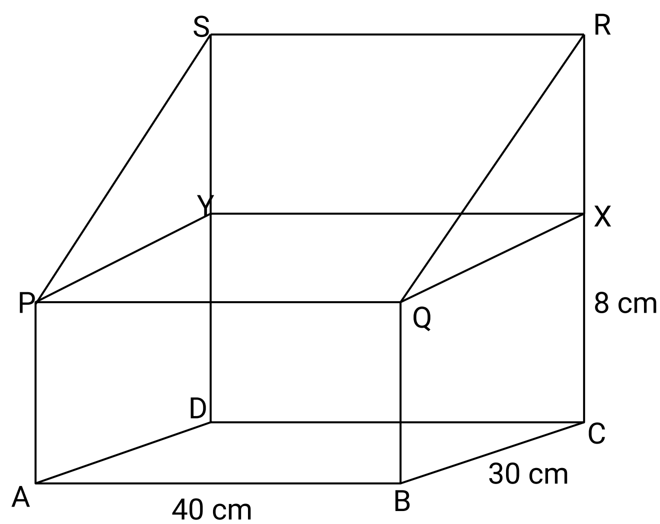
v) QCS and BCRQ

vi) DPR and ADSP

d) angle QAR

Q.2

The figure below shows a cuboid ABCDYPQX onto which a wedge PQXYSR of vertical height 10 cm sits on. Given that AB = 40 cm and BC = 30 cm, CX = 8 cm.



Find the;

a)lengths QR and PR

b) angle QRC

c) angle between the planes ABCD and PQRS

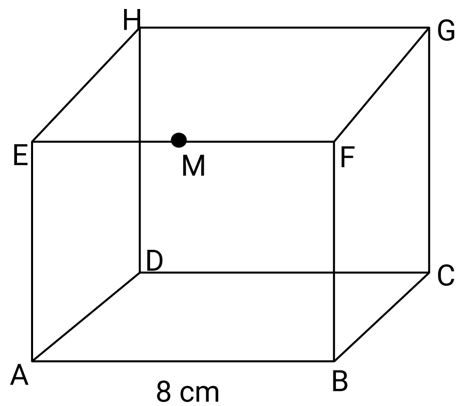
d) angle of inclination of PR to the horizontal

e) angle of inclination of AR to the horizontal

Q.3

The figure below shows a cube ABCDEFGH of side 8 cm and

EM = MF. A tetrahedron AMHE is cut off from the cube.



Calculate the;

i)area of triangle HAM

ii) angle between planes HAM and AEHD

iii) volume of the remaining part of the cube after the tetrahedron has been cut off

MATRIX TRANSFORMATIONS

Summary:

1. *A transformation describes the relation between any point and its image point*

2. *In transformation, the original shape is called the object and the transformed shape is called the image*

3. *Transformation* $\times$ *object = image* ***(T × O = I)***

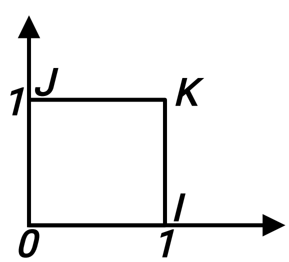
4. *From* ***T × O = I,*** *it follows that* $O = T^{-1} \times I$. *Thus the inverse of transformation maps the image back to the object.*

5. *Image area = object area* $\times$ *det of transformation. Thus the ratio of ratio of object area to image area =* ***1: det of transformation***

6. *Two successive transformations* T_1 *followed by* T_2 *can be written as a single matrix* $T = T_2 T_1$. *The multiplication is in reverse order because* T_1 *first maps the object onto an intermediate image then* T_2 *gives the final image*

7. *A unit square is one with vertices at points* ***O(0, 0), I(1, 0), J(0, 1)*** *and* ***K(1, 1)***

as shown:



The images of ***I(1, 0)*** *and* ***J(0, 1)*** *can be used to obtain certain types of transformation matrices.*

EXAMPLES:

1. *A transformation matrix* $\begin{pmatrix} 3 & 2 \\ -1 & 2 \end{pmatrix}$ *maps point* ***P(–1, 2)*** *onto its image* $P^{/}$ ***.***

*Find the**:***

(i) *coordinates of* $P^{/}$

(ii) *transformation matrix which maps* $P^{/}$ *onto* ***P***

2. *A transformation matrix* $\begin{pmatrix} 2 & b \\ c & 3 \end{pmatrix}$ *maps point* ***P(–1, 3)*** *on to its image*

$P^{/}(10,\ 8)$.

Find the values of ***b*** *and* ***c.***

3. *A transformation matrix* $\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$ *maps point* ***Q*** *onto* $Q^{/}(9,\ 6)$. *Find the*

coordinates of ***Q***

4. *A Triangle with vertices* ***A(2, 3), B(4, 5)*** *and* ***C(6, 3)*** *is mapped onto triangle*

$A^{/}B^{/}C^{/}$ *by a transformation represented by matrix* $T = \begin{pmatrix} 2 & 1 \\ 5 & 4 \end{pmatrix}$. *Find the**:***

(i) *coordinates of* $A^{/}$, $B^{/}$ *and* $C^{/}$

(ii) *area of triangle* ***ABC.*** *Hence find the area of triangle* $A^{/}B^{/}C^{/}$

(iii) *transformation matrix which maps* $A^{/}B^{/}C^{/}$ *back to* ***ABC***

5. (i) *Find the image of a unit square under the transformation matrix* $\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$.

(ii) *plot the unit square and its image in **(i)** above on the same axes.*

6. *A transformation represented by matrix* $T = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$ *maps triangle **ABC** onto its image with vertices* $A^{/}(6,\ 2),\quad B^{/}(4,\ 0)$ *and* $C^{/}(6,\ -2)$. *Find the:*

(i) *coordinates of **A, B** and **C***

(ii) *area of triangle **ABC.** Hence find the area of triangle* $A^{/}B^{/}C^{/}$

7. *A triangle with vertices **A(–4, –2), B(–2, –2)** and **C(–2, –4)** is mapped onto its image by a transformation to give* $A^{/}(6,\ 2),\quad B^{/}(4,\ 0)$ *and* $C^{/}(6,\ -2)$. *Find the:*

(i) *matrix for the transformation.*

(ii) *ratio of the area of triangle **ABC** to that of triangle* $A^{/}B^{/}C^{/}$

(iii) *transformation matrix which maps* $A^{/}B^{/}C^{/}$ *back to **ABC***

8. *An object of areas* $12\ cm^2$ *is transformed by the matrix* $\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. *Find the area of its image*

9. *A transformation matrix* $\begin{pmatrix} x & -1 \\ x+1 & x+3 \end{pmatrix}$ *maps an object of areas* $5\ cm^2$ *onto an* image of area $65\ cm^2$. find the values of **x.**

10. A transformation matrix $T = \begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix}$ maps the line **y = 3x – 5** onto line **L.** Find the equation of the line **L.**

11. Two transformations are represented by the matrices $T_1 = \begin{pmatrix} 5 & 6 \\ 1 & 4 \end{pmatrix}$ and $T_2 = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$. Find the matrix of a single transformation which represents:

(i) T_1 followed by T_2

(ii) T_2 followed by T_1

12. A Triangle with vertices **A(2, 3), B(4, 5)** and **C(6, 3)** is mapped onto triangle

$A^/B^/C^/$ after two successive matrix transformations $T_1 = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ followed by $T_2 = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$. Find the:

(i) matrix of a single transformation which maps **ABC** onto $A^/B^/C^/$

(ii) coordinates of $A^/$, $B^/$ and $C^/$

(ii) area of triangle **ABC.** Hence find the area of triangle $A^/B^/C^/$

(iii) matrix transformation that will map $A^/B^/C^/$ back to **ABC**

13. A triangle with vertices **P(0, 2), Q(1, 4)** and **R(2, 2)** is mapped on its image $P^/Q^/R^/$ by the matrix transformation $T_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Triangle $P^/Q^/R^/$ is then mapped onto $P^{//}Q^{//}R^{//}$ by another matrix transformation

$T_2 = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$. *Find the:*

(i) *coordinates of* $P^/Q^/R^/$ *and* $P^{//}Q^{//}R^{//}$

(ii) *matrix transformation that will map* $P^{//}Q^{//}R^{//}$ *back to* ***PQR***

(iii) *ratio of the area of triangle* ***PQR*** *to that of triangle* $P^{//}Q^{//}R^{//}$

EXERCISE:

1. *A transformation matrix* $\begin{pmatrix} 1 & c \\ b & -4 \end{pmatrix}$ *maps point* ***P(3, –2)*** *on to its image* $P^/(-1, 17)$. *Find the values of* ***b*** *and* ***c.***

2. *A transformation matrix* $T = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ *maps a triangle with vertices* ***P(3. 1), Q(5, 5)*** *and* ***R(7, 1)*** *onto its image. Find the area of the image triangle*

3. *A transformation matrix* $\begin{pmatrix} 2 & x \\ 1 & x^2 \end{pmatrix}$ *maps an object of areas* $1 \cdot 5\ cm^2$ *onto an*

image of area $4 \cdot 5\ cm^2$. *find the values of* ***x.***

4. *A transformation matrix* $T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ *maps the line* ***2y = x – 1*** *onto line* ***L****. Find the equation of the line* ***L.***

5. *Find the image of a unit square under the transformation matrix* $\begin{pmatrix} 2 & 1 \\ 2 & -6 \end{pmatrix}$

6. *Find the image of point* ***P(5, 3)*** *after a transformation matrix* $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ *followed by* $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$

7. *A transformation matrix maps point* ***(1, 2)*** *onto* $(-1, 4)$ *and* ***(2, 3)*** *onto* $(-1, 7)$. *Find the:*

(i) *matrix for the transformation.*

(ii) *image of point* ***P(–3, –2)*** *under the transformation above*

8. *A triangle with vertices* ***P(2, 1) Q(2, 3)*** *and* ***R(4, 1)*** *is mapped on its image* $P'Q'R'$ *by a translation* $T = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. *Triangle* $P'Q'R'$ *is then mapped onto* $P''Q''R''$ *by a matrix transformation* $T_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(i) *Find the coordinates of* $P'Q'R'$ *and* $P''Q''R''$

(ii) *Plot triangle* ***PQR*** *and its image* $P'Q'R'$ *on the same axes.*

(iii) *By joining triangle* ***PQR*** *and its image* $P'Q'R'$, *find the volume of the resulting figure formed*

9. *A transformation represented by matrix* $T = \begin{pmatrix} 4 & 6 \\ 1 & 2 \end{pmatrix}$ *maps triangle* ***ABC*** *onto its image with vertices* $A'(6, 2)$, $B'(16, 7)$ *and* $C'(22, 9)$. *Find the:*

(i) *coordinates of* ***A, B*** *and* ***C***

(ii) *area of triangle* ***ABC.*** *Hence find the area of triangle* $A'B'C'$

10. *A transformation represented by matrix* $\begin{pmatrix} 6 & -4 \\ 2 & -1 \end{pmatrix}$ *maps triangle* ***ABC*** *onto its image with vertices* $A'(8, 3)$, $B'(32, 11)$ *and* $C'(2, 2)$. *Triangle*

$A^{/}B^{/}C^{/}$ is then mapped onto $A^{//}B^{//}C^{//}$ by another matrix transformation $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Find the:

(i) *coordinates of* ***ABC*** *and* $A^{//}B^{//}C^{//}$

(ii) *matrix of a single transformation which maps* $A^{//}B^{//}C^{//}$ *back to* ***ABC***

11. *A Triangle with vertices* ***(–2, 1), Q(3, 1)*** *and* ***R(0, 3)*** *is mapped onto its image triangle by the matrix transformations* $T_1 = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$ *followed by* $T_2 = \begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$. *Find the:*

(i) *coordinates of the final image*

(ii) *matrix transformation that will map the final image back to the object*

(i) *area of the final image*

12. *A triangle with vertices* ***P(2,0), Q(1, –3)*** *and* ***R(–2, 1)*** *is mapped on its image* $P^{/}Q^{/}R^{/}$ *by the matrix transformation* $T_1 = \begin{pmatrix} 4 & 0 \\ 1 & 1 \end{pmatrix}$ *Triangle* $P^{/}Q^{/}R^{/}$ *is then mapped onto* $P^{//}Q^{//}R^{//}$ *by another matrix transformation* $T_2 = \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$. *Find the:*

(i) *coordinates of* $P^{/}Q^{/}R^{/}$ *and* $P^{//}Q^{//}R^{//}$

(ii) *matrix transformation that maps* ***PQR*** *onto* $P^{//}Q^{//}R^{//}$

(iii) *ratio of the area of triangle* **PQR** *to that of triangle* $P^{//}Q^{//}R^{//}$

14. *A Triangle with vertices* ***A(1, 3), B(2, –1)*** *and* ***C(4, 0)*** *is mapped onto its image by a matrix transformation to give* $A^{/}(10, -5)$, $B^{/}(-1, 11)$ *and* $C^{/}(4, 16)$. *Find the:*

(i) *transformation matrix*

(ii) *area of triangle* ***ABC.*** *Hence find the area of triangle* $A^{/}B^{/}C^{/}$

(iii) *transformation matrix that will map* $A^{/}B^{/}C^{/}$ *back to* ***ABC***

REFLECTION

Summary:

1. *In reflection:*

(i) *the image is formed using a mirror line*

(ii) *the image is as far behind the mirror as the object is in front of it*

(iii) *one figure is the mirror image of the other*

2. (i) *Reflection is described by stating the mirror line*

(ii) *The most common reflections can be described by a* $\mathbf{2 \times 2}$ *matrix using the images of* ***I(1, 0)*** *and* ***J(0, 1)*** *of a unit square*

EXAMPLES:

1. *Use the points* ***I(1, 0)*** *and* ***J(0, 1)*** *to find the matrix corresponding to:*

(i) *a reflection in the line* ***y = 0 (x–axis)***

(ii) *a reflection in the line* ***x = 0 (y–axis)***

(iii) *a reflection in the line* $\boldsymbol{x + y = 0}$ $\boldsymbol{(y = -x)}$

(iv) *a reflection in the line* $\boldsymbol{x - y = 0}$ $\boldsymbol{(y = x)}$

2. *A triangle with vertices* ***P(1, 4), Q(3, 2)*** *and* ***R(5, 3)*** *is mapped onto its image by a reflection in the line* $\boldsymbol{y = 0}$

(a) *Write down the matrix for the reflection*

(b) *Find the coordinates of the image of* ***PQR:***

(i) *using matrices*

(ii) *by construction*

3. *A triangle with vertices* ***P(2, 3), Q(5, 4)*** *and* ***R(5, 6)*** *is mapped onto its image by a reflection in the line* $\boldsymbol{x = 0}$

(a) *Write down the matrix for the reflection*

(b) *Find the coordinates of the image of* ***PQR:***

(i) *using matrices*

(ii) *by construction*

4. *A triangle with vertices* ***P(2, –4), Q(6, –3)*** *and* ***R(3, –1)*** *is mapped onto its image by a reflection in the line* $\boldsymbol{x - y = 0}$

(a) *Write down the matrix for the reflection*

(b) *Find the coordinates of the image of* ***PQR:***

(i) *using matrices*

(ii) *by construction*

5. *A triangle with vertices* ***P(2, 3), Q(5, 4)*** *and* ***R(5, 6)*** *is mapped onto its* *image by a reflection in the line* ***x + y = 0***

(a) *Write down the matrix for the reflection*

(b) *Find the coordinates of the image of* ***PQR:***

(i) *using matrices*

(ii) *by construction*

6. *Find the coordinates of the image of point* ***P(3, 2)*** *under a transformation* $M = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$ *followed by a reflection in a line* ***y = –x***

7. *Find the coordinates of the image of a triangle with vertices* ***A(2, 1) B(2, 3)*** *and* ***C(4, 1)*** *under a translation* $T = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ *followed by a reflection in the* ***x–*** *axis*

8.(a) (i) *Find the coordinates of the image of a triangle with vertices* ***A(1, 4) B(1, 1)*** *and* ***C(2, 1)*** *under a transformation matrix* $L = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(ii) *Draw triangle* ***ABC*** *and its image* $A^{/}B^{/}C^{/}$ *on the same axes*

(iii) *Describe the matrix transformation* ***L.*** *Hence deduce the matrix transformation which would map triangle* $A^{/}B^{/}C^{/}$ *onto triangle* ***ABC.***

(b) *Triangle* $A^{/}B^{/}C^{/}$ *is mapped onto triangle* $A^{//}B^{//}C^{//}$ *by a matrix transformation* $M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(i) *Find the coordinates of* $A^{//}B^{//}C^{//}$

(ii) *Draw triangle* $A^{//}B^{//}C^{//}$ *on the same axes in* ***a(ii)*** *above.*

(iii) *Use your graph to describe a single transformation that will map triangle*

ABC *onto triangle* $A^{//}B^{//}C^{//}$. *Hence find the single matrix transformation which maps triangle* ***ABC*** *onto triangle* $A^{//}B^{//}C^{//}$.

9. *Find the equation of the image of the line* ***y = 2x –1*** *when reflected in the line* ***x + y = 0***

10. *A triangle with vertices* ***P(3, 2), Q(1, 4)*** *and* ***R(5, 3)*** *is mapped onto its image* $P^{/}(-3, 2)$, $Q^{/}(-1, 4)$ *and* $R^{/}(-5, 3)$ *by a transformation* ***T.*** *Triangle* $P^{/}Q^{/}R^{/}$ *is then mapped onto triangle* $P^{//}Q^{//}R^{//}$ *by another matrix transformation* $M = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

(i) *Draw triangle* ***PQR*** *and its image* $P^{/}Q^{/}R^{/}$ *on the same axes*

(ii) *Describe fully the transformation which maps* ***PQR*** *onto* $P^{/}Q^{/}R^{/}$. *Hence write down the matrix for the transformation*

(iii) *Find the coordinates of* $P^{//}Q^{//}R^{//}$

(iv) *Find the single matrix of transformation which maps* ***PQR*** *onto* $P^{//}Q^{//}R^{//}$

EER:

1. *The image of* ***P(3, 2)*** *after a reflection is* $P^{/}(-2, -3)$. *Use a graph paper to construct the line of reflection and state its equation*

2. *A triangle with vertices* ***P(2, –4), Q(6, –3)*** *and* ***R(3, –1)*** *is reflected in the* *line* $\boldsymbol{x - y = 0}$ *to get triangle* $P^{/}Q^{/}R^{/}$. *Triangle* $P^{/}Q^{/}R^{/}$ *is then reflected in the line* $\boldsymbol{x = 0}$ *to get triangle* $P^{//}Q^{//}R^{//}$.

(i) *Draw the three triangles on the same axes.* ***[****Use a scale of* ***2cm*** *to* ***1*** *unit****]***

(ii) *Write down the coordinates of* $P^{/}Q^{/}R^{/}$ *and* $P^{//}Q^{//}R^{//}$.

3. *A triangle with vertices* ***P(3, 2), Q(1, 4)*** *and* ***R(5, 3)*** *is mapped onto its image* $P^{/}(3, -2)$, $Q^{/}(1, -4)$ *and* $R^{/}(5, -3)$ *by a transformation*. *Triangle* $P^{/}Q^{/}R^{/}$ *is then mapped onto triangle* $P^{//}Q^{//}R^{//}$ *by another matrix transformation* $M = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

(i) *Draw triangle* ***PQR*** *and its image* $P^{/}Q^{/}R^{/}$ *on the same axes*

(ii) *Describe fully the transformation which maps* ***PQR*** *onto* $P^{/}Q^{/}R^{/}$***.*** *Hence write down the matrix for the transformation*

(iii) *Find the coordinates of* $P^{//}Q^{//}R^{//}$

(iv) *Find the single matrix of transformation which maps* ***PQR*** *onto* $P^{//}Q^{//}R^{//}$

ENLARGEMENT

Summary:

1. (i) *Enlargement is a transformation which changes the size of an object*

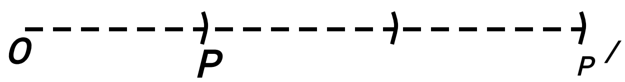
(ii) *Enlargement is described by stating its centre and the scale factor*

2. *The point about which enlargement occurs is called the centre of* *enlargement*

3. ***(i)*** *By definition, scale factor* $= \dfrac{\textit{Image distance from the centre}}{\textit{Object distance from the centre}}$

(ii) *The scale factor tells us by how much the object has been enlarged. Thus a scale factor of **3** means that the image is three times the size of the object*

(iii) *The sketch below shows how to locate the image* $P^{/}$ *of point **P** after an enlargement with centre **O** and scale factor **3:***



In the above sketch, distance $OP^{/} = 3 \times OP$

(iv) *A positive scale factor means that both the object and its image are on the same side of the centre*

(v) *A negative scale factor means that the object and its image are on the opposite sides of the centre*

4. *Under enlargement, Image area = Object area* $\times\ (\textit{scale factor})^{2}$

EXAMPLES:

1. *A triangle with vertices **P(–1, –2), Q(–3, –4)** and **R(–5, –1)** is mapped onto its image* $P^{/}Q^{/}R^{/}$ *by an enlargement with centre **C(–6, –3)** and scale factor **3.** Find the:*

(i) *coordinates of* $P^{/}Q^{/}R^{/}$

(ii) *area of triangle **PQR.** Hence find the area of triangle* $P^{/}Q^{/}R^{/}$

2. *A quadrilateral with vertices **P(3, –9), Q(5, –7), R(3, –6)** and **S(1, –6)** is mapped onto its image* $P^{/}Q^{/}R^{/}S^{/}$ *by an enlargement with centre **C(2, –4)** and scale factor **–3.** Find the:*

(i) *coordinates of* $P^{/}Q^{/}R^{/}S^{/}$

(ii) *area of quadrilateral* ***PQRS .*** *Hence obtain the area of* $P^{/}Q^{/}R^{/}S^{/}$

3. *A triangle with vertices* ***P(–6, –2), Q(–2, –2)*** *and* ***R(–2, –6)*** *is mapped onto its image* $P^{/}Q^{/}R^{/}$ *by an enlargement with centre* ***C(0, 4)*** *and scale factor* ***0·5****. Find the:*

(i) *coordinates of* $P^{/}Q^{/}R^{/}$

(ii) *area of triangle* ***PQR.*** *Hence obtain the area of triangle* $P^{/}Q^{/}R^{/}$

4. *A triangle with vertices* ***P(–6, –2), Q(–2, –2)*** *and* ***R(–2, –6)*** *is mapped onto its image* $P^{/}Q^{/}R^{/}$ *by an enlargement with centre* ***C(0, 2)*** *and scale factor* ***1·5****. Find the:*

(i) *coordinates of* $P^{/}Q^{/}R^{/}$

(ii) *area of triangle* ***PQR .*** *Hence obtain the area of triangle* $P^{/}Q^{/}R^{/}$

ENLARGEMENT ABOUT THE ORIGIN

Summary:

If the centre of enlargement is located at the origin, then the general matrix for the enlargement is $E = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$*, where* ***k*** *is the scale factor of enlargement*

EXAMPLES:

1. *A triangle with vertices* ***P(1, 1), Q(3, 1)*** *and* ***R(1, 4)*** *is mapped onto its* image $P^{/}Q^{/}R^{/}$ by an enlargement with centre **O(0, 0)** and scale factor **3.**

(a) Write down the matrix for the enlargement

(b) Find the:

(i) coordinates of $P^{/}Q^{/}R^{/}$

(ii) area of triangle **PQR .** Hence obtain the area of triangle $P^{/}Q^{/}R^{/}$

2. A triangle with vertices **P(2, 0), Q(4, 1)** and **R(2, 2)** is mapped onto its image $P^{/}Q^{/}R^{/}$ by an enlargement with centre **O(0, 0)** and scale factor **–2.**

(a) Write down the matrix for the enlargement

(b) Find the:

(i) coordinates of $P^{/}Q^{/}R^{/}$

(ii) area of triangle **PQR .** Hence obtain the area of triangle $P^{/}Q^{/}R^{/}$

3. A triangle with vertices **A(1, 0), B(0, 1)** and **R(2, 1)** is mapped onto its image by a transformation to give $A^{/}(-2, 0)$, $B^{/}(0, -2)$ and $C^{/}(-4, -2)$.

(i) Find the matrix for the transformation.

(ii) Describe the matrix for the transformation

FINDING THE CENTRE AND SCALE FACTOR OF ENLARGEMENT

Summary:

1. When the object and its image are given, the centre of enlargement is located as follows:

(i) Join up a point and its image

(ii) *Repeat for another point and its image*

(iv)*These lines meet at the centre of enlargement*

2. *To find the scale factor, use scale factor*

$$= \frac{Image \quad distance \quad from \ the \ centre}{Object \quad distance \quad from \ the \ centre}$$

EXAMPLES:

1. *Find the scale factor and centre of enlargement that maps a triangle with vertices* ***P(3, 3), Q(5, 2)*** *and* ***R(5, 4)*** *onto its image* $P^{/}(3, 7)$, $Q^{/}(7, 5)$ *and* $R^{/}(7, 9)$

2. *Find the scale factor and centre of enlargement that maps a line with end points* ***P(3, –9)*** *and* ***Q(5, –7)*** *onto its image* $P^{/}(-1, 11)$ *and* $Q^{/}(-7, 5)$

3. *Under an enlargement of scale factor* ***3,*** *the image of point* ***P(3, 4)*** *is* $P^{/}(5, 10)$. *Find the coordinates of the center of enlargement*

Soln:

Let ***(x, y)*** *be the required centre*

If scale factor $= \frac{Image \quad distance}{Object \quad distance}$

$$\Rightarrow \quad \frac{x-5}{x-3} = 3 \quad also \quad \frac{y-10}{y-4} = 3$$

$\therefore$ *x = 2, y = 1*

Required centre = ***(2, 1)***

4. *Under an enlargement of scale factor* ***–2,*** *the image of point* ***P(4, –7)*** *is* $P^{/}(1, 2)$. *Find the coordinates of the centre of enlargement*

Soln:

*Let **(x, y)** be the required centre*

$$\Rightarrow \frac{x-1}{x-4} = -2 \quad \text{also} \quad \frac{y-2}{y-{}^{-}7} = -2$$

$\therefore$ *x = 3, y = –4*

Required centre = ***(3, –4)***

5. *Under an enlargement of scale factor **–2,** point **P(4, 5)** is mapped on to $P^{/}(-2, -4)$ and **Q(3, 6)** is mapped on to $Q^{/}$. Find the coordinates of:*

(i) *the centre of enlargement*

(ii) $Q^{/}$

Soln:

(i) *Let **(x, y)** be the required centre*

$$\Rightarrow \frac{x-{}^{-}2}{x-4} = -2 \quad \text{also} \quad \frac{y-{}^{-}4}{y-5} = -2$$

$\therefore$ *x = 2, y = 2*

Required centre = ***(2, 2)***

(ii) *By graphical method,* $Q^{/}(0, -6)$

EER:

1. *Under an enlargement of scale factor **3,** the image of point **P(–2, 3)** is $P^{/}(2, 7)$. Find the coordinates of the center of enlargement.*

2. *A triangle with vertices **P(2, 4), Q(4, 3)** and **R(4, 5)** is mapped onto its*

*image $P^/Q^/R^/$ by an enlargement with centre **C(2, 2)** and scale factor **–2.***
Find the:

***(i)** coordinates of $P^/Q^/R^/$*

***(ii)** area of triangle **PQR.** Hence obtain the area of triangle $P^/Q^/R^/$*

3.** Point **Q(1, 3)** is mapped onto its image $Q^/$ by an enlargement with centre **O(0, 0)** and scale factor **2.

***(i)** Write down the matrix of enlargement.*

***(ii)** find the coordinates of $Q^/$*

***4.** A triangle with vertices **P(–2, 1), Q(3, 1)** and **R(0, 3)** is mapped on its image $P^/Q^/R^/$ by the matrix transformation $T_1 = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$. Triangle $P^/Q^/R^/$ is then mapped onto $P^{//}Q^{//}R^{//}$ by another matrix transformation $T_2 = \begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$.*

***(i)** Find the coordinates of $P^/Q^/R^/$ and $P^{//}Q^{//}R^{//}$*

***(ii)** Describe fully the single matrix transformation that **PQR** onto $P^{//}Q^{//}R^{//}$*

***(iii)** Find the area of triangle $P^{//}Q^{//}R^{//}$*

5.** A triangle with vertices **P(1, 1), Q(3, 2)** and **R(2, 4)** is mapped onto its image $P^/Q^/R^/$ by an enlargement with centre **C(–1, –1)** and scale factor **2.
Find the:

***(i)** coordinates of $P^/Q^/R^/$*

***(ii)** area of triangle **PQR.** Hence obtain the area of triangle $P^/Q^/R^/$*

*6. A triangle with vertices **P(0, 2), Q(1, 4)** and **R(2, 2)** is reflected in the line **x – y = 0** to give triangle $P'Q'R'$. Triangle $P'Q'R'$ is then mapped onto $P''Q''R''$ by an enlargement with centre **O(0, 0)** and scale factor **–2.***

***(a)** Write down the matrix for the:*

***(i)** reflection* ***(ii)** enlargement*

***(b)** Find the coordinates of $P'Q'R'$ and $P''Q''R''$.*

***(c)** Find the matrix of a single transformation which would map **PQR** onto $P''Q''R''$.*

*7. Find the scale factor and centre of enlargement that maps a triangle with vertices **P(1, 2), Q(3, 3)** and **R(0, 3)** onto its images $P'(5, 4)$, $Q'(11, 7)$ and $R'(2, 7)$*

*8. Under an enlargement of scale factor **3,** the image of point **P(–1, 2)** is $P'(9, 0)$. Find the coordinates of the center of enlargement.*

*9. A triangle with vertices **P(3, 3), Q(2, 5)** and **R(2, 2)** is mapped onto its image $P'(-6, 3)$, $Q'(-4, -1)$ and $R'(-4, 5)$ by an enlargement. Find the scale factor and centre of enlargement*

*10. Under an enlargement of scale factor **3,** the image of point **P(0, 3)** is $P'(4, 5)$. Find the coordinates of the center of enlargement.*

*11. Find the scale factor and centre of enlargement that maps a line with end points **P(2, 0)** and **Q(3, 1)** onto its image $P'(-2, 2)$ and $Q'(1, 5)$*

*12. A quadrilateral with vertices **P(3, –9), Q(5, –7), R(3, –6)** and **S(1, –6)** is mapped onto its image $P'(-1, 11)$, $Q'(-7, 5)$, $R'(-1, 2)$ and $S'(5, 2)$ by an enlargement. Find the:*

***(i)** scale factor and centre of enlargement*

(ii) *area of* ***PQRS .*** *Hence obtain the area of* $P^{/}Q^{/}R^{/}S^{/}$

ROTATION

Summary:

1. *In rotation, an object is turned about a point called the centre of rotation*

2. (i) *Rotation is described by stating its centre, angle of rotation and direction of rotation.*

3. (i) *Anticlockwise rotation is positive and clockwise rotation is negative*

(ii) *A* ***90°*** *anticlockwise rotation is called a positive quarter turn*

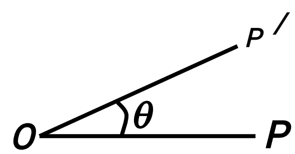
(iii) *A* ***90°*** *clockwise rotation is called a negative quarter turn*

(iv) *A* ***180°*** *rotation is called a half turn*

(v) *A rotation of* ***360°*** *(full rotation) takes the object back to its original position*

(vi) *A rotation of* ***270°*** *in any given direction is the same as a rotation of* ***90°*** *in the opposite direction*

4. *The sketch below shows how to locate the image* $P^{/}$ *of point* ***P*** *after an anticlockwise rotation through an angle* θ *about point* ***O:***



In the above sketch, distance $OP = OP^{/}$ *and* $\angle POP^{/} = \theta$

EXAMPLES:

1. *A line segment with end points* ***P(4, 5)*** *and* ***Q(0, 7)*** *is mapped onto its image by a positive quarter turn about point* ***(2, 1).*** *Find the coordinates of the image of* ***PQ***

2.** Find the coordinates of the image of a line segment with end points **P(–2, 4)** and **Q(1, 7)** after a negative quarter turn about point **(3, 2)

3.** Find the coordinates of the image of a line segment with end points **P(5, –6)** and **Q(4, –1)** after a half turn about point **(1, –2)

4.** Find the coordinates of the image of point **P(5, 4)** after a clockwise rotation of **270 °** about point **(3, 2)

5.** Find the coordinates of the image of a triangle with vertices **P(5, 4), Q(8, 4)** and **R(5, 7)** after a positive quarter turn about point **(3, 2)

ROTATION ABOUT THE ORIGIN

Summary:

*If the centre of rotation is located at the origin, then this rotation can be described by a **2 × 2** matrix using the images of **I(1, 0)** and **J(0, 1)** of a unit square*

EXAMPLES:

***1.** Use the points **I(1, 0)** and **J(0, 1)** to find the matrix corresponding to:*

***(i)** a positive quarter turn about the origin*

***(ii)** a negative quarter turn about the origin*

***(iii)** a half turn about the origin*

2.** Given that **H** denotes a half turn about the origin and **X** denotes a reflection in the x–axis, find a single matrix transformation equivalent to **XH

***3.** A triangle with vertices **P(3, 2), Q(8, 4)** and **R(5, 7)** is mapped onto its image by a positive quarter turn about the origin.*

***(a)** Write down the matrix for the rotation*

(b) *Find the coordinates of the image of* ***PQR***

4*. Find the coordinates of the image of a triangle with vertices* ***P(3, 2), Q(6, 2)*** *and* ***R(6, 5)*** *after a negative quarter turn about the origin*

5*. Find the coordinates of the image of a triangle with vertices* ***P(8, 6), Q(4, 10)*** *and* ***R(2, 6)*** *after a half turn about the origin*

6*. A triangle with vertices* ***P(2, 1), Q(4, 4)*** *and* ***R(2, 4)*** *is reflected in the line* ***y = 0*** *to get triangle* $P'Q'R'$. *Triangle* $P'Q'R'$ *is then given a negative quarter turn about the origin to get triangle* $P''Q''R''$.

(i) *Draw the three triangles on the same axes. [Use a scale of* ***2cm*** *to* ***1*** *unit]*

(ii) *Write down the coordinates of* $P'Q'R'$ *and* $P''Q''R''$.

(iii) *Use your graph to describe fully the transformation which maps* $P''Q''R''$ *back onto* ***PQR.***

FINDING THE CENTRE AND ANGLE OF ROTATION

Summary:

1. *When the object and its image are given, the centre of rotation is located as follows:*

(i) *Join two corresponding points of the object and the image shape*

(ii) *Construct a perpendicular bisector of this line*

(iii) *Repeat for another pair of corresponding points*

(iv)*The perpendicular bisectors meet at the centre of rotation*

2. *To find the angle of rotation, you measure the angle formed by joining corresponding points to the centre of rotation. This angle may be positive or negative depending on the direction of rotation*

EXAMPLES:

1. *A triangle with vertices* ***P(5, 4), Q(8, 4)*** *and* ***R(5, 7)*** *is mapped onto its image $P^{/}(1, 4)$, $Q^{/}(1, 7)$ and $R^{/}(-2, 4)$ after a rotation.*

(i) *Draw the two triangles on the same axes. [Use a scale of* ***1cm*** *to* ***1*** *unit]*

(ii) *Find the centre and angle of rotation*

2. *A line segment with end points* ***P(–6, 2)*** *and* ***Q(–8, 4)*** *is mapped onto its image $P^{/}(-2, 4)$ and $Q^{/}(0, 6)$ after a rotation. The image of* ***PQ*** *is further rotated through a half turn to give the image $P^{//}Q^{//}$.*

(i) *Draw the two lines on the same axes. [Use a scale of* ***1cm*** *to* ***1*** *unit]*

(ii) *Find the centre and angle of rotation*

(iii) *Find the coordinates of $P^{//}Q^{//}$.*

2. *A line segment with end points* ***P(–6, 2)*** *and* ***Q(–8, 4)*** *is mapped onto its image $P^{/}(-2, 4)$ and $Q^{/}(0, 6)$ under a rotation. The image of* ***PQ*** *further undergoes a clockwise rotation of* ***60°*** *to give the image $P^{//}Q^{//}$.*

(i) *Draw the two lines on the same axes. [Use a scale of* ***1cm*** *to* ***1*** *unit]*

(ii) *Find the centre and angle of rotation*

(iii) *Find the coordinates of $P^{//}Q^{//}$. State the angle formed between* ***PQ*** *and $P^{//}Q^{//}$.*

EER:

1. *Find the coordinates of the image of a triangle with vertices* ***P(8, 6), Q(4, 10)*** *and* ***R(2, 6)*** *after a half turn about point* ***(3, 2).***

2. *A triangle with vertices* ***P(1, 1), Q(2, 4)*** *and* ***R(4, 0)*** *undergoes a positive rotation of* ***90°*** *about the origin to give triangle* $P^{/}Q^{/}R^{/}$. *Triangle* $P^{/}Q^{/}R^{/}$ *is then reflected in the line* ***y = –x*** *to give triangle* $P^{//}Q^{//}R^{//}$.

(i) *Draw the three triangles on the same axes. [Use a scale of* ***1cm*** *to* ***1*** *unit]*

(ii) *Use your graph to describe fully a single transformation which is equivalent to the two successive transformations*

(iii) *Find the matrix of a single transformation which maps* ***PQR*** *onto* $P^{//}Q^{//}R^{//}$

3. *A triangle with vertices* ***P(3, 2), Q(8, 4)*** *and* ***R(5, 7)*** *is mapped onto its image by a positive quarter turn about the origin.*

(a) *Write down the matrix for the rotation*

(b) *Find the coordinates of the image of* ***PQR:***

(i) *using matrices*

(ii) *by construction*

4. *A triangle with vertices* ***P(2, 3), Q(2, 2)*** *and* ***R(4, 2)*** *is mapped onto its image* $P^{/}(-1, 2)$, $Q^{/}(0, 2)$ *and* $R^{/}(0, 4)$ *under a rotation. The image of triangle* ***PQR*** *further undergoes a rotation of* ***52°*** *to give the image* $P^{//}Q^{//}R^{//}$.

(i) *Draw the triangle* ***PQR*** *and its image* $P^{/}Q^{/}R^{/}$ *on the same axes,*

*[Use a scale of **2cm** to **1** unit]*

(ii) *Find the centre and angle of rotation*

(iii) *Find the coordinates of* $P^{//}Q^{//}R^{//}$. *State the angle formed between* ***PQR*** *and* $P^{//}Q^{//}R^{//}$.

5. (a) *Use the points* ***I(1, 0)*** *and* ***J(0, 1)*** *to find the matrix corresponding to:*

(i) *a reflection in the line* ***x + y = 0***

(ii) *a positive quarter turn about the origin*

(b) *Find the coordinates of the image when the points:*

(i) P(2, 2) *and* ***Q(4, 2)*** *undergo a reflection in the line* ***x + y = 0*** *to give* $P^{/}$ *and* $Q^{/}$

(ii) $P^{/}$ *and* $Q^{/}$ *undergo a positive quarter turn about the origin to give* $P^{//}$ *and* $Q^{//}$

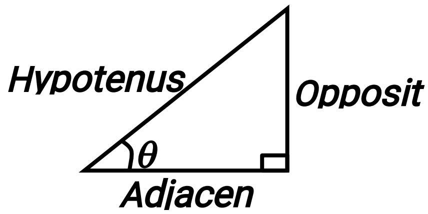
(c) *By plotting* ***PQ*** *and its images on the same axes, describe a single transformation that would map* $P^{//}Q^{//}$ *back onto* ***PQ***

TRIGONOMETRY

Summary:

1. *Trigonometry deals with the relationships between the sides and angles of a triangle*

2. *A right angled triangle has the following sides relative to angle* θ*:*



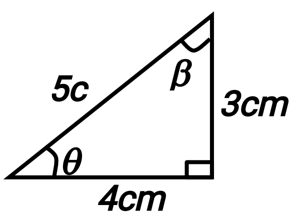
3. *For any angle* θ *in a right angled triangle:*

(i) $Sin\ \theta = \frac{Opp}{Hyp}$ **(ii)** $Cos\ \theta = \frac{Adj}{Hyp}$ **(iii)** $Tan\ \theta = \frac{Opp}{Adj}$

4. *To remember the above ratios, use the relation* ***SOH–CAH–TOA***

EXAMPLES:

1. *Study the triangle below:*



Write down the ratios for:

(i) $Sin\ \theta$ **(ii)** $Cos\ \theta$ **(iii)** $Tan\ \theta$ **(iv)** $Sin\ \beta$ **(v)** $Cos\ \beta$ **(vi)** $Tan\ \beta$

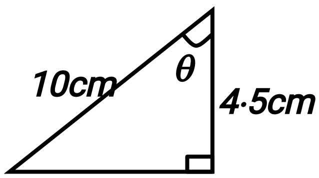
2. *Using a calculator, find the value of:*

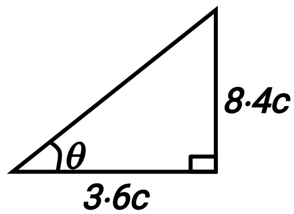
(i) Sin 30° (ii) Cos 45° (iii) Tan 60° (v) Sin 50° (vi) Cos 75° (vii) Tan 45°

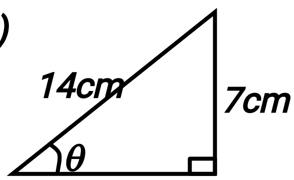
3. *Using a calculator, find the value of* $\boldsymbol{\theta}$ *if:*

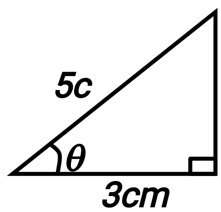
(i) $\text{Sin } \theta = 0 \cdot 5$ **(ii)** $\text{Sin } \theta = 0 \cdot 8039$ **(iii)** $\text{Cos } \theta = 0 \cdot 5$ **(iv)** $\text{Cos } \theta = 0 \cdot 6519$
(v) $\text{Tan } \theta = 1$ **(vi)** $\text{Tan } \theta = 0 \cdot 4738$

4. *Find the size of the angle marked* $\boldsymbol{\theta}$ *in the following triangles:*

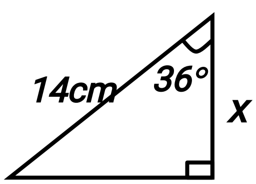
(i)


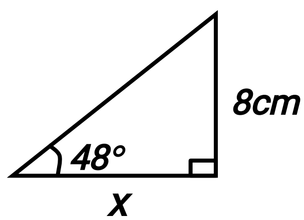
(ii)


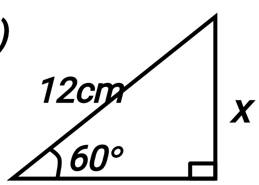
(iii)


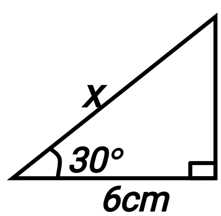
(iv)


5. *Find the length of the side marked* **x** *in the following triangles:*

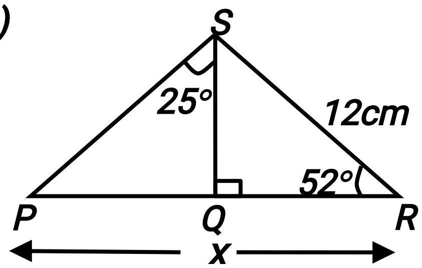
(i)


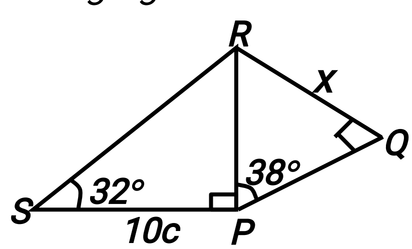
(ii)


(iii)


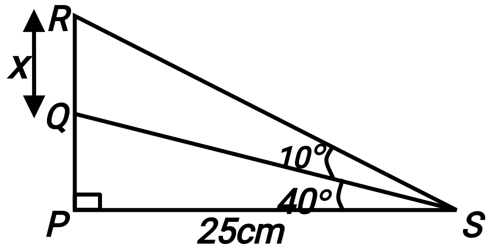
(iv)


6. *Find the length marked* **x** *in the following figures:*

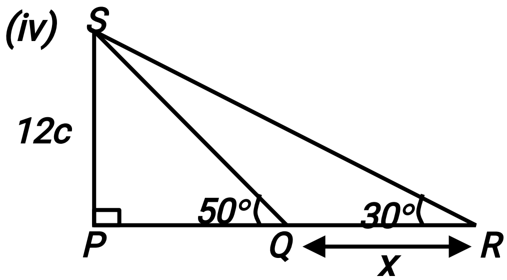
(i)


(ii)


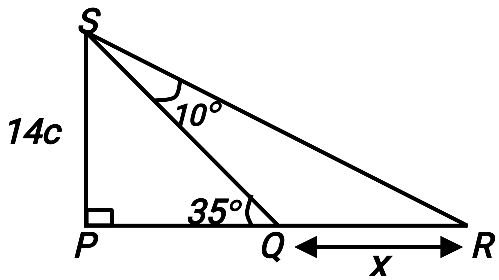
(iii)



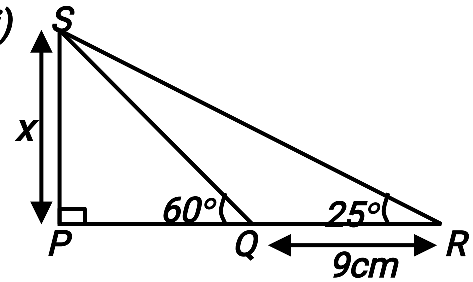
(iv)



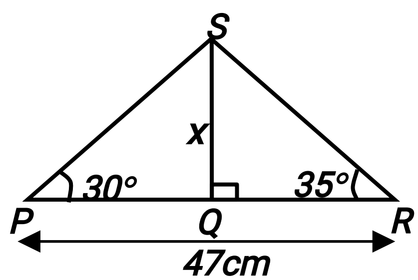
(v)



(vi)



(vii)



*7. A ladder **12m** long leans against a wall and makes an angle of **30°** with the wall. Find:*

(i) how high up the wall does the ladder reach

(ii) how far from the wall is the foot of the ladder

*8. Find the area of a rectangle whose diagonal is **10cm** long and makes an angle of **55°** with one of the sides*

*9. An isosceles triangle has a base of **16cm** and a vertical angle of **64°**. Find the height and area of the triangle*

*10. Find the area of a regular pentagon of side **10cm***

***Hint:** Divide the figure into **5** triangles and find the angle of each at the centre Required area = 5 × area of one triangle*

__12.__ Find the length of the shadow of a vertical pole __60cm__ tall when the sun is at __56°__ to the horizontal

__13.__ A ship sails __35km__ on a bearing of __042°.__ Calculate:
__(i)__ how far east has it travelled
__(ii)__ how far north has it travelled

__14.__ A ship sails __35km__ on a bearing of __243°.__ Calculate:
__(i)__ how far south has it travelled
__(ii)__ how far west has it travelled

__15.__ A ladder __9m__ long leans against a wall with its foot __5m__ away from the wall. Find the angle between the ladder and the wall

__16.__ An isosceles triangle has sides of length __8cm, 8cm__ and __5cm.__ Find the angle between the two equal sides

__17.__ Find the acute angle between the diagonals of a rectangle whose sides are __5cm__ and __8cm__

__18.__ A man walks from town __P 9 km__ due north then __12km__ due east to town __Q.__ Calculate the distance and bearing of __P__ from __Q__

__19.__ A boat sails __15km__ on a bearing of __000°.__ It then sails __8km__ due East. Calculate the distance and bearing of the ship from its starting point

__20.__ Two ships set off from port __P__ at the same time. One ship sails __8km__ on a

bearing of __030°__ to reach point __Q__ and the other ship sails __15km__ on a bearing

of __120°__ to reach point __R.__ Calculate the:

__(i)__ distance and bearing of __R__ from __Q__

__(ii)__ area of the figure bounded by __P QR__

EER:

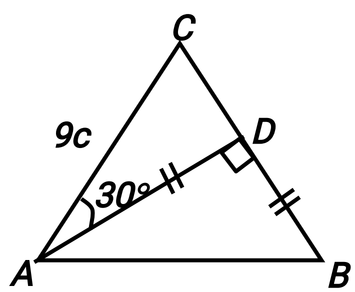
1. A road is inclined at ***30°*** *to the horizontal. Find the vertical height climbed when a car travels* ***800m*** *up the road*

2. The stretched string of a kite is ***120m*** *long and makes an angle of* ***30°*** *with the horizontal. Find the height of the kite above the ground*

3. Find the volume of a cone whose vertical angle is ***60°*** *and slant side* ***14cm*** *long*

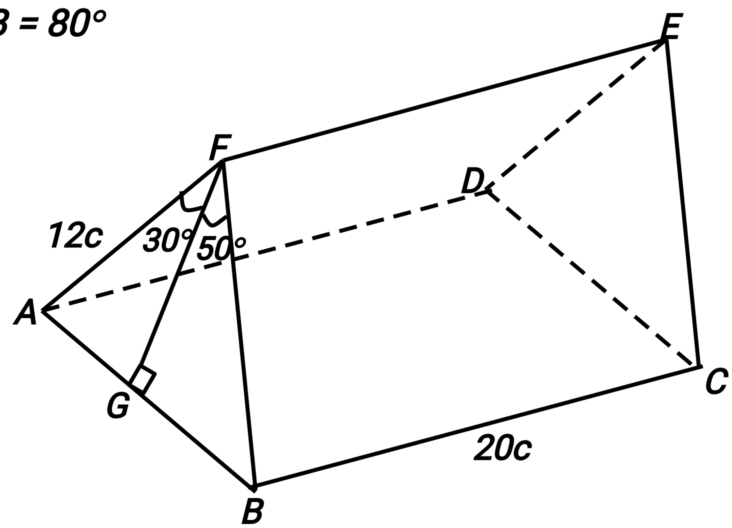
4. Find the area of a regular nonagon of side ***10cm***

5. In the figure below, ***AD*** *is perpendicular to* ***BC, AD = DB, AC = 9cm*** *and angle* ***CAD = 30°***



Find the length of ***AB***

6. The figure below shows a prism ***ABCDEF*** *of uniform triangular cross–section* ***ABF*** *in which* ***FG*** *is perpendicular to* ***AB*** *such that* ***∠AFG = 30°, ∠AFB = 80°***



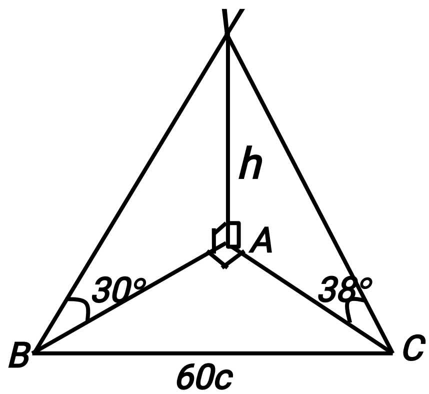
*Given that **AF = 12cm** and **BC = 20cm,** find the volume of the prism*

***6. ABCD** is a quadrilateral in which **AB = 6cm, AC = 10cm, CD = 5cm,** angles **ABC** and **CDA** are **90°** each. Calculate the:*

*(**i**) length of **BC***

*(**ii**) size of angle **ACD***

7.** In the figure below, the angles **VAC, VAB** and **BAC** are all **90°**. Find the value of **h,** if **BC = 60cm

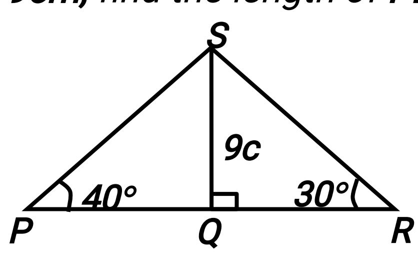


***8.** A ladder **13m** long leans against a wall with its foot **5m** away from the wall. Find:*

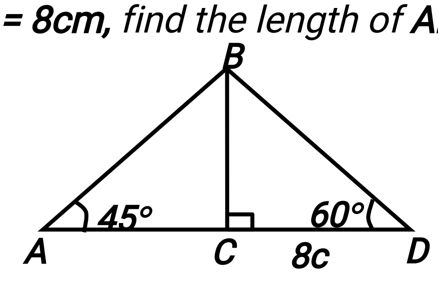
*(**i**) how high up the wall does the ladder reach*

(ii) the angle between the ladder and the wall

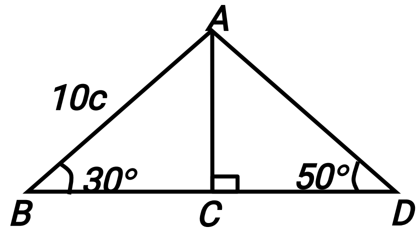
***9.** If **SQ = 9cm,** find the length of **PR** in the diagram below*



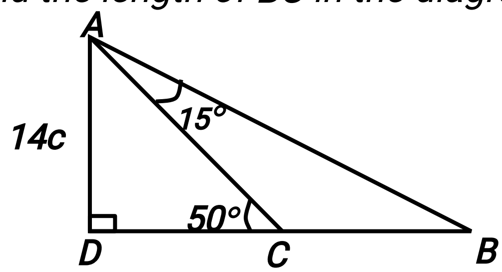
***10.** If **CD = 8cm,** find the length of **AB** in the diagram below*



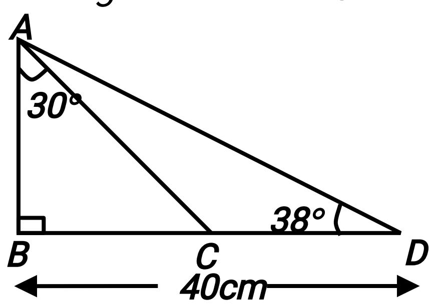
11. *If **AB = 10cm,** find the length of **AD** and **BD** in the diagram below*



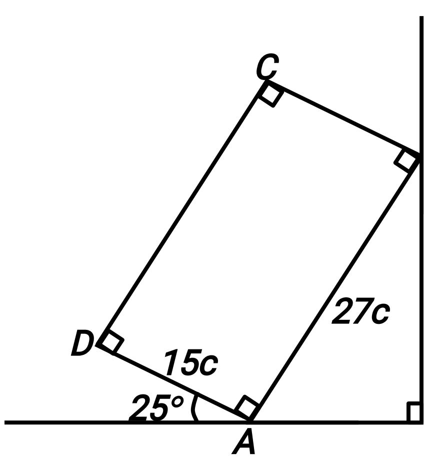
12. *If **AB = 14cm,** find the length of **BC** in the diagram below*



13. *If **BD = 40cm,** find the length of **AB** and **CD** in the diagram below*

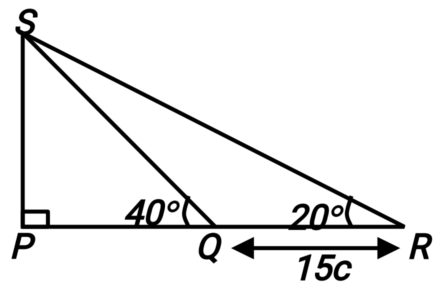


14. *A rectangular piece of cardboard measuring **27cm** long and **15cm** wide rests against a vertical wall as shown below*



*Given that angle **DAY = 25°,** find the height of **C** above the ground*

***15.** If **QR = 15cm,** find the length of **PS** and **PQ** in the diagram below*

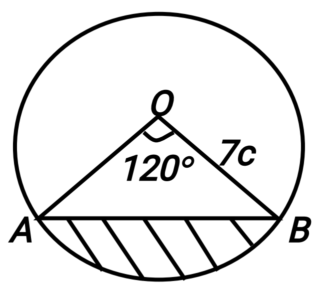


***16.** A regular hexagon of side **8cm** form the cross section of a prism **20cm** long. Find the:*

***(i)** area of the cross section of the prism*

***(ii)** volume of the prism*

17.** In the figure below, chord **AB** subtends an angle of **120°** at the centre **O** of the circle whose radius is **7cm



Find the:

***(i)** shortest distance of chord **AB** from the centre*

***(ii)** perimeter of the shaded segment*

***(iii)** perimeter of the region enclosed between chord **AB** and the major arc*

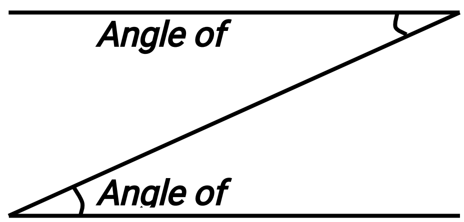
ANGLES OF ELEVATION AND DEPRESSION

Summary:

(i) *Angle of elevation is the angle above the horizontal to see an object upwards*

(ii) *Angle of depression is the angle below the horizontal to see an object downwards*

(iii) *The above angles are illustrated as follows:*



EXAMPLES:

1. *The angle of elevation of the top of the tower from a point* ***12m*** *away from its foot is* ***60º.*** *Calculate the height of the tower*

2. *The angle of depression of a boat from the top of a tower* ***25m*** *high is* ***30º.*** *Find how far is the boat from the foot of the tower*

3. *A man* ***1·5 m*** *tall is* ***20 m*** *away from a tower* ***30 m*** *high. Find the angle of elevation of the top of the tower*

4 *When a man* ***1·8 m*** *tall is* ***12 m*** *away from a tower, the angle of elevation of the top of the tower is* ***30º.*** *Find the height of the tower.*

5. *From a point* ***40m*** *away from the foot of the building, the angles of elevation of the top and bottom of a flagpole on top of a building are* ***58º*** *and* ***50º*** *respectively.   Find the:*
(i) *height of the building*

(ii) *height of the flagpole*

6. *The angles of elevation of the top of the tower from points **P** and **Q 15m** apart on the same side of the tower are **60º** and **40º** respectively. Calculate the:* ***(i)*** *height of the tower*
(ii) *distance of **P** from the foot of the tower*

7. *The angles of elevation of the top of the tower from points **P** and **Q 54m** apart on either side of the tower are **45º** and **30º** respectively. Calculate the:*
(i) *height of the tower*
(ii) *distance of **P** from the foot of the tower*

8. *Building **A 12·5 m** high is **43 m** away from building **B**. When a man **1·5 m** tall stands on top of **A** and **B** respectively, the angles of depression of a point **P** between the buildings are **30º** and **40º**. Find the height of building **B**.*

9. *From the top of a building **100m** high the angles of depression of the top and bottom of a tower are **30º** and **60º** respectively. Find the height of the tower*

10. *From the top of a tower **50m** high the angles of elevation and depression of the top and bottom of a building are **20º** and **27º** respectively. Find the height of the building*

11. *From the bottom and top of a tower **60m** high the angles of elevation of the top of a building are **60º** and **30º** respectively. Find the height of the building*

12. *A flag mast slants towards the west at an angle of **13º** the vertical. From a point **Q** to the east and **20 m** away from the foot, **F** of the mast, the angle of elevation of the top **T** of the mast is **35º**. From another point **R** to the west of the mast, the angle of elevation of the top **T** is **22º**.If **Q, F** and **R** are on level ground, Find the:*

(i) *vertical distance of the top **T** from the ground*

(ii) *distance of the foot of the mast **F** from **R***

(iii) *length* ***TF***

EER:

1. *The angle of depression of a boat from the top of a tower* ***40m*** *high is* ***30º.*** *Find how far is the boat from the foot of the tower*

2. *Find the length of the shadow of a vertical pole* ***60m*** *tall when the angle of elevation of the sun is* ***56° .***

3. *The angle of elevation of the top of a tower to a man* ***1·7 m*** *tall and* ***20 m*** *away from the tower is* ***43º.*** *Find the height of the tower.*

4. *The angles of elevation of the top of the tower are* ***30º*** *and* ***50º*** *from two points* ***10m*** *apart on the same side of the tower. Find how tall is the tower*

5. *The shadow of a vertical post increases by* ***10m*** *when the angle of elevation of the sun changes from* ***45°*** *to* ***30° .*** *Find the height of the post.*

6. *The angles of elevation of the top of a tower* ***80m*** *high to two men standing on either sides of the tower are* ***45º*** *and* ***60º*** *respectively. Find the distance between the two men*

7. *Find the height of a vertical post that casts a shadow* ***20m*** *long when the angle of elevation of the sun is* ***53° .***

8. *The angle of elevation of the top of a tower* ***50m*** *high from the foot of a hill is* ***30º*** *and angle of elevation of the top of a hill from the foot of a tower is* ***60º.*** *Calculate the height of the hill*

9. *From the top of a tower* ***90m*** *high, the angles of depression of two ships on either sides of the tower are* ***30º*** *and* ***45º.*** *Find the distance between the two ships*

10. *From the top of a house* ***40m*** *high, the angles of depression of the top and bottom of a tower are* ***30º*** *and* ***60º.*** *Find the height of the tower*

__11.__ From the top of a house __80m__ high, the angles of elevation and depression of the top and bottom of a hill are __60º__ and __30º__ respectively. Find the height of the hill

__12.__ On a shore running from east to west are two ports __P__ and __Q__ which are __18km__ apart. Town __R__ on an island on the same level as __P__ and __Q__ is on a bearing of __230°__ from __P__ and __140°__ from __Q__ respectively. A pilot flying a plane above port __P__ observes town __R__ at an angle of depression of __6°__. Calculate the:

__(i)__ distances __PR__ and __QR__

__(ii)__ vertical height of the plane above __P__

__(iii)__ angle of elevation of the plane from port __Q__

__13.__ Three towns __A, B__ and __C__ lie on the same level ground. Town __B__ is __15km__ away from town __C.__ The bearings of towns __B__ and __C__ from __A__ are __060°__ and __150°__ respectively. The bearing of __C__ from __B__ is __200°__. To a pilot flying an aircraft above __A,__ the angle of depression of __C__ is __7·5°__. Calculate the:

__(i)__ distances __AB__ and __AC__

__(ii)__ vertical height of the aircraft above __A__

__(iii)__ angle of elevation of the aircraft from __B__

__14.__ From the top of a tower __50m__ high, the angles of depression of two boats are __45º__ and __30º__ respectively. Find the distance between the boats, if they are: __(i)__ on the same side of the tower

__(ii)__ on either sides of the tower

__15.__ The angle of elevation of the top of a tower from a point __P__ due south of the tower is __38º__ and from another point __Q__ due east of the tower is __29º__. Find the height of the tower, if distance __PQ = 50m.__

TRIGONOMETRY OF SPECIAL ANGLES

Summary:

(i) *The exact trigonometric ratios of* ***30° 45°*** *and* ***60°*** *can be obtained without a calculator*

(ii) *An equilateral triangle split into two right angled triangles can be used to work out the sine, cosine and tangent of* ***30°*** *and* ***60°***

(iii) *A right angled isosceles triangle can be used to work out the sine, cosine and tangent of* ***45°***

EXAMPLES:

1. *Without using tables or a calculator, find the value of:*
(i) Sin 30° (ii) Cos 30° (iii) Tan 30° (v) Sin 60° (vi) Cos 60° (vii) Tan 60° Soln
Hint: *Use an equilateral triangle with sides* ***2*** *units long*

2. *Without using tables or a calculator, find the value of:*
(i) Sin 45° (ii) Cos 45° (iii) Tan 45°

Soln
Hint: *Use a right angled isosceles triangle with two sides of unit length*

3. *Without using tables or a calculator, find the value of:*
(i) $3\cos^2 30^\circ + 7\sin^2 30^\circ$ ***(ii) 2Sin 30° + 3Cos 60° − Tan 45°***

(iii) $\left(\frac{\sin 60^\circ}{\sin 30^\circ} + \tan 60^\circ\right)^2$

4. Use the fact that $Cos30^{\circ} = \frac{\sqrt{3}}{2}$*, to express* $\frac{1 + Cos30^{\circ}}{1 - Cos30^{\circ}}$ *as a surd and simplify*

EER:

1. *With the help of an equilateral triangle and a right angled isosceles triangle, copy and complete the table below:*

θ	30°	45°	60°
$Sin\theta$			
$Cos\theta$			
$Tan\theta$			

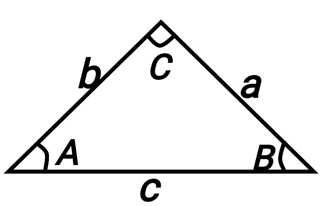
2. *Express as a surd and simplify:* $\frac{1 + Cos30^{\circ}}{1 - Sin60^{\circ}}$

3. *With the help of an equilateral triangle, show that* $\sin 60^{\circ} = \frac{\sqrt{3}}{2}$*. Hence without using tables or calculators, find the value of* $\left(\frac{Sin\ 60^{\circ}}{Sin\ 30^{\circ}} + tan\ 60^{\circ}\right)^{2}$

THE SINE AND COSINE RULE

Summary:

1. *The sine and cosine rules are used to solve problems involving any triangle* ***2.*** *A general triangle has the following sides relative to its angles:*



3. (i)*The sine rule relates the sides and angles of any triangle as follows:*

$$\frac{a}{Sin\ \ A} = \frac{b}{Sin\ \ B} = \frac{c}{Sin\ \ C}$$

(ii)*The sine rule is used when we are given an angle and its opposite side*

4. (i)*The cosine rule relates the sides of any triangle and one of its angles as follows:*

$$a^2 = b^2 + c^2 - 2bcCosA$$

$$b^2 = a^2 + c^2 - 2acCosB$$

$$c^2 = a^2 + b^2 - 2abCosC$$

(ii)*The cosine rule is used when we are given two sides and the included angle or three sides*

5. *The area of any triangle* $= \frac{1}{2}abSin\ \ C$ *Or* $\frac{1}{2}bcSin\ \ A$ *Or* $\frac{1}{2}acSin\ \ B$

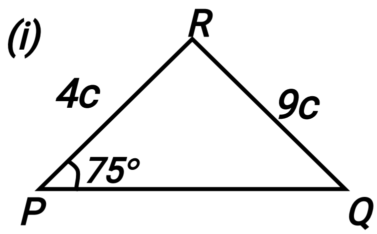
6. *In any triangle:*
(i) *The three angles add up to* ***180°***
(ii) *The largest angle is always opposite the longest side*
(ii) *The smallest angle is always opposite the shortest side*

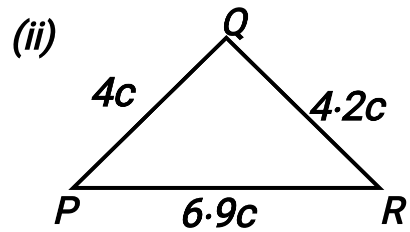
7. *The radius of a circle circumscribing any triangle is obtained using the relation:*

$$\frac{a}{Sin\ \ A} = \frac{b}{Sin\ \ B} = \frac{c}{Sin\ \ C} = 2R$$

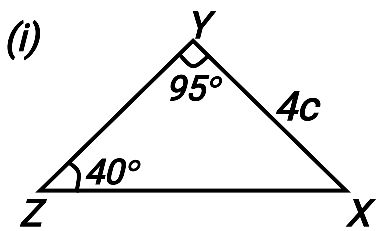
EXAMPLES:

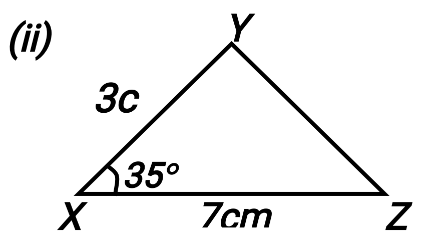
1. *Find the size of angle* ***PRQ*** *in the following diagrams:*





2. *Find the length of side* ***YZ*** *in the following diagrams:*

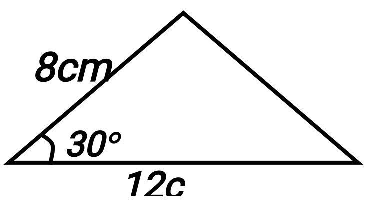




The angles in a triangle add to ***180°***

$\Rightarrow$ ***X = ∠ 45°***

3. *Find the area of the given triangle below:*



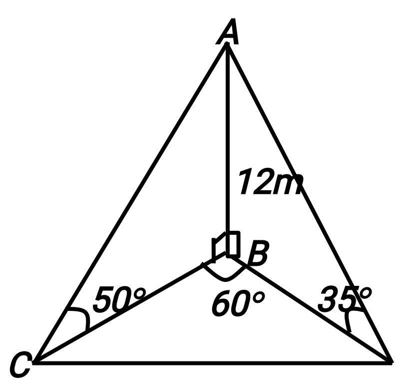
4. *A triangle has sides of length* ***3cm, 5cm*** *and* ***7cm.*** *Find the:*

(i) *size of its largest angle*

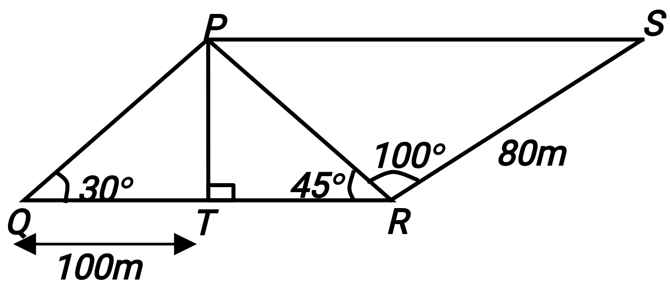
(ii) *area of the triangle*

(iii) *radius of the circle circumscribing the triangle.*

5. *Find the length of* ***CD*** *in the given figure below*



6. *The figure* ***PQRS*** *below represents a quadrilateral piece of land divided into three triangular plots such that* ***QT = 100m, RS = 80m,*** *angle* ***PQT = 30º,*** *angle* ***PRT = 45º*** *and angle* ***PRS = 100º***



Calculate the:

(i) *length of* ***PT*** *and* ***PS*** *correct to* ***4*** *significant figures*

(ii) *perimeter of the land*

(iii) *area of the land*

7. *The points* ***P,Q*** *and* ***R*** *are on level ground. A vertical flagpole* ***ST*** *stands between* ***P*** *and* ***Q*** *such that* ***Q*** *is* ***15 m*** *away from* ***S****, the base of the pole. The angles of elevation of* ***T*** *from* ***P*** *and* ***Q*** *are* ***48º*** *and* ***36º*** *respectively. If angle* ***PQR = 35º*** *and* ***QR = 13m,*** *calculate the:*

(i) *height of the flagpole* ***ST,***

(ii) *length* ***PQ,***

(iii) *angle of elevation of* ***T*** *from* ***R.***

8. *Two ships set off from port* ***P*** *at the same time. One ship sails* ***70km*** *on a*

bearing of ***050°*** *to reach point* ***Q*** *and the other ship sails* ***150km*** *on a bearing*

of ***110°*** *to reach point* ***R.***

(a) *Calculate the:*

(i) *distance and bearing of* ***R*** *from* ***Q***

*(ii) area of the figure bounded by **P QR***

*(b) If both ships take **t** hours to reach their destination and the speed of the*

faster ship is $60kmh^{-1}$*, find the:*

*(i) value of **t***

(ii) speed of the slower ship

***9.** Port **B** is **25 km** east of port **C**. A navigator observes that the bearing of **C** from*

*his ship is **310º** and that of **B** is **018º**.*

(a) Calculate the:

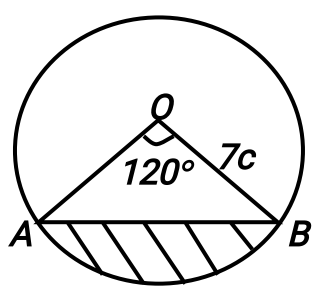
*(i) distance and bearing of the ship from **B***

*(ii) distance and bearing of the ship from **C***

(b) If the ship begins to sail at a speed of $10\ kmh^{-1}$ *on the bearing of **240º**,*

*determine the distance and bearing of the ship from **C** after **48 minutes.***

10.** In the figure below, chord **AB** subtends an angle of **120°** at the centre **O** of the circle whose radius is **7cm

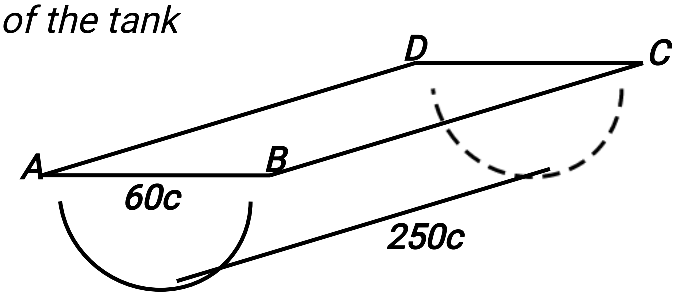


Find the:

*(i) length of chord **AB***

(ii) area of the shaded segment

***11.** The figure below shows a water trough cut from a horizontal cylindrical tank of length **250cm** and radius **70cm. AB** and **CD** are chords **60cm** long and below the centre of the tank*



Find the:

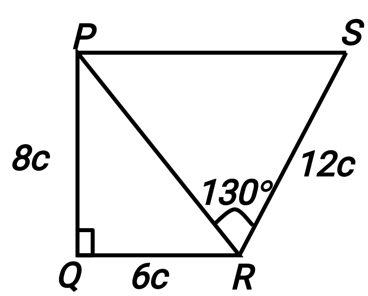
(i) area of the cross section of the trough

(ii) volume of water in litres required to fill the trough

EER:

1.** Calculate the area of triangle **ABC** in which **AB = 5cm, AC = 4cm** and angle **BAC = 150°.

2.** In the figure below, **PQ = 8cm, QR = 6cm, RS = 12cm,** angle **PQR = 90°** and angle **PRS = 130°.



*Calculate the area of the quadrilateral **PQRS***

3.** In triangle **ABC,** angle **BAC = 120°, BC = 20cm** and **AC = 8cm.** Find the* *size of angle* ***ABC

4*. A boat sails* ***7km*** *on a bearing of* ***306°*** *and then a further* ***11km*** *on a bearing of* ***070°.*** *Calculate the distance and bearing of the return journey*

5*. A triangle has sides of length* ***4cm, 4·2cm*** *and* ***6·9cm.*** *Find the:*
(i) *size of its largest angle*
(ii) *area of the triangle*

6. *Calculate the area of a triangle with sides of length* ***5cm, 7cm*** *and* ***9cm.***

7. *A boat sails* ***22km*** *on a bearing of* ***042°*** *and then a further* ***30km*** *on a bearing of* ***090°.*** *Calculate the distance and bearing of the return journey*

8. *Calculate the area of a triangle whose sides are* ***5cm, 7cm*** *and the angle between them is* ***135°.***

9. PQR *represents a triangular plot of land in which* ***PQ = 36m, PR = 44m*** *and*

angle ***QPR = 68°.*** *Calculate the*

(i) *length of* ***QR***

(ii) *size of angle* ***PRQ***

(iii) *area of the plot*

10. *From port* ***P,*** *ship* ***Q*** *lies* ***11km*** *away on a bearing of* ***041°*** *and* ***R*** *lies* ***8km*** *away*

on a bearing of ***341°.*** *Calculate the:*

(i) *distance and bearing of* ***Q*** *from* ***R***

(ii) *area of the figure bounded by* ***P QR***

11. ABCD *is a quadrilateral in which* ***AB = 4cm, BC = 5cm, CD = 10cm,*** *angle*

ABC = 80° *and angle* ***ACD = 30°.*** *Calculate the:*

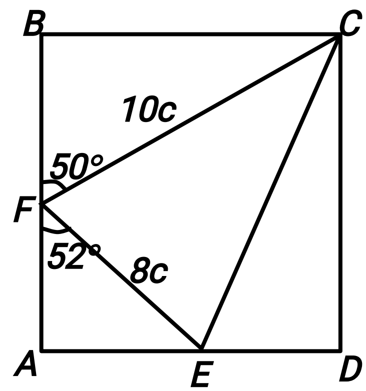
(i) *length of* ***AC***

(ii) *size of angle* ***ACB***

(iii) *length of* ***AB***

(iv) *area of quadrilateral* ***ABCD***

12.*In the diagram below****, ABCD*** *is a rectangle with* ***CF = 10cm, EF = 8cm,*** *angle* ***BFC = 50°*** *and angle* ***EFA = 52°.***



*Calculate the****:***

(i) *length of* ***BC*** *and* ***AB***

(ii) *area of triangle* ***CEF***

13. *A point* ***P*** *is* ***10km*** *due north of* ***Q.*** *A man walks from* ***Q*** *on a bearing of* ***030°.*** *Calculate how far he travels before he is****:***
(i) *equidistant from* ***P*** *and* ***Q***
(ii) *as close as possible to* ***P***
(iii) *north east of* ***P***

14. ABCD *is a quadrilateral in which* ***AB = 7cm, BC = 6cm, DA = 4cm,*** *angle*

BAD = 60° *and angle* ***BCD = 90°.*** *Calculate the:*

(i) *length of* ***BD*** *and* ***CD***

(ii) *size of angle* ***ADC***

(iii) *length of* ***AB***

(iv) *area of quadrilateral* ***ABCD***

15*. In a triangle* ***ABC, AC = 8 cm, BC = 7cm*** *and* ***AB = 12cm.*** *Find the:*

(i) *largest angle of the triangle,*

(ii) *size of angle* ***QPR***

(iii) *area of triangle.*

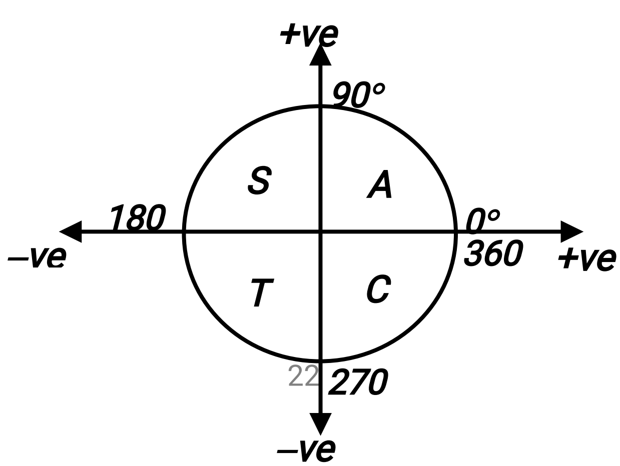
(iv) *radius of the circumcircle of the triangle.*

16*. An equilateral triangle is inscribed in a circle of radius* ***6 cm.*** *Calculate the length of the side of the triangle.*

SOLVING TRIGONOMETRIC EQUATIONS

Summary:

1. *Trigonometric equations are solved using both a calculator and a quadrant diagram*
2. *A quadrant diagram is a circle centred at the origin in the x–y plane and marked as follows:*



***A =** All the ratios are positive,* ***C =** Only cosine is positive,*
***T =** Only tangent is positive,* ***S =** Only sine is positive*

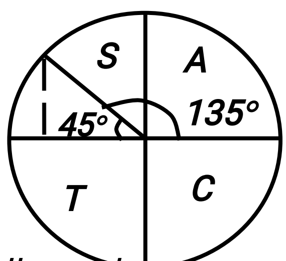
***3.** The term **ACTS** is used to remember the quadrants*

***4.** Angles are measured from the positive x–axis in an anticlockwise direction*

***5.** The acute angle subtended with the x–axis in any quadrant is called a reference angle*

6.** Angles greater than **90°** and their corresponding reference angles have the same trigonometric function values **(**The signs may differ depending on the quadrant enclosing the reference angle**)

7.** In the quadrant diagram below**, 45°** is the reference angle of **135°



*From **(6)** above it follows that:*

$$\textbf{(i)}\ Sin135^{\circ} = Sin45^{\circ} = \frac{1}{\sqrt{2}}$$

$$\textbf{(ii)}\ Cos135^{\circ} = -Cos45^{\circ} = \frac{-1}{\sqrt{2}}$$

$$\textbf{(iii)}\ Tan135^{\circ} = -Tan45^{\circ} = -1$$

EXAMPLES:

1.** Without using tables or a calculator**,** find the value of**:

(i) Sin 150° ***(ii) Cos 150°*** ***(iii) Tan 150°*** ***(v) Sin 315°***
(vi) Cos 315° ***(vii) Tan 315°*** ***(vii) Sin 240°*** ***(viii) Cos 240°***

(ix) Tan 240° ***(x) Sin 390°*** ***(xi) Cos 390°*** ***(xii) Sin 510°***
(xiii) Cos 510° ***(xiv) Tan 510°*** ***(xv) Cos780°*** ***(xvi) tan765°***

Soln

Hint: *Express the given angle in terms of its reference angle on a quadrant diagram*

2. *Given that* ***sinθ = 0·5*** *for* ***0° < θ < 360°,*** *find the two possible values of* ***θ***

Soln

Hint: *The first angle is got from a calculator and the rest from a quadrant diagram*

3. *Solve the equation* ***2cosθ – 1 = 0*** *for* ***0° < θ < 360°***

4. *Solve the equation* $2\sin\theta - \sqrt{3} = 0$ *for* ***0° < θ < 360°***

5. *Solve the equation* $3\sqrt{2}\cos\theta - 3 = 0$ *for* ***0° < θ < 360°***

6. *Given that* ***sinθ = –0·5*** *for* ***0° < θ < 360°,*** *find the two possible values of* ***θ***

Soln

Hint: *First ignore the negative sign and find the reference angle from a calculator*

Use this angle in the quadrants where the sine function is negative

7. *Solve the equation* ***2cosθ + 1 = 0*** *for* ***0° < θ < 360°***

8. *Solve the equation* ***tanθ = – 1*** *for* ***0° < θ < 360°***

9. *Given that* $2\cos^2\theta - 1 = 0$ *for* ***0° < θ < 360°,*** *find the four possible values of* ***θ***

10. *Given that* $\sin\theta = \frac{3}{5}$ *and* ***θ*** *is obtuse, find without using tables or a calculator the value of:*

(i) cosθ

(ii) tanθ

Soln

Hint: *The sine function is positive in both quadrants **A** and **S** but since **θ** is obtuse, then quadrant **A** is eliminated*

11. *Given that* $Tan\ \theta = \frac{-3}{4}$ *for **0° < θ < 270°,** find without using tables or a calculator the value of:*

(i) sinθ

(ii) cosθ

(iii) sinθ − cosθ

(iv) $sin^2\ \theta + cos^2\ \theta$

Soln

Hint: *The tangent function is negative in both quadrants **S** and **C** but since **θ** lies between **0°** and **270°,** then quadrant **C** is eliminated*

12. *Given that* $Tan\ \theta = \frac{5}{12}$ *for **90° < θ < 270°,** find without using tables or a calculator the value of **sinθ − cosθ***

Soln

Hint: *The tangent function is positive in both quadrants **A** and **T** but since **θ** lies between **90°** and **270°,** then quadrant **A** is eliminated*

13. *Given that* $Cos\ \theta = \frac{-8}{17}$ *for **180° < θ < 360°,** find without using tables or a calculator the value of:*

(i) sinθ

(ii) tanθ

(iii) cosθ − sinθ

Soln
Hint: *The cosine function is negative in both quadrants **S** and **T** but since **θ** lies between **180°** and **360°,** then quadrant **S** is eliminated*

14. *Given that $Tan\ \theta = -\sqrt{3}$ and **θ** is reflex**,** find without using tables or a calculator the values of:*

(i) $\frac{1}{Cos\ \theta}$

(ii) $\frac{1}{Sin\ \theta}$

Soln
Hint: *The tangent function is negative in both quadrants **S** and **C** but since **θ** is reflex**,** then quadrant **S** is eliminated*

15. *Given that $Tan\ \theta = \frac{5}{12}$, find without using tables or a calculator the possible values of **cosθ − sinθ***

Soln
Hint: *The tangent function is positive in both quadrants **A** and **T.** Thus both quadrants are valid since **θ** has no restriction*

16. *Given that $Sin\ A = \frac{3}{5}$ and $Tan\ B = \frac{4}{3}$, where **A** and **B** are both acute angles**,** find without using tables or a calculator the value of*

$Sin \quad A\ Cos \quad B - SinB \quad CosA$

Soln

Hint: *Angles **A** and **B** are both acute in quadrant **A***

EER:

1. *Without using tables or a calculator, find the value of:*

(i) Sin 225° ***(ii) Cos 225°*** ***(iii) Tan 225°*** ***(v) Sin 300°***

(vi) Cos 300° ***(vii) Tan 300°*** ***(vii) Sin 240°*** ***(viii) Cos 750°***

(ix) Tan 210° ***(x) Sin 210°*** ***(xi) Cos 480°*** ***(xii) Sin 330°***

2. *Solve the equation* $2\cos\theta - \sqrt{3} = 0$ *for* ***0° < θ < 360°***

3. *In triangle **ABC,** angle **BAC = 20°, BC = 3cm** and **AC = 5cm.** Find the two possible values of angle **ABC***

4. *If **cos x = – 0·634** for **90° < x < 270°,** find the two possible values of **x***

5. *If **3tan x – 2 = 4cos35°** for **0° < x < 360°,** find the two possible values of **x***

6. *Given that **tan35° = 0·7,** without using tables or a calculator, find the value of:* ***(i) tan145°*** ***(ii) 2tan215° + 10tan325°***

7. *Find the two possible angles in the range **0°** to **360°** which satisfy the equations:* ***(i) sinx = 0·4210*** ***(ii) cosx = –0·8660*** ***(iii) tanx = 2·106***

8. *Given that* $4\sin^2\theta - 3 = 0$ *for **0° < θ < 360°,** find the four possible values of **θ***

9. *Given that* $Sin \quad \theta = \frac{12}{13}$ *and **θ** is acute, find without using tables or a calculator the values of **cosθ** and **tanθ***

10. *If **cos θ = – 0·5** for **0° < θ < 360°,** find the two possible values of **θ***

11. *Given that* $Tan\ \theta = \frac{-1}{\sqrt{3}}$ *for* ***180° < θ < 360°,*** *find without using tables or a calculator the values of* ***sinθ*** *and* ***cosθ***

12. *Without using tables or calculators find the values of the following in surd form*

(i) cos780 ***(ii) sin315*** ***(iii) tan585***

GRAPHING TRIGONOMETRIC CURVES

Summary:

1. (i) *The sine and cosine curves have hills and valleys in continuous form*

(ii) *The height of such hills is called the amplitude*

(iii) *The distance from the top of a hill to the next is called the period*

(iv) *The maximum and minimum values of the function occur at its turning points*

(v) *By drawing suitable lines, the drawn graphs can be used to solve related equations*

2. *Consider a general function* ***y = ASinωx:***

(i) *Amplitude =* $|A|$

(ii) *Period* $T = \frac{2\pi}{\omega}$.

EXAMPLES:

1. *Find the amplitude and period of the function* ***y = –3Sin2x***

2. (a) *Draw a graph of* ***y = Sinx*** *for* ***0° ≤ x ≤ 360°*** *using intervals of* ***30°***

(use a scale of ***1cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 1 unit*** *on the* ***y-axis)***

(b) *Use your graph to solve the equations:*

(i) Sinx = 0

(ii) Sinx = 0·5

(iii) Sinx = –0·5

(c) *State the amplitude and period of* ***y = Sinx***

3. (a) *Draw a graph of* ***y = Cosx*** *for* ***0° ≤ x ≤ 360°*** *using intervals of* ***30°***

(use a scale of ***1cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 1 unit*** *on the* ***y-axis)***

(b) *Use your graph to solve the equations:*

(i) Cosx = 0

(ii) Cosx = 0·5

(iii) Cosx = –0·5

(c) *State the amplitude and period of* ***y = Cosx***

3. (a) *Draw a graph of* ***y = 3Cosx – 4Sinx*** *for* ***0° ≤ x ≤ 360°*** *using intervals of* ***30°. (****use a scale of* ***1cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 1 unit*** *on the* ***y-axis)***

(b) (i) *State the minimum and maximum values of* ***3Cosx – 4Sinx***

(ii) *State the value of* ***x*** *at which the maximum value of* ***y*** *occur*

(c) *Use your graph to solve the equations:*

(i) 3Cosx – 4Sinx = 0

(ii) 3Cosx – 4Sinx + 1 = 0

(d) *State the range of values of* ***x*** *for which* ***3Cosx – 4Sinx < –4***

4. (a) *Draw on the same axes, the graphs of* ***y = Sin2x*** *and* ***y = 3Cosx – 2*** *for*

0° ≤ x ≤ 360° *using intervals of* ***30°. (****use a scale of* ***1cm : 30°*** *on the* ***x-axis***

and ***2cm : 1 unit*** *on the* ***y-axis)***

(b) *Use your graphs to solve the equation* ***3Cosx – 2 = Sin2x***

(c) *State the amplitude of* ***y = 3Cosx – 2***

5. (a) *Draw on the same axes, the graphs of* ***y = 4Co2x*** *and* ***y = 2Sin(2x +30°)*** *for*

0° ≤ x ≤ 180° *using intervals of* ***15°. (****use a scale of* ***1cm : 15°*** *on the* ***x-axis***

and ***2cm : 1 unit*** *on the* ***y-axis)***

(b) *Use your graphs to solve the equation* ***4Cos2x – 2Sin(2x +30°) = 0***

(c) (i) *State the amplitude of* ***y = 4Cos2x***

(ii) *State the period of* ***y = 2Sin(2x +30°)***

EER:

1. (a) *Copy and complete the table below for the function* $y = Sinx - Cosx$

x	0	30	60	90	120	150	180	210	240
Sinx		0·5			0·87			–0·5	
–Cosx		–0·87			0·5			0·87	
y		–0·3			1·3			0·37	

		7			7				

(b) *Draw a graph of* $y = Sinx - Cosx$ *for* $0° \le x \le 240°$

(use a scale of ***2cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 0·5 units*** *on the* ***y-axis)***

(c) *Use your graph to solve the equations:*

(i) Sinx – Cosx = 0

(ii) Sinx – Cosx = 1·2

2. (a) *Draw a graph of* $y = Sinx + Cosx$ *for* $0° \le x \le 360°$ *using intervals of* ***30°. (****use a scale of* ***1cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 1 unit*** *on the* ***y-axis)***

(b) *Use your graph to solve the equations:*

(i) Sinx + Cosx = 0

(ii) Sinx + Cosx = 1

(iii) 2Sinx = 1 – 2Cosx

3. (a) *On the same axes, draw the graphs of* ***y = 3Cosx + 2Sinx*** *and* $y = 3 + \frac{x}{90^o}$ *for* $0° \le x \le 90°$ *using intervals of* ***15°. (****use a scale of* ***2cm : 15°*** *on the* ***x-axis*** *and* ***2cm : 0·5 units*** *on the* ***y-axis)***

(b) *State the minimum and maximum values of* ***3Cosx + 2Sinx***

(c) *Use your graphs to solve the equations:*

(i) 3Cosx + 2Sinx = 2·5

(ii) $3Cosx + 2Sinx = 3 + \frac{x}{90^o}$

4. (a) *Draw on the same axes, the graphs of* ***y = 2Cosx*** *and* ***y = Sin(x +30°)*** *for*

0° ≤ x ≤ 360° *using intervals of* ***30°.*** ***(****use a scale of* ***1cm : 30°*** *on the* ***x-axis***

and ***1cm : 0·5 units*** *on the* ***y-axis)***

(b) *Use your graphs to solve the equation* ***Sin(x +30°) – 2Cosx = 0***

(c) *State the amplitudes of the functions:*

(i) y = Sin(x +30°)

(ii) y = 2Cosx

VECTORS

Summary:

1. *A vector has both magnitude and direction.*

2. $OP = \begin{pmatrix} x \\ y \end{pmatrix}$ *is the position vector of point* $P(x, \ y)$.

3. *The magnitude* ***or*** *length* ***or*** *modulus of vector* ***OP*** *is denoted by*

$$|OP| = \sqrt{x^2 + y^2}.$$

4. *To add two vectors we add the corresponding numbers*

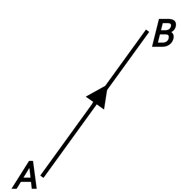
5. *To subtract two vectors we subtract the corresponding numbers*

6. *A scalar* ***k*** *multiplied by vector* $OP = \begin{pmatrix} x \\ y \end{pmatrix}$ *is treated as follows:*

$$kOP = k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

7. *A displacement vector* ***AB*** *is represented by a directed line segment* ***AB*** *as*

shown:

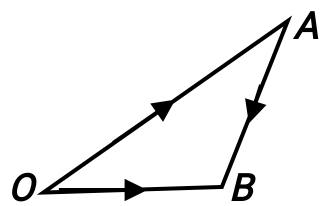


The vectors ***AB*** *and* ***BA*** *are equal in length but opposite in direction*

$\therefore$ ***BA = –AB***

*8. In the triangle **OAB,** the displacement **OA** followed by **AB** is equal to a single*

*displacement **OB.***



$$OB = OA + AB$$

$\therefore$ $AB = OB - OA$ *"The vector triangle equation"*

*9. If vector **AB** is parallel to **CD,** then* $AB = kCD$

*10. If **ABCD** is a parallelogram, then the two opposite sides are parallel and also equal in length **(AB = DC** and **AD = BC).***

*11. If **AB** is parallel to **BC** with a common point **B,** then the points **A, B** and **C** are collinear (* $AB = kBC$ *)*

EXAMPLES:

*1. Given the points **A(4, 1)** and **B(12, 16),** find the:*

*(i) column vector **AB***

*(ii) length of **AB***

*2. The position vectors of **P** and **Q** are* $\begin{pmatrix} -2 \\ 13 \end{pmatrix}$ *and* $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ *respectively, find the* *magnitude of* ***PQ***

3. *Find the distance between the points* ***P(–8, 2)*** *and* ***Q(4, 7)***

4. *Given that* $OA = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ *and* $AB = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$, *find the:*

(i) *position vector of* ***B***

(ii) $|OB|$

5. *Given that* $OB = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$ *and* $AB = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$, *find the:*

(i) *coordinates of* ***A***

(ii) *modulus of* ***OA***

6. *Given the points* ***P(–2, 3)*** *and* ***Q(3, 6),*** *find the coordinates of* ***R,*** *if*

$OR = 3OP + \frac{1}{3}OQ.$

7. *Given the points* ***A(3, 4)*** *and* ***B(9, 2),*** *find the coordinates of* ***T,*** *if*

$OT = OA + \frac{1}{2}AB.$

8. *Given that* $\mathbf{a} = \begin{pmatrix} -2 \\ -9 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$ *and* $\mathbf{m} = \mathbf{a} + 2\mathbf{b}$, *find the magnitude of* ***m.***

9. *Given the vectors* $\mathbf{a} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$ *and* $\mathbf{c} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$, *find the length of*

$\mathbf{a} + 2\mathbf{b} + \mathbf{c}.$

10. *Given the vectors* $AB = \begin{pmatrix} 9 \\ -7 \end{pmatrix}$ *and* $BC = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$, *find the:*

(i) *column vector* ***AC***

(ii) *modulus of* ***AC***

11. *Given the vectors* $PQ = \begin{pmatrix} 13 \\ 4 \end{pmatrix}$ *and* $RQ = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$, *find:*

(i) *vector* ***PR***

(ii) *the length of* ***PR***

12. *Given the vectors* $AB = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$ *and* $AC = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, *find the magnitude of* ***BC.***

13. *If* $p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $q = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ *and* $r = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$, *find the values of* ***a*** *and* ***b*** *such that*

$ap + bq = r.$

14. *If* $a = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $b = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ *and* $c = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$, *find the values of* ***x*** *and* ***y*** *such that*

$xa + yb = c.$

15. *If* $u = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ *and* $w = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$, *find the values of* ***x*** *and* ***y*** *such that*

$xu + yv = w.$

15. ***ABCD*** *is a parallelogram with* ***A(–2, –2), B(6, –2)*** *and* ***C(2, 1).*** *Find the coordinates of* ***D.***

16. ***ABCD*** *is a parallelogram with* ***A(2, 1), B(3, 4)*** *and* ***C(–1, 2).*** *Find the coordinates of* ***D***

17. ***PQRS*** *is a parallelogram with* ***P(1, 1), Q(5, 3)*** *and* ***R(7, 7)****. Find the:*
 (i)*column vector* ***PS***
 (ii) *coordinates of* ***S.***

18. ***ABCD*** *is a quadrilateral with* ***A(4, 1), B(2, –2, C(–2, 0)*** *and* ***D(0, 3).*** *Show that* ***ABCD*** *is a parallelogram*

19. *The vectors* $\mathbf{a} = \begin{pmatrix} 2 \\ \lambda \end{pmatrix}$ *and* $\mathbf{b} = \begin{pmatrix} 8 \\ -12 \end{pmatrix}$ *are parallel to each other. Find the*
 value of λ

20. *The vectors* $\mathbf{p} = \begin{pmatrix} \lambda \\ 2 \end{pmatrix}$ *and* $\mathbf{q} = \begin{pmatrix} 15 \\ 6 \end{pmatrix}$ *are parallel to each other. Find the*
 value of λ

21. *Show that the points* ***A(–1, 3), B(2, 1)*** *and* ***C(8, –3)*** *are collinear.*

22. *Show that the points* ***P(–1, –5), Q(0, –2)*** *and* ***R(2, 4)*** *are collinear.*

23. *Given the points* ***A(–2, –1), B(1, 5)*** *and* ***C(2, 7),*** *find the value of* ***k*** *such that*
 $AB = kAC$, *hence state the ratio* $AB : AC$.

24. *In the vector triangle* ***OAB, M*** *is a point on* ***AB*** *such that* ***AM: AB = 2:5.***
 Express:
 (i) AM *in terms of* ***AB***
 (ii) MB *in terms of* ***AB***
 (iii) AB *in terms of* ***AM***
 (iv) AB *in terms of* ***MB***
 (v) OM *in terms of* ***OA*** *and* ***AB***
 (vi) OM *in terms of* ***OB*** *and* ***AB***

25. *In the vector triangle* ***OAB, K*** *is a point on* ***AB*** *such that* ***3AK = 2KB.***
Express:
(i) AK *in terms of* ***AB***
(ii) KB *in terms of* ***AB***
(iii) AK *in terms of* ***KB***
(iv) KB *in terms of* ***AK***
(v) OK *in terms of* ***OA*** *and* ***AB***
(vi) OK *in terms of* ***OB*** *and* ***AB***

26. *In the vector triangle* ***OAB, N*** *is the midpoint of* ***AB.*** *Express:*
(i) ON *in terms of* ***OA*** *and* ***AB***
(ii) ON *in terms of* ***OB*** *and* ***AB***

27. *The position vectors of the points* ***A*** *and* ***B*** *are* $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ *and* $\begin{pmatrix} 12 \\ -11 \end{pmatrix}$

respectively. Point ***M*** *is on* ***AB*** *such that* ***AM: AB = 2 : 3,*** *find the:*

(i) *column vector* ***AB***

(ii) *column vector* ***AM***

(iii) *position vector of* ***M.***

28. *Given that* $OA = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, $OB = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ *and* ***M*** *is a point on* ***AB*** *such that*

3AM = 2MB, *find the:*

(i) *coordinates of* ***M***

(ii) *magnitude of* ***OM***

29. *Given that* $OA = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$, $OB = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ *and* ***M*** *is the midpoint of* ***AB,*** *find the:*

(i) *column vector* ***AB***

(ii) *position vector of* ***M***

30. *Given that* $OA = \begin{pmatrix} -1 \\ -15 \end{pmatrix}$, $OB = \begin{pmatrix} 7 \\ -11 \end{pmatrix}$ *and point* ***E*** *divides* ***AB*** *in the ratio*

1 : 3, *find the position vector of* ***E.***

31. *Given that* $OA = a$, $OB = b$ *and* ***M*** *is the midpoint of* ***AB,***

(a) *Draw a vector diagram showing vector* ***AB***

(b) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) AB

(ii) AM

(iii) OM

32. *In a triangle* ***OAB,*** $OA = a$, $OB = b$ *and point* ***K*** *divides* ***AB*** *in the ratio* ***1:2,***

Express the following vectors in terms of ***a*** *and* ***b:***

(i) AB

(ii) AK

(iii) OK

33. *In a triangle* ***OAB,*** $OA = a$, $OB = b$ *and* ***N*** *is a point on* ***AB*** *such that*

2AB = 3NB. *Express vector* ***ON*** *in terms of* ***a*** *and* ***b.***

34. *In a triangle* ***OAB,*** $OA = a$, $OB = b$, *point* ***C*** *divides* ***AB*** *in the ratio* ***2:3***

and

D is the midpoint of OC.

(a) Express the following vectors in terms of **a** *and* **b**:

(i) AB
(ii) OC
(iii) BD

(b) Taking **O** *as the origin, point* **A(–15, 20)** *and* **B(10, 0),** *find the:*

(i) position vector of **C** *in* **(a)(i)** *above.*
(ii) coordinates of **C.**
(iii) length of **OC.**

35. *In a triangle* **OAB, M** *and* **N** *are midpoints of* **OA** *and* **OB** *respectively.* $OA = a$, $ON = b$ *and* **P** *is a point on* **AB** *such that* **4AP = 3AB.**

(a) Express the following vectors in terms of **a** *and* **b**:

(i) AB
(ii) OP
(iii) MB
(iv) NP

(b) Show that **AB** *is parallel to* **MN.**

36. *In a triangle* **OAB, M** *and* **N** *are midpoints of* **AB** *and* **OB** *respectively.* $OA = a$, $ON = b$ *and* **P** *is a point on* **OM** *such that* **3OP = 2OM.**

(a) Express the following vectors in terms of **a** *and* **b**:

(i) AB
(ii) OM
(iii) PB
(iv) AP

(b) (i) Show that the points **A, P** *and* **N** *are collinear.*
(ii) Find the ratio in which **P** *divides* **AN.**

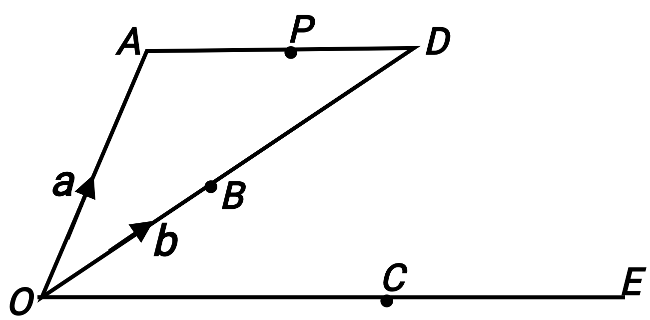
37. *In a triangle* **OAB,** $OA = a$, $OB = b$, **P** *and* **Q** *are points on* **OA** *and* **AB** *respectively such that* **3OP = PA, AQ = 2QB** *and* **N** *is the midpoint of* **OQ. ANM** *is a straight line which is such that* **AN = 5NM.** *Given also that* $OM = h\,OB$, *where* **h** *is a scalar.*

(a) *Express the following vectors in terms of* **a** *and* **b**:

(i) *OQ*
(ii) *AN*
(iii) *PN*
(iv) *NB*

(b) *Show that the points* **P, N** *and* **B** *are collinear*
(c) *Find the value of* **h.**

38. *In the figure below,* **P** *is a point on* **AD** *such that* **PD : AP = 1 : 2,** $OA = a$, $OB = b$, **3OB = 2BD** *and* **OC = 3CE = 3AP.**



(a) *Express the following vectors in terms of* **a** *and* **b**:

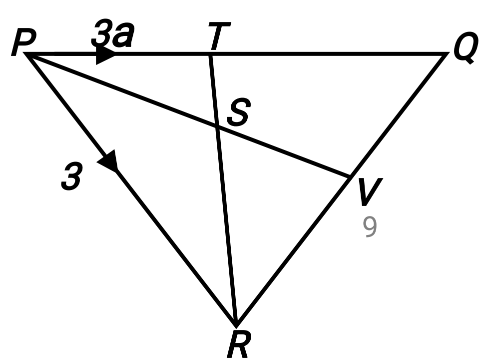
(i) *AD*

(ii) *BP*

(iii) *DC*

(b) *Show that* **AD : OE = 3 : 8**

39. *In the figure below,* $PT = 3a$, $PR = 3b$, $PQ = 4PT$, $2PS = PV$ *and* $3RS = 2RT$.



(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

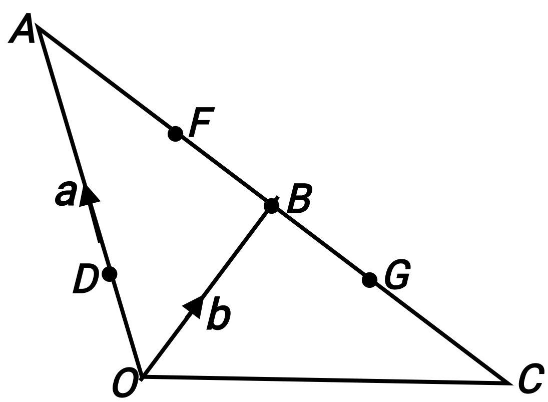
(i) RS

(ii) PV

(iii) RQ

(b) *Find the ratio of* **RV** *to* **RQ.**

37. *In the figure below,* $OA = \mathbf{a}$, $OB = \mathbf{b}$, ***F*** *and* ***G*** *are points on* ***AC*** *such that* ***AF : AB = 3 : 4*** *and* ***AG : AC = 2 : 3.*** *Point* ***D*** *is on* ***OA*** *such that* ***OD : DA = FB : BG = 1 : 2.***



(a) *Express* ***AG*** *and* ***AC*** *in terms of* ***AB.*** *Hence find the following vectors in*

terms of ***a*** *and* ***b:***
(i) AB

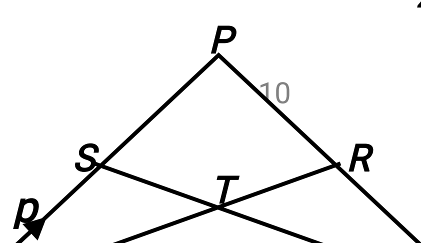
(ii) AC

(iii) DG

(iv) OF

(b) *Find the ratio* ***DG : OC***

39. *In the figure below,* $OP = \mathbf{p}$, $OQ = \mathbf{q}$, $OS = \frac{3}{4}OP$ *and* ***PR : RQ = 2 : 1***



(a) *Express the following vectors in terms of* **p** *and* **q:**

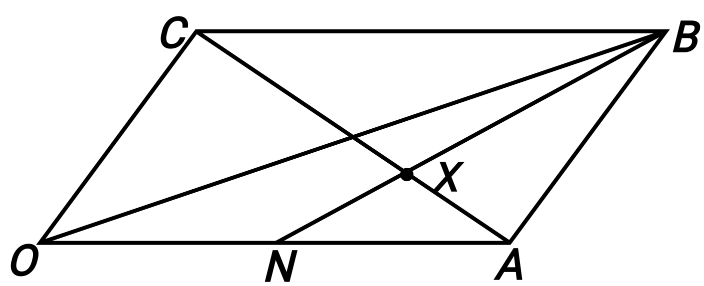
(i) PQ

(ii) OR

(iii) SQ

(b) *Line* ***OR*** *and* ***SQ*** *meet at point* ***T*** *such that* ***OT = hOR*** *and* ***ST = kSQ.***
(i) *By expressing* ***OT*** *in two different ways, find the values of* **h** *and* **k**
(ii) *Determine the ratio in which* ***T*** *divides* ***SQ***

40. *In the figure below,* ***OABC*** *is a parallelogram where* ***OA = a*** *and* ***AB = b.*** *Point* ***N*** *is on* ***OA*** *such that* ***ON:NA = 1:2.***



(a) *Express the following vectors in terms of* **a** *and* **b:**

(i) AC

(ii) BN

(b) *Line* ***AC*** *and* ***BN*** *meet at point* ***X*** *such that* ***AX = hAC*** *and* ***BX = kBN***

(i) *By expressing* ***OX*** *in two different ways, find the values of* **h** *and* **k**

*(ii) Determine the ratio in which **X** divides **AC***

35. *In a triangle **OAB,** $OA = a,\quad OB = b,$ **N** and **M** are points of **AB** and **OB** respectively. Line **ON** and **AM** meet at point **T** such that **AT = TM** and $OT = \frac{3}{4}ON.$ Given that **OM = XOB** and **AN = yAB,** Express the vectors:*

*(i) **AM** and **OT** in terms of **a, b** and **x.***

*(ii) **ON** and **OT** in terms of **a, b** and **y,** hence find the values of **x** and **y.***

EER:

1. *Given the points **A(3, 4)** and **B(9, 1),** find the coordinates of **P,** if*

$OP = OA + \frac{1}{3}AB.$

2. *Given the vectors $a = \begin{pmatrix} 1 \\ 3 \end{pmatrix},\ b = \begin{pmatrix} 1 \\ m \end{pmatrix}$ and $c = \begin{pmatrix} n \\ -2 \end{pmatrix},$ find the values of **m** and **n** for which $4a + 2b = 3c.$*

3. *Given the vectors $p = \begin{pmatrix} 1 \\ 1 \end{pmatrix},\ q = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $r = \begin{pmatrix} 8 \\ 2 \end{pmatrix},$ find the values of **a** and **b** for which $ap + bq = r.$*

4. *Given that vector $OA = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $OB = \begin{pmatrix} 7 \\ 4 \end{pmatrix},$ find the magnitude of vector $P = OA + \frac{2}{3}AB.$*

5. *Given that vector $p = \begin{pmatrix} x \\ x+2 \end{pmatrix},$ find the possible values of **x** for which*

$|\mathbf{p}| = 10.$

6. *The position vectors of the points* **A** *and* **B** *are* $\begin{pmatrix} -1 \\ -15 \end{pmatrix}$ *and* $\begin{pmatrix} 7 \\ -11 \end{pmatrix}$ *respectively. If point* **E** *divides* **AB** *in the ratio* **1 : 3,** *find the position vector of* **E.**

7. *Find the distance between the points* **P(–8, 2)** *and* **Q(4, 7)**

8. *Given the points* **A(–1, 2), B(2, 8), C(–2, –5)** *and* **D(4, y),** *find the value of* **y** *for which* **AB** *is parallel to* **CD.**

9. *Given the vectors* $\mathbf{p} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ *and* $\mathbf{r} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$, *find the values of* **a** *and* **b** *for which* $a\mathbf{p} + b\mathbf{q} = \mathbf{r}$.

10. *Given the vectors* $\mathbf{AB} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$ *and* $\mathbf{CB} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, *find the:*

(i) *vector* **AC**

(ii) *magnitude of* **AC**

11. ABCD *is a parallelogram with* $\mathbf{CB} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$, $\mathbf{CD} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$ *and point* **C** *is* **(–5, 2).** *Find the:*

(a) *coordinates of:*

(i) B **(ii) D** **(iii) A**

(b) length of the diagonal ***AC***
(c) point of intersection of the diagonal ***AC*** and ***BD***

12. The vectors $OP = \begin{pmatrix} -1 \\ -15 \end{pmatrix}$, $OQ = \begin{pmatrix} 7 \\ -11 \end{pmatrix}$ *and* $PN = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

(a) *Find the:*

(i) *position vector of* ***N*** ***(02 marks)***

(ii) *length of* ***ON*** ***(02 marks)***

(iii) *coordinates of point* ***E,*** *where* ***E*** *divides* ***PQ*** *in the ratio* ***1:3.*** ***(03 marks)***

(b) *Use the vector method to show that* ***N*** *lies on* ***PQ.*** *Hence state the ratio* ***PN : PQ.*** ***(05 marks)***

13. *Given that* ***OA = a, OB = b*** *and* ***C*** *is the midpoint of* ***AB,***

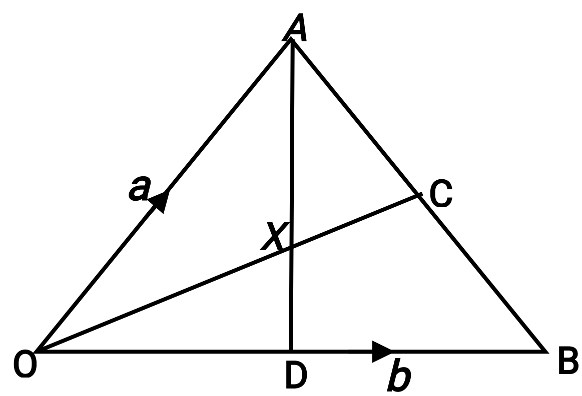
(a) *Draw a vector diagram showing vector* ***AB****.* ***(01 mark)***

(b)*Express in terms of* ***a*** *and* ***b*** *the vectors:*

(i) AB ***(01 mark)***

(ii) OC ***(02 marks)***

14. *In the triangle* ***OAB,*** $OA = \mathbf{a}$, $OB = \mathbf{b}$, ***C*** *is a point on* ***AB*** *such that* ***AC:AB = 1:3*** *and* ***D*** *is the midpoint of* ***OB.***



(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) AB

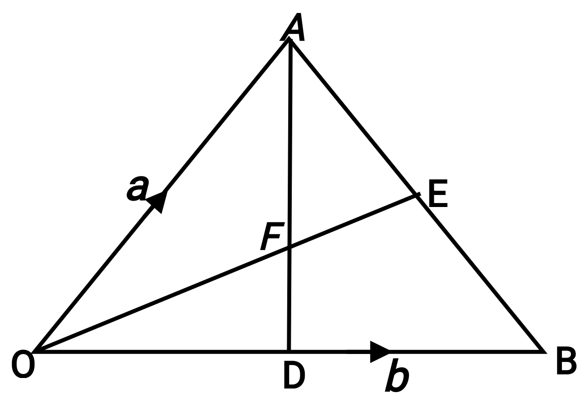
(ii) OC
(iii) AD

(b) X *is a point on* ***AD*** *such that* ***AX : AD = 1 : 2.*** *Find in terms of* **a** *and* **b** *the vectors:*

(i) AX
(ii) OX

(c) *Find in simplest form the ratio* ***OX : OC.***

15. *In the triangle* ***OAB,*** $OA = \mathbf{a}$, $OB = \mathbf{b}$, ***D*** *is a point on* ***OB*** *such that* ***OD:OB = 2:5*** *and* ***E*** *is the midpoint of* ***AB.***



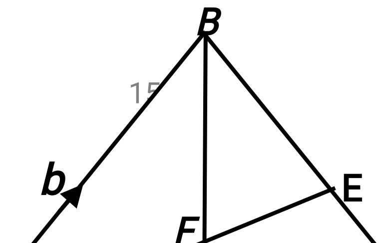
(a) *Express the following vectors in terms of* **a** *and* **b***:*

(i) OE
(ii) AD

(b) *Given that* ***AF = tAD*** *and* ***OF = hOE,*** *find the values of* ***t*** *and* ***h***

(c) *Show that the points* ***O, F*** *and* ***E*** *are collinear*

16. *In the triangle* ***ABC,*** $AB = \mathbf{b}$, $AC = \mathbf{c}$, ***D*** *is a point on* ***AC*** *such that* ***AD:DC = 3:2*** *and* ***E*** *is the midpoint of* ***BC.***



(a) *Express the following vectors in terms of* ***a*** *and* ***b***:

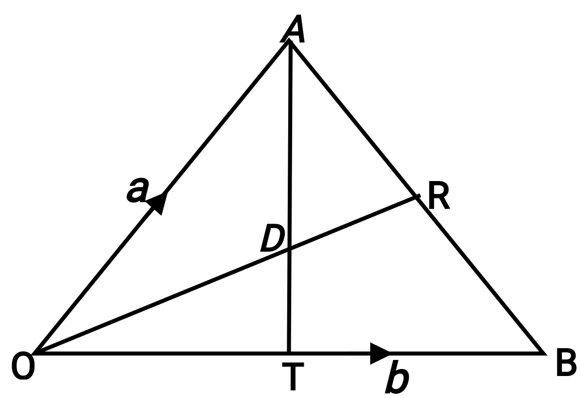
(i) BD

(ii) AE

(b) *Given that* ***BF = tBD*** *and* ***AF = nAE,*** *find the values of* ***t*** *and* ***n***

(c) *State the ratio of* ***BD*** *to* ***BF***

17. *In the triangle* ***OAB,*** $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, *Point* ***R*** *divides* ***AB*** *in the ratio* ***2:5*** *and point* ***T*** *divides* ***OB*** *in the ratio* ***1:3.***



(a) *Express the following vectors in terms of* ***a*** *and* ***b***:

(i) BT

(ii) OR

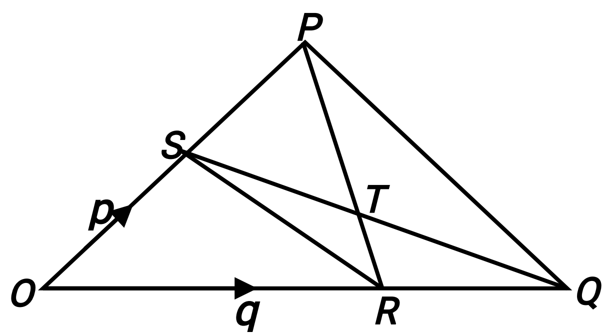
(iii) AT

(b) *Given that* ***AD = kAT*** *and* ***RD = hRO,*** *find the values of* ***k*** *and* ***h.*** *Hence express vector* ***AD*** *in terms of* ***a*** *and* ***b***

18. *In the triangle* ***OPQ,*** $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$, $\overrightarrow{OS} = \frac{1}{3}\overrightarrow{OP}$, $\overrightarrow{OR} = \frac{1}{3}\overrightarrow{OQ}$ *and point* ***T***

is

on ***QS*** *such that* $QT = \frac{3}{4}QS$



(a) *Express the following vectors in terms of* **p** *and* **q**:

(i) SR ***(ii) QS*** ***(iii) PT*** ***(iv) TR***

(b) *Show that the points* ***P, T*** *and* ***R*** *are collinear*

TRANSLATION

Summary:

1. *Translation deals with movement of an object to a new position*

2. *A translation* $T = \begin{pmatrix} a \\ b \end{pmatrix}$, *means that an object is moved a distance* ***a*** *in the* ***x***–*direction and a distance* ***b*** *in the* ***y***–*direction*

3. *A translation* $T = \begin{pmatrix} a \\ b \end{pmatrix}$, *moves point* ***P(x, y)*** *to a new position* $P^{I}(x + a,\ y + b)$. *Thus ,* ***Translation + object = image***

EXAMPLES:

1. *A translation* $T = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, *maps the points* ***P(3, 7)*** *and* ***Q(6, 1)*** *onto the* *points P^{I} and Q^{I} respectively. Find the coordinates of P^{I} and Q^{I}*

2. *A triangle with vertices **A(2, 1) B(2, 3)** and **C(4, 1)** is mapped onto its image by a translation $T = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find the coordinates of the image of the triangle **ABC.***

3. *A translation $T = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ maps point **P** onto $P^{I}(-1, 4)$. Find the coordinates of point **P***

4. *Find the translation that maps point A**(2, 6)** onto $A^{I}(3, 8)$.*

5. *A translation **T** maps point **P(2, 5)** onto $P^{I}(3, 2)$. Find the image of **Q(5, 7)** under translation **T***

6. *A triangle with vertices **A(1, 2) B(3, 4)** and **C(5, 2)** is mapped onto its image by a translation $T_1 = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ followed by a translation $T_2 = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.*

*Find: **(i)** a single translation representing the two successive translations*

(ii)** the coordinates of the image of the triangle **ABC.

7. *A triangle with vertices **A(2, 0) B(1, –3)** and **C(–2, 1)** under goes a translation $T_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ to give triangle $A^{I}B^{I}C^{I}$. Triangle $A^{I}B^{I}C^{I}$ is then mapped onto triangle $A^{II}\ B^{II}\ C^{II}$ by a translation $T_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.*

***(a)** Find the coordinates of the vertices of:*

(i) *triangle* $A^{I} B^{I} C^{I}$

(ii) *triangle* $A^{II}\ B^{II}\ C^{II}$

(b) *Plot triangle* ***ABC*** *and its images on the same axes.*

8. *A translation* $T = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, *maps the line* ***y = 2x + 1*** *onto its image. Find the equation of the image line*